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ROBUSTNESS OF MULTIMODAL TRANSPORTATION NETWORKS

MODEL OCENY ODPORNOŚCI MULTIMODALNYCH SIECI TRANSPORTOWYCH*

This paper describes a declarative approach to modeling a multimodal transportation network (MTN) composed of multiple connecting transport modes, such as bus, tram, light rail, subway and commuter rail, where within each mode, service is provided on separate lines or routes. The considered model of a network of multimodal transportation processes (MTPN) provides a framework to address the needs for transportation networks robustness while taking into account their capacity and demand requirements. Therefore the work focuses on evaluation of the network robustness allowing distinguished multimodal processes to continue in order to accomplish trips following an assumed set of multimodal chains connecting transport modes between origins and destinations. Consequently, a solution to the problem of prototyping robust transits on a given multimodal network is implemented and tested. The conditions that guarantee the network robustness, taking into account disruptions of supply and demand as well as operational control, are provided. The aim of investigations is to provide a tool for evaluating the robustness of a network of multimodal transportation processes as well as different travel modes through a transportation network.

Keywords: *multimodal networks, transportation systems, cyclic scheduling, robustness, multimodal processes, state space, cyclic steady state.*

Dynamiczny rozwój infrastruktury komunikacji miejskiej obejmującej linie autobusowe, trolejbusowe, tramwajowe, linie metra, kolei podmiejskiej, itp. składające się na tzw. Multimodalne Sieci Transportowe (MST) rodzi wiele nowych problemów. Wśród ważniejszych z nich warto wymienić problemy planowania obsługi ruchu pasażerskiego w sytuacjach związanych z awariami elementów infrastruktury, wypadkami losowymi czy też z obsługą imprez masowych. Wiadomo, że istnienie rozwiązań dopuszczalnych gwarantujących zakładaną przepustowość infrastruktury warunkuje tzw. odporność MST na ww. zakłócenia. W tym kontekście, niniejsza praca przedstawia pewien deterministyczny model multimodalnej sieci transportowej złożonej z połączonych stacjami przesiadkowymi, linii komunikacji miejskiej. Składające się na sieć, pracujące w zamkniętych cyklach, linie komunikacji miejskiej pozwalają obsłuchiwać ruch pasażerski na wybranych kierunkach np. północ-południe. Obsługiwane strumienie pasażerów modelowane są jako tzw. multimodalne procesy transportowe. Wprowadzone miary odporności MST, umożliwiające ocenę rozważanych wariantów infrastruktury, pozwalają na wyznaczenie warunków spełnienie, których gwarantuje dopuszczalną jakość obsługi ruchu pasażerskiego. Umożliwiają, zatem zarówno planowanie obsługi pasażerów na wybranych trasach, jak i kształtowanie struktury rozbudowywanej i/lub modernizowanej sieci komunikacji miejskiej.

Słowa kluczowe: *sieci multimodalne, systemy transportowe, harmonogramowanie cykliczne, odporność na zakłócenia, procesy multimodalne, przestrzeń stanów, cykliczne przebiegi ustalone.*

1. Introduction

Multimodal route planning that aims to find an optimal route between the source and the target of a trip while utilizing several transportation modes including different passenger/cargo transportation systems, e.g. ship, airline, AGV systems, train and subway networks, are of significant and fast growing importance [9, 11, 14, 16, 20]. Multimodal transportation process (MTP), i.e. a set of transport modes which provide connection from the origin to the destination, executed in a multimodal transportation network (MTN) can be seen as passengers and/or goods flow transferred between different modes to reach their destination [5].

MTPs planning problems, i.e. taking into account MTPs routing and scheduling can be found in different application domains (such as manufacturing, intercity fright transportation supply chains, multimodal passenger transport network combining several unimodal networks as well as data and supply media flows, e.g., cloud computing, oil pipeline and overhead power line networks**)** [1, 3, 5, 6, 8, 15, 16]. The problems concerning multimodal routing of freight flows and scheduling of multimodal transportation processes (MTPN) scheduling, are NP-hard [7, 12].

The local transportation processes serviced by different transportation modes, while executed along unimodal networks (lines), are usually cyclic. Hence, MTPs supported by them also have the periodic character. That means that the periodicity of MTPN depends on periodicity of unimodal (local) processes executed in MTN. Of course, the MTPN throughput is maximized by the minimization of its cycle time.

Apart from such typically used objectives as the maximization of a user's (e.g. a passenger) benefits and/or the minimization of a provider's (e.g. public transport service bureau) costs, the present paper discusses the importance of a network structure in assuring a robust network. In other words, a network structure design, that is efficient at handling traffic in normal conditions and provides spare capacity in exceptional situations, is of our main interest.

Therefore, the considered problem can be seen as a problem of robust MTPN designing where the assumed demand objectives are satisfied. That is, assuming each line is serviced by a set of stream-like moving transportation means (vehicles) and operation times required for travelling between subsequent stations as well as semaphores ensuring vehicles mutual exclusion on shared stations are given, the main question concerns MTPN timetabling, for instance guaranteeing the shortest time of passengers' itinerary following a given direction. MTN capacity determines a maximum traffic flow obtainable with

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** use of all available lines and roads. In turn, a network demand reflects its users' perspective, i.e. MTPs encompassing traveling passengers itineraries and conditions, taking into account factors such as the quality of transport options available and their prices. Depending on different supply and demand as well as operation control disruptions the MTPN timetabling and, consequently, the time of the passengers trip, following different itineraries, may dramatically differ. In that context, special attention is devoted to disruptions causing MTPNs' deadlock occurrence while threatening MTP by congestions arise.

The declarative models employing the constraint programming techniques implemented in modern platforms such as OzMozart, ILOG, [2, 3, 4] seems to be well suited to cope with MTPs planning problems. Since a problem of robust MTPNs design remains still open, the sufficient conditions guaranteeing the assumed level of MTN's robustness are of primary importance. Therefore, constraints stating the sought conditions can be formulated in terms of MTPN declarative model as well.

To the best of our knowledge, there is no research paper on cyclic scheduling of MTPs subjected to assumed robustness of MTPN modelled in terms of SCCPs. The existing approach to solving the SCCPs scheduling problem is based upon the simulation models, e.g. the Petri nets [17], the algebraic models [18] upon the (max,+) algebra or the artificial intelligent methods [10]. The SCCP driven models, assuming a unique process execution along each cyclic route while allowing to take into account the stream-like flow of local cyclic processes, e.g. buses servicing a given city line, studied in [5], do not take into account the MTP robustness factor. Therefore, this work can be seen as a continuation of our former investigations conducted in [2, 3, 4, 5, 19]. In that context, our paper provides contribution to a time and/or distance robust path-finding problem [13, 14] within the environment of multimodal transportation network as well as its possible implementation in the route advisory systems solving the Multi-Criteria, Multi-Modal Shortest Path Problem [9].

The rest of the paper is organized as follows. We start by introducing a concept of multimodal transportation network (MTN) and then provide its representation in terms of a system of concurrently flowing cyclic processes (SCCP) encompassing a network of multimodal transportation processes (MTPN) while allowing for multimodal transportation processes (MTPs) modeling. The MTPN robustness issues in different contexts of supply and demand as well as operation control disruptions (disruptions leading to the deadlocks) are discussed.

Next, we present the formulation of the problem of robust MTPN designing where network's capacity and demand objectives are simultaneously taken into account. Afterwards, we discuss the implications of adopting different robustness measures and following from them network robustness conditions. Then, we discuss and compare the results obtained through the model for an arbitrarily chosen set of MTPs. In the final section, we briefly summarize our results and provide some concluding results.

2. Multimodal Transportation Network

Multimodal Transportation Network (MTN) concerning the organization of city traffic and the network of public transportation can be modelled with focus on the network of city serviced lines and/or routes. Subway or tram lines as well as bus routes form cycles interconnected via common shared interchange stations or closely situated (short walk-distance) transportation mode specific stations. The means of transportation servicing a particular line mode can be seen in turn as transportation processes enabling passengers to move along their destination route.

2.1. Structure of Multimodal Processes Network

The MTPN seen as a network of vehicles periodically circulating along cyclic routes (see Fig. 1a) can be modeled in terms of Systems of Concurrently flowing Cyclic Processes (SCCP) shown in Fig. 1b). Vehicles are used for the transport of passengers following two directions: north-south (blue line – mP_1) and east-west (red line – mP_2). These routes, setting the courses of multimodal processes, are composed of fragments of the local mode transportation lines (trams and busses). In the considered case, there are six means of transportation: trams (P_1, P_3, P_5) and busses (P_2, P_2' , P_4).

The SCCP is assumed to include two types of processes:

- *local processes* (representing modes of transport – P_1, P_2, P_3, P_4, P_5), whose operations are cyclically repeated along the set routes (sequences of successively visiting stations). For the system from Fig. 1b), the line linking stations R_1, R_2, R_8, R_9 provide two buses that can be modeled by two streams, P_2 and P_2', respectively. The routes of local processes are defined as follows: $p_1=(R_7,\ R_2,\ R_3)$, $p_2=p_2'=(R_1,\ R_2,\ R_8,\ R_9)$, $p_3=(R_1,\ R_5,\ R_4)$, $p_4=(R_3, R_4, R_6)$, $p_5=(R_8, R_{10}, R_9)$.
  The i-th operation (executed on resource R_k) of the local process P_j (or its stream) is denoted by $o_{i,j}$ and $t_{i,j}$ denotes the time of its execution.
- *multimodal processes* (mP_1, mP_2) representing streams of passengers. Operations of the multimodal processes are implemented cyclically along routes being compositions of fragments of routes of local processes representing resources used for transporting materials along a given route. For the system from Fig. 1b), the routes of multimodal processes (i.e. itineraries of passenger streams) are defined as follows:

$mP_1=((R_{10},\ R_9),(R_9,\ R_1,\ R_2),(R_2,\ R_3),(R_3,\ R_4,\ R_6))$, $mP_2=((R_7,\ R_2,\ R_3),(R_3, R_4,),(R_4, R_1, R_5))$.

Similarly as before the i-th operation of the multimodal process, mP_j is denoted by $mo_{i,j}$ and $mt_{i,j}$ denotes the time of its execution.

Process operation are implemented on two kinds of resources: local resources (each of them is used by only one process of a given kind – R_5, R_6, R_7, R_{10}) and shared resources (each of them is used by more than one process of a given kind: R_1–R_4, R_8, R_9).

The local processes use resources that are shared in the mutual exclusion mode, i.e. in a given moment only one local process operation of a given kind can be implemented on a resource (in other words one station can be occupied by only one transportation mode).

The access to shared resources of local processes, is given in the sequence determined by the dispatching rules Θ. It is assumed that $\Theta=\{\sigma_1,\ldots,\sigma_k,\ldots,\sigma_{lk}\}$, where σ_k – is the sequence whose elements determine the order in which the processes (or their streams) are provided with access to the resource R_k. In case of the system from Fig. 1b), the access to shared resources is determined by the following rules:

$$\sigma_1=(P_2, P_3, P_2'), \sigma_2=(P_1, P_2, P_2'), \sigma_3=(P_1, P_4), \sigma_4=(P_3, P_4), \sigma_8=(P_5, P_2, P_2'),$$
$$\sigma_9=(P_5, P_2, P_2').$$

The subsequent operation starts right after the current operation is completed, providing that the resource indispensible to its implementation is available. While waiting for the busy resource, the process does not release the resource which was assigned for implementing the previous operation [4]. Moreover, an assumption is made that processes are of non-expropriation nature, and the times and sequence of operations performed by the processes do not depend on external interferences.

The parameters described above constitute the structure of SCCP that determines its behavior. Formally, the structure of SCCP is defined as the following tuple [4]:

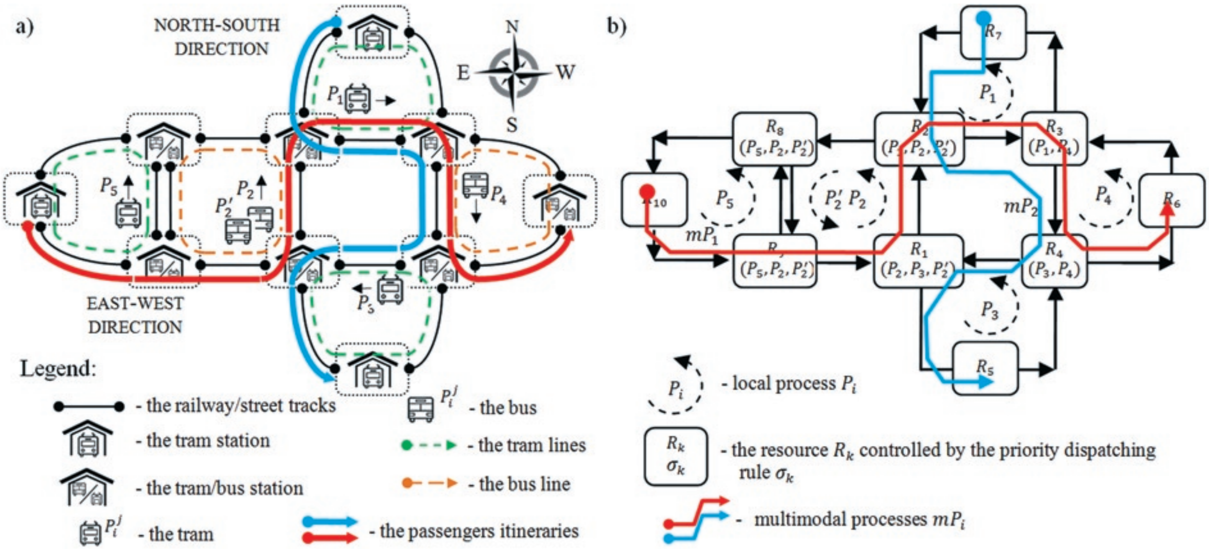

Fig. 1. An example of MTPN a) and corresponding SCCP model b)

$$SC=((R,SL),SM)\ , \tag{1}$$

where: $R=\{R_k \mid k=1,...,lk\}$ – set of resources,

$SL=(P,U,O,T,\Theta)$ – structure of local processes, where:

$P=\{P_i \mid i=1\dots ln\}$ – set of local processes (streams), P_i – i-th process,

$U=\{p_i=(p_{i,1},\dots,p_{i,j},\dots,p_{i,lr(i)}) \mid i=1\dots ln\}$ – set of routes of local processes, p_i – i-th route, $p(i,j)\in R$ – resource required for implementing j-th operation of the process P_i,

$O=\{O_i=(o_{i,1},..,o_{i,j},...,o_{i,lr(i)}) \mid i=1\dots ln\}$ – set of sequences of operations, $o_{i,j}$ – j-th operation of the process P_i,

$T=\{T_i=(t_{i,1},\dots,t_{i,j},\dots,t_{i,lr(i)})) \mid i=1\dots ln\}$ – set of sequences of operation performance times , $t_{i,j}$ – time of performing an operation $o^h_{i,j}$,

$\Theta=\{\sigma_k=(s_{k,1},...,s_{k,d},...,s_{k,lh}) \mid k=1\dots lk\}$ – set of dispatching rules, σ_k – dispatching rule for the resource R_k, $s_{k,d}$ – local process, lh– length of the rule σ_k,

$SM=(mP,mU,mO,mT)$ – structure of multimodal processes, where:

$mP=\{mP_i \mid i=1\dots lw\}$ – set of multimodal processes mP_i, lw- number of the processes

$mU=\{mp_i=(mp_{i,1},\dots,mp_{i,j},\dots,mp_{i,lm(i)}) \mid i=1\dots lw\}$ – set of routes of multimodal processes, mp_i – i-th route,

$mO=\{mO^h_i=(mo_{i,1},...,mo_{i,j},...,mo_{i,lm(i)}) \mid i=1\dots lw\}$ – set of sequences of operations, $mo_{i,j}$ – j-th operation of the process mP_i,

$mT=\{mT_i=(mt_{i,1},\dots,mt_{i,j},\dots,mt_{i,lm(i)}) \mid i=1\dots lw\}$ – set of sequences of operation times, $mt_{i,j}$ – time of operation performance $mo_{i,j}$.

2.2. Behavior of Multimodal Processes Network

In the systems of concurrent cyclic processes, the behavior is usually presented [2, 3, 4], as schedules determining the moments of initiating all the operations implemented within them. Fig. 2b) provides an example of such a schedule that determines the way of implementing the processes of *SC* structure from Fig. 2a).

The presented schedule is an example of the cyclic behavior, i.e. the successive states of the processes are reachable within the constant period (the operations of local and multimodal processes are repeated within the period α=7 time units [t.u.]).

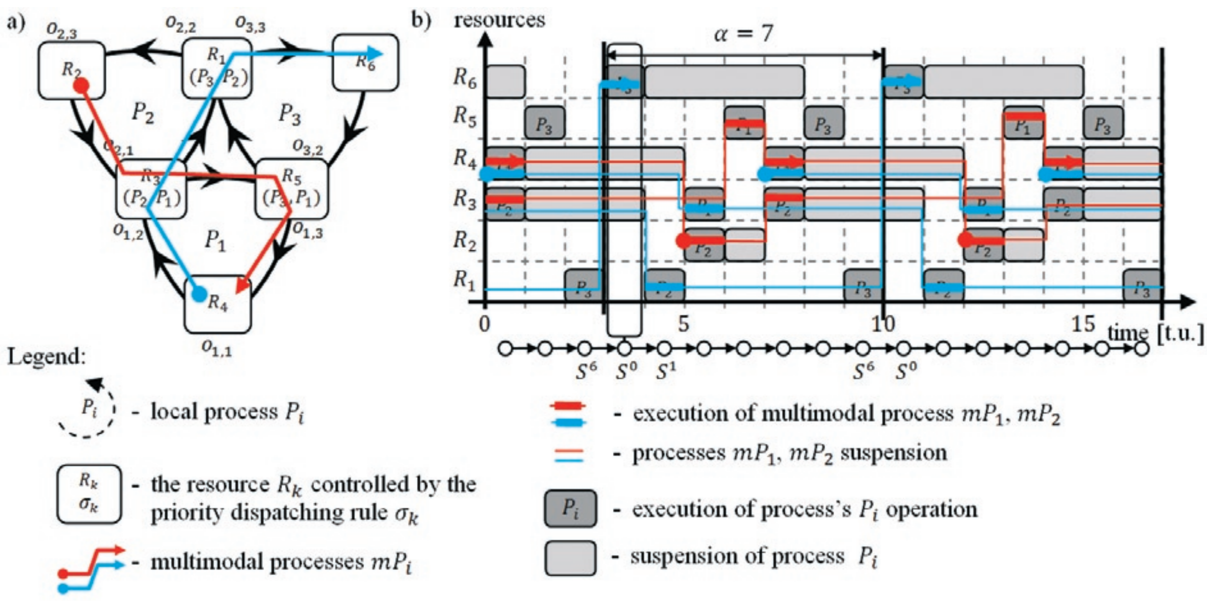

Fig. 2. Example of SCCP structure a) and the corresponding cyclic schedule b)

In this approach, each behavior can be represented by a sequence of successive states (subsequent allocations of processes, as well successively changing, according to the rules Θ of access rights). In case of the schedule from Fig. 2b), it is a sequence of 7 states S^0, S^1, …, S^6. Formally, the SCCP state is defined as follows [4]:

$$S^r=(Sl^r, mS^r), \quad (2)$$

where Sl^r means the r^{th} state of local processes:

$$Sl^r=(A^r,Z^r,Q^r),$$

$A^r=(a_1^r,a_2^r,\ldots,a_k^r,\ldots,a_{lk}^r)$ – allocation of local processes in the r-th state, $a_k^r \in P\cup\{\Delta\}$; $a_k^r=P_i$ – **allocation** meaning that the resource R_k occupied by the process P_i, and $a_k^r=\Delta$ – means that the resource R_k is unoccupied.

$Z^r=(z_1^r,z_2^r,\ldots,z_k^r,\ldots,z_m^r)$ – sequence of semaphores of the r-th state, $z_k^r \in P$ – **semaphore** determining the process (an element of rule σ_k), which has an access to the resource R_k next in the sequence, i.e. $z_k^r=P_i$ means that process P_i is the next to access the resource R_k.

$Q^r=(q_1^r,q_2^r,\ldots,q_k^r,\ldots,q_m^r)$ – **sequence of semaphore indexes** of the r-th state, q_k^r – **index** determining the position of the semaphore value z_k^r in the dispatching rule σ_k, $q_k^r \in \mathbb{N}$. For example, if a semaphore z_2^r indicates the process P_1: $z_2^r=P_1$ which is the second element of the dispatching rule σ_2, then $q_2^r=2$.

mS^r – means the r-th state of multimodal processes:

$$mS^r=\left(mA_1^r,\ldots,mA_i^r,\ldots,mA_l^r\right),$$

$mA_i^r=\left(ma_{i,1}{}^r,ma_{i,2}{}^r,\ldots,ma_{i,k}{}^r,\ldots,ma_{i,m}{}^r\right)$ – sequence of allocations of a multimodal process mP_i in the r-th state, $ma_{i,k}{}^r \in \{mP_i,\Delta\}$, $ma_{i,k}{}^r=mP_i$ – allocation means that the resource R_k is occupied by the process mP_i, and $ma_k^r=\Delta$ – means that the resource R_k is unoccupied.

Behaviors of the system characterized by various sequences of subsequently reachable states S^r(2) can be illustrated in a graphical form as the state space $\mathcal{P}$. Fig. 3a) shows an example illustrating this possibility for the system from Fig. 2a). If we take the graph-theoretical interpretation of the space $\mathcal{P}$, the diagraph corresponding to it is represented by the pair $\mathcal{P}=(\mathbb{S},\mathbb{E})$, where $\mathbb{S}$ means a set of admissible SCCP states [4], $\mathbb{E}\subseteq\mathbb{S}\times\mathbb{S}$ means a set of arcs representing transitions between SCCP states (transitions take place according to the function $S^f=\delta(S^e)$ described in [4].

Cyclic behaviors of SCCP are connected with the presence of cycles (e.g. cycle in digraph G_1) in the space $\mathcal{P}$. A sequence of states being part of a cycle is called as a **cyclic steady state**.

Formally, the cyclic steady state is the sequence $D_C=\left(S^{d_1},\ldots,S^{d_i},S^{d_{i+1}},\ldots,S^{d_{ld}}\right)$ of various admissible states $S^{d_i},S^{d_{i+1}}\in\mathbb{S}$, in which each pair of states satisfies the expression $S^{d_{i+1}}=\delta\left(S^{d_i}\right)$, $i=1\ldots(ld-1)$ and $S^{d_1}=\delta^{lp}\left(S^{d_{ld}}\right)$.

The states of space $\mathcal{P}$ leading to the shared cyclic steady state D_C constitute a coherent digraph called **Whirlpool** $W(D_C)$ (Fig. 3b).

$$W(D_C)=G(D_C)\dot{\cup}\left(\dot{\bigcup_{\forall D_T\in DT(D_C)}}G(D_T)\right) \quad (3)$$

where: $G(D_C)$ – digraph consisting of cyclic steady state D_C,

$G(D_T)$ – digraph consisting of sequence of states D_T leading to the cyclic steady state D_C, $D_T\in DT(D_C)$, where: $DT(D_C)$ – set of all sequences of states leading to D_C,

$G_1\dot{\cup}G_2$ – sum of digraphs $G_1=(V_1,V_1)$ and $G_2=(V_2,E_2)$:

$$G_1\dot{\cup}G_2=(V_1\cup V_2,E_1\cup E_2),$$

$$\dot{\bigcup}_{G_i\in G^*}G_i=G_1\dot{\cup}G_2\dot{\cup}\ldots\dot{\cup}G_a, \text{ for } G^*=\{G_1,G_2,\ldots,G_a\}$$

An example of a whirlpool is presented in Fig. 3b). It shows clearly that the initiation of process implementation of any state belonging to this whirlpool consequently results in cyclic state D_C.

It must be emphasized that not all digraphs of the state space $\mathcal{P}$ result in a cyclic steady states D_C. Some states lead to deadlock states S^* (marked with the symbol ⊗), which means system interruption caused by a closed-loop resource request occurrence.

An example of a deadlock caused by a closed-loop resource request is illustrated in Fig. 3a). In the state S^*, the process P_2 waits for releasing the resource R_3 by the process P_1, the process P_1 waits for releasing the resource R_5 by the process P_3 and the process P_3 waits for the access to resource R_1. The access to resource R_5 is possible only for the process P_2; yet, it cannot reach the resource as it is

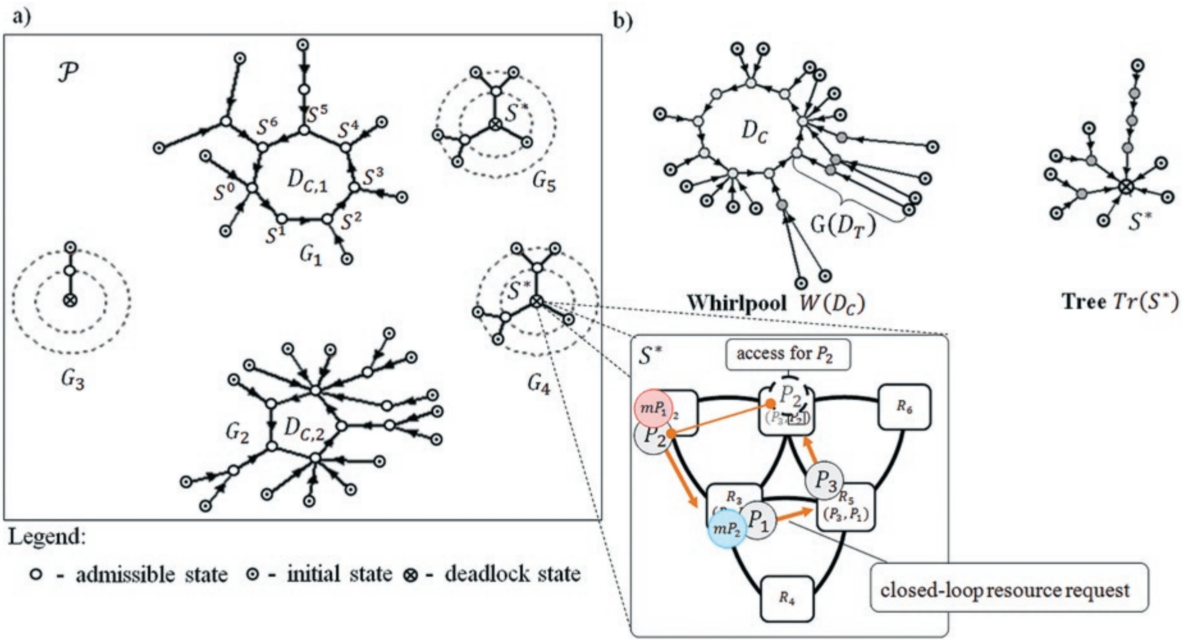

Fig. 3. The states space $\mathcal{P}$ determined by structure from Fig 2a), and the basic components of $\mathcal{P}$ b)

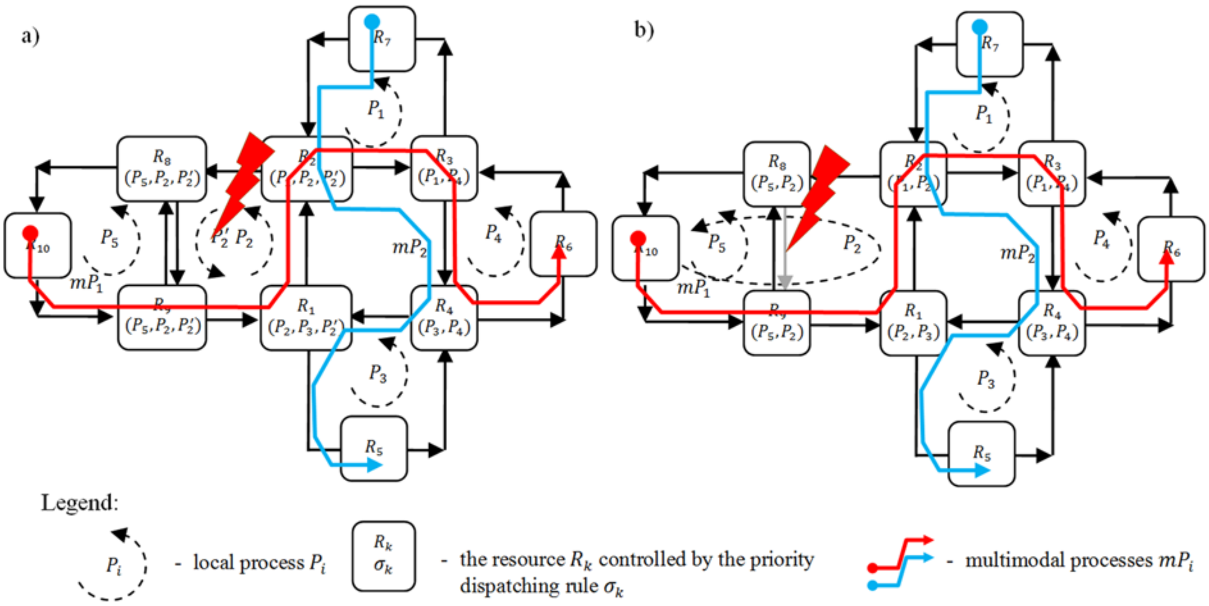

Fig. 4. Example of structural disruption in system from Fig. 1, stream failure P_2' a) , connection failure $R_8 - R_9$ b)

blocked by P_1. In practice, we face such situations when buses (trams) queue up in the order of service different that that required in a given station. In the considered case, bus P_2 is the last in the queue though it is the first to be served. As a result, such deadlocks stop the work of the system.

States causing deadlocks constitute the other type of behavior digraphs: **Tree** (Fig. 3b):

$$Tr\left(S^*\right)=\left(\dot{\bigcup_{\forall D_T \in DT\left(S^*\right)}} G\left(D_T\right)\right), \tag{4}$$

where: $G(D_T)$ – a digraph consisting of sequence of states D_T leading to the deadlock state S^*, $DT \in DT(S^*)$; $DT(S^*)$ – a set of all sequences of states leading to the deadlock state S^*.

Whirlpools and trees are the two basic components of the state space $\mathcal{P}$. Whirlpools make it possible to estimate the presence of cyclic steady states (determining the collision-free and deadlock-free transport of passengers in MTN). Trees enable determining dangerous states that lead to deadlocks (e.g. traffic congestions).

2.3. Disruptions in Supply and Demand

Determining the state spaces is the subjects of numerous investigations [2, 3, 4]. One of the properties of the considered SCCPs is the fact that once obtained behavior (cyclic steady state D_C that guarantees meeting the requirements of a user, deadlock S^*) does not change

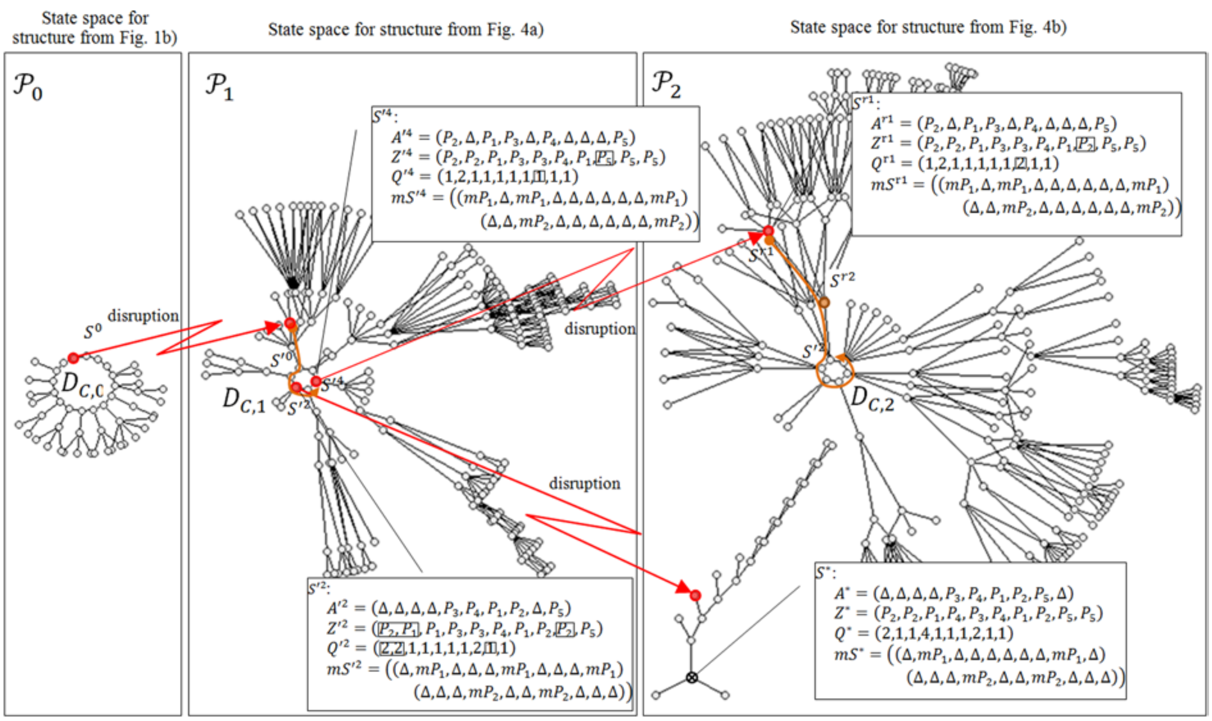

Fig. 5. The change of the state space generated by Fig. 4 while cussed by structural disruptions

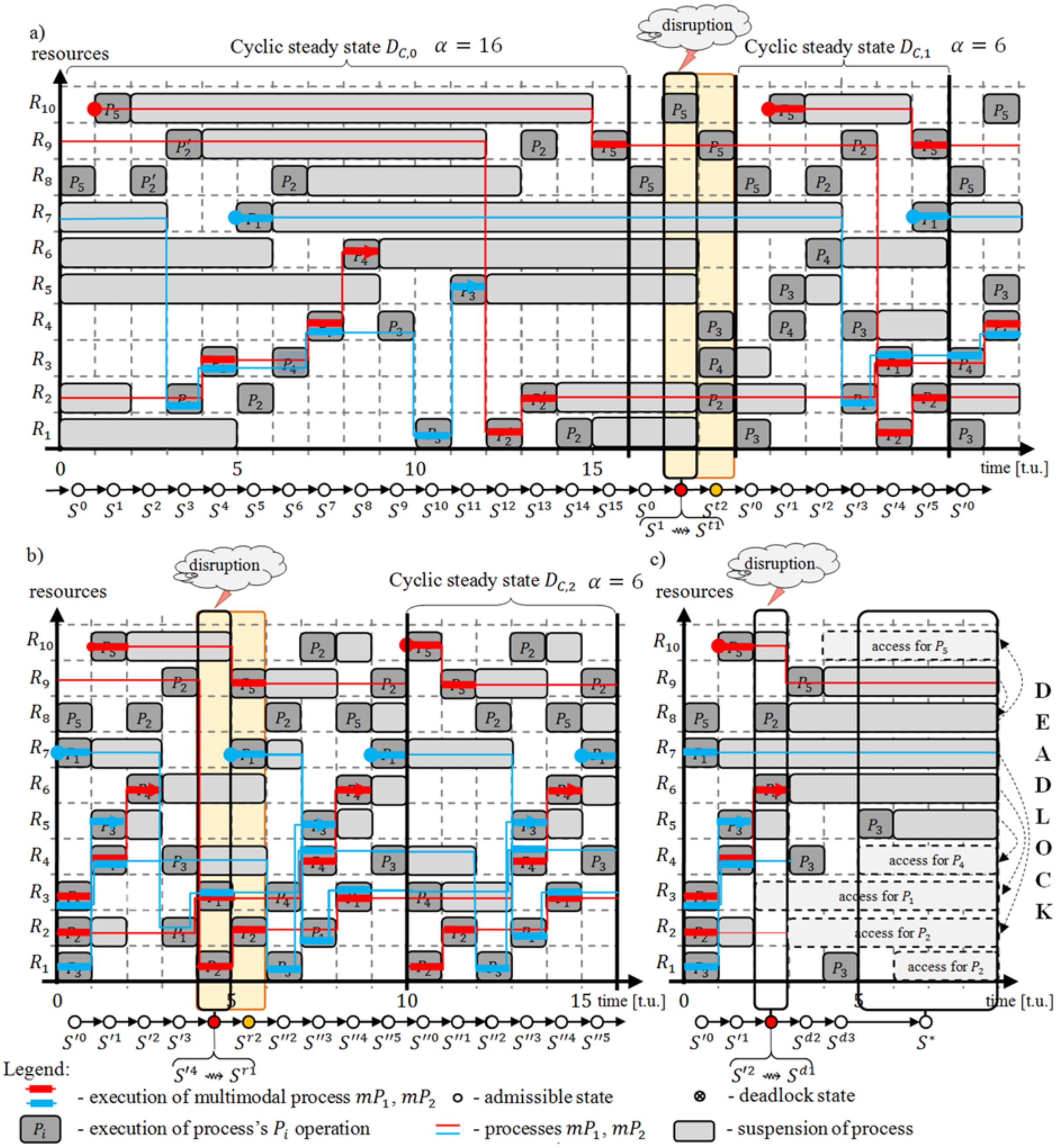

Fig. 6. Schedules illustrating changes of SCCP behavior caused by structural disruptions: leading to the cyclic steady state – disruption from Fig 4a) a), leading to the cyclic steady state – disruption from Fig 4b) b), leading to the deadlock c)

until the work conditions of the system are changed (e.g. change in the parameters of structure *SC*). An alteration in conditions of this kind may be caused by a number of external disruptions. Among disruptions in supply and demand, two types of disruptions can be distinguished:

– structural disruption: disruptions related to changes in the structure *SC* including, among others, addition or removal of a process (e.g. a new bus) – see Fig. 4a), changes in process routes caused by failures of transport lines (the railway/street tracks) – see Fig. 4b), resources failure (the tram/bus stations), etc.

– behavioral malfunction: disruptions related to changes in the way processes are implemented (disruptions of the system state S^r) including:
  - delays in the course of operation implementation (changes in duration of operation $t_{i,j}$). SCCPs systems subjected to this kind of disruptions have the ability to return to cyclic steady states unaided [5],
  - disruption in operation control resulting in changes in the current access rights to the shared resources (changes in values of semaphores Z^r and indexes Q^r related to them). Such disruptions include failures of traffic lights, railway signal, etc.

Examples of structural damages are presented in Fig. 4. We are considering a situation when the removal of stream P_2' (line P_2' bus broke down) – see Fig. 4a), and next there was a failure in the connection between resources R_8 and R_9 – see Fig. 4b). Consequently, the route of the local process P_2 was changed into $p_2=(R_1,R_2,R_8,R_{10},R_9)$; in practice it means that bus P_2, changed its itinerary. When such a series of disturbances occurs the question arises whether it is possible to maintain the cyclic behavior of the system.

Figure 5 shows state spaces $\mathcal{P}_0$, $\mathcal{P}_1$, $\mathcal{P}_2$ generated by the structures of SCCP affected by subsequent failures. In case of the first failure,

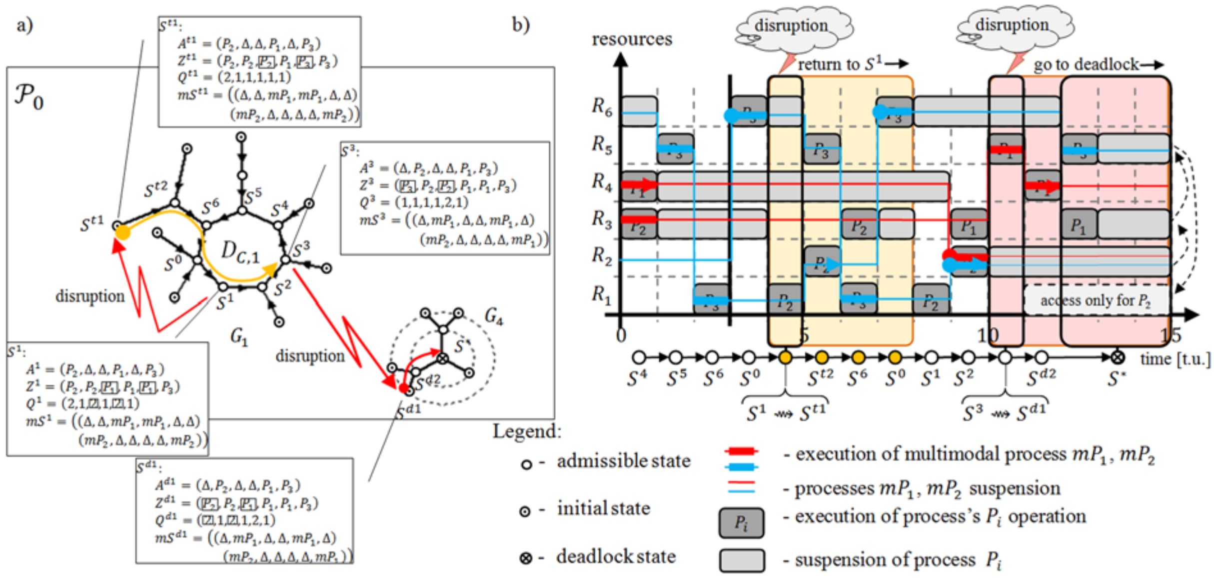

Fig. 7. The state space $\mathcal{P}_0$ generated by the system from Fig. 2 due to disruptions in operation control a), Gantt's diagram illustrating the changes in a system behavior b)

changing the state space from $\mathcal{P}_0$ to $\mathcal{P}_1$results in the transition of the system from the state S^0 to S^{r1}, which leads to cyclic steady state $D_{C,1}$. Generally, disruptions of this kind (removing processes) lead back to the cyclic steady state; these disruptions were subjected to investigations described in [5].

In case of the next disruption (removal of the connection $R_8 - R_9$), the behavior of the system is not that obvious. Figure 5 presents two possible scenarios for the further behavior of the system, depending on the state in which the disruption occurred. The occurrence of a disruption is equivalent to changing the state space to $\mathcal{P}_2$, where the state S^{4} passes to the state S^{r1}, and the state S^{2} passes to the state S^{d1} (as a result of changing the itinerary of the process P_2, some values of semaphores and indexes change – they are marked with a frame ☐).

It is clear that the occurrence of a disruption in the state S^4 results in disturbances in the behavior of the system manifested by the states (i.e. S^{r1}, S^{r2}) leading to a new cyclic steady state $D_{C,2}$. On the other hand, the disruption in the state S^2 (see Fig. 6c) results in states leading to the deadlock S^*. Schedules illustrating the described-above transitions are presented in Fig. 6.

As the described example shows, the occurrence of the structural disruption in SCCP causes a change of states and, consequently, the current state of the system. Further behavior of the system depends on the fact whether the newly obtained state is a part of a whirlpool (leading to a cycle, as S^{r1}) or a tree (leading to a deadlock, as S^{d1}).

2.4. Disruptions in Operation Control

A change in the structure *SC* caused by the occurence of structural disruption leads to the change in the state space (e.g. a change from $\mathcal{P}_0$ to $\mathcal{P}_1$ and $\mathcal{P}_1$ to $\mathcal{P}_2$). However, such situations do not happen in case of behavioral malfunction, and especially in case of disruptions in operation control. Disruptions of this kind do not lead to the physical damage of the structure (connection failure, removal of a process, etc.) and, consequently, they do not cause changes in the space state. It means that the change of the current state space, resulting from a disruption, occurs in the same space.

An example illustrating a situation of this kind is shown in Fig. 7. The considered disruption is of the operation control type. Its idea is to change the current access rights to resources (by changing semaphores Z^1 and corresponding indexes Q^1). For example, the disruption in the state S^1 consists of changing the access rights to the resource R_3 (from $\mathcal{P}_1$ to $\mathcal{P}_2$) and R_5 (from $\mathcal{P}_1$ to $\mathcal{P}_3$). The change of this kind results in a transition of the system into the state S^{t1} which, subsequently (through the states S^{t1} and S^6) can return to the cyclic steady state $D_{C,1}$. In practice, such a disruption may mean a signaling system failure in the stations R_3 and R_5, which results in the change of servicing order of trams and buses in these stations. Another example, this time leading to deadlock (see Fig. 7), is the disruption in the state S^3.

Similarly as in case of structural damages, the system's ability to return to cyclic steady state after the disruption of operation control depends on the system's ability to pass to the state included in the whirlpool.

3. Problem formulation

The existence of states leading to cyclic steady state, among states resulting from a disruption, proves the system's ability to self-organize. The system's robustness results from this ability. It is assumed that the robustness is expressed as:

$$Rob(dist) = \frac{NC(dist)}{NT(dist)} \tag{5}$$

where:

$Rob(dist)$ – robustness of SCCP to disruption dist; $Rob(dist) \in [0,1]$; $Rob(dist)=0$ – means the lack of robustness, i.e. disruption dist will always lead to a deadlock, $Rob(dist)=1$ – means full robustness to disruption *dist*, regardless of the disruption state, the system always returns to the cyclic steady state,

$\mathcal{P}_{dist}$ – state space imposed by disruption *dist*,

$NC(dist)$ – number of states leading to cyclic steady states contained in $\mathcal{P}_{dist}$,

$NT(dist)$ – number of states contained in $\mathcal{P}_{dist}$, $NT(dist)=|\mathbb{S}_{dist}|$.

According to (5), in all the cases discussed so far, the system's robustness to disruption *dist* should be regarded as the ratio of the number of whirlpool's states to states of the space $\mathcal{P}_{dist}$. The value, obtained in this way, determines the natural robustness (denoted as Rob_0 (*dist*) determined by the structure of the system SC(1).

Owing to the fact that in many cases occurring in practice [2, 3, 4] the state space $\mathcal{P}_{dist}$ includes mainly digraphs of the tree type, the value Rob_0 (*dist*) does not exceed 0.5. That means, that over 50% of possible disruptions lead to stoppage of the system (deadlocks).

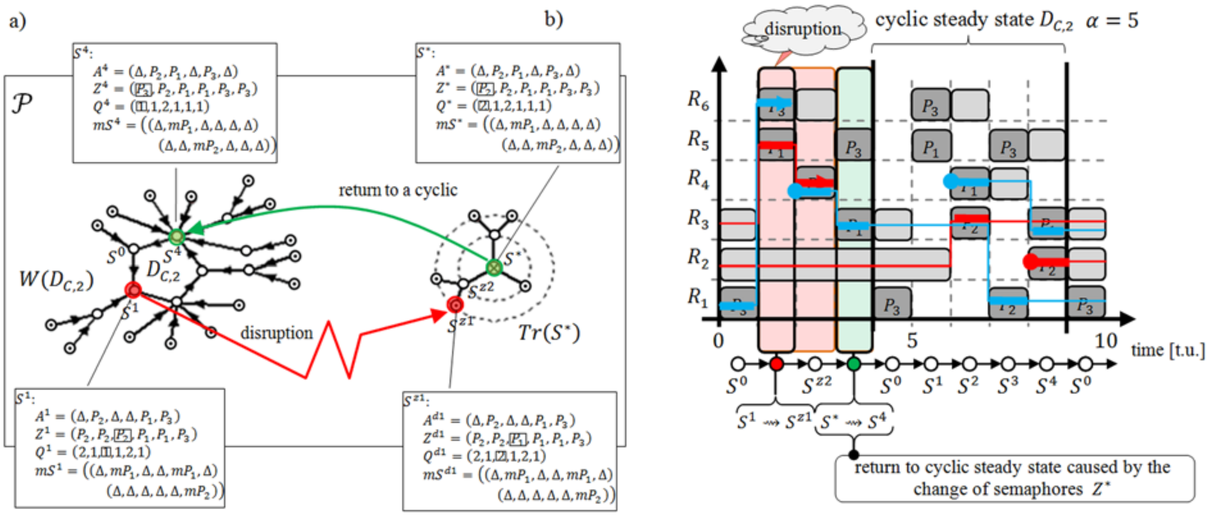

Fig. 8. An example of a return to cyclic steady state in the state space from Fig. 3

Therefore, investigations are carried out aiming at MTN structure design robustness of which is higher than Rob_0 (*dist*). In general case the robustness of a MTN depends on its structural features (such as redundancy, density, and so on) and on control mechanism employed in course of synchronization of concurrently executed flows. In that context, the present work attempts to determine these conditions for disruptions in operation control. Within this approach, the considered problem takes the following form:

There is a given MTN network modelled by SCCP with the structure SC (1). The robustness of the system Rob_0 (dist) to disruptions in operation control is known. The answer to the following question is sought: Are there any conditions (e.g. determining the methods of controlling the system) guaranteeing $Rob(dist)>Rob_0$ (dist)?

4. Robustness Conditions

In our further considerations, let's make an assumption that we are constraining to disruptions in operation control (e.g. disruptions in signaling – see Fig. 7). According to the previously made annotation (see point 2.4), it means that, as a result of a disruption, the state space does not change: $\mathcal{P}_{dist}=\mathcal{P}$.

In such a space, only states belonging to whirlpools make it possible to return to cyclic steady state. (see Fig. 7). It means that the increase in robustness (5) comes down to determining whether it is possible to reach cyclic steady states from states that do not belong to whirlpools. In other words, it is crucial to answer the following question: Is the transition between structures of the whirlpool type and the tree type possible in space $\mathcal{P}$ (Fig. 8a)?

As Fig. 8a) shows, that this kind of transition depend on the existence of a state that is, at the same time, an element of both the tree and the whirlpool. However, the presence of such states in space $\mathcal{P}$ is not acceptable [3]. Transitions of this kind may occur only as a result of modification of elements (semaphores and indexes) of the proper states (e.g. S^* and S^4). For this purpose, the properties referring to states possessing the same allocation are used.

Figure 8b) shows the implementation of process operations of the system from Fig. 2 caused by the disruption in state S^{z1} (the disruption consists in change of access rights to the resource R_3). As a result of the disruption, the system passes to the state S^{z1}, which leads to a deadlock (state S^*). It is noticeable that the allocation of local processes of state S^* is identical as the allocation in the state S^4: $A^*=A^4$. In practice, it means that in the state S^* means of transport (buses and trams) are in the same stations as in the state S^4. Hence, the deadlock is caused by improper assignment of access rights to resources (signaling that controls the order of service in stations). It means that it is enough to change the access rights to resources and the system will return to cyclic steady state. In the considered case, such a change comes down to changing semaphores from Z^* to Z^4 (on the resource R_1 the access P_2 is changed into the access for P_3).

The example above shows that in certain situations (e.g. concerning structures of the tree type) it is possible to return to cyclic steady state as a result of changing current values of semaphores (signaling). However, a condition must be fulfilled that the states, between which the transition happens, are characterized by the same allocation of processes. This observation leads to the following property:

Property 1

If in the space $\mathcal{P}$ there are two states $S^a \in V_{Tr}(S^*)$, $S^b \in V_W(D_{C,2})$ (where: $V_{Tr}(S^*)$ –set of states belonging to the tree $Tr(S^*)$ (4), $V_W(D_{C,2})$ – set of states belonging to the whirlpool $W(D_{C,2})$ (3)) possessing the same allocation of local processes $A^a = A^b$, then the whirlpool $W(D_{C,2})$ (and, consequently, cyclic steady state $D_{C,2}$) is reachable from the tree $Tr(S^*)$.

□

The presence of states that make it possible to return to cyclic steady state increases the system's robustness to *disruptions in operation control*. The evaluation of the presence of such states (as well as determining their number) requires searching through the states of $\mathcal{P}$ in order to identify the same allocation of processes. In the situation when the cyclic steady state $D_{C,1}$ (being a part of the whirlpool $W(D_{C,1})$ and the tree $Tr(S^*)$ are known, determining the states possessing the same allocation comes down to searching through all the admissible pairs of states, i.e. elements of the set $V_W(D_{C,1})\times V_{Tr}(S^*)$. The proper algorithm has the following form:

Algorithm 1

function StatesCoAll $\left(W(D_{C,1})\right)=\left(V_W(D_{C,1}),E_W(D_{C,1})\right),Tr\left(S^*\right)=\left(V_{Tr}(S^*),E_{Tr}(S^*)\right)\Big)$

```
AC ← ∅
forall S^a ∈ V_Tr(S^*)
  forall S^b ∈ V_W(D_C,1)
    if A^a = A^b then AC ← AC ∪ (S^a,S^b)
    end
  end
end
return AC
end
```

where: $W(D_{C,1})=(V_W(D_{C,1}),E_W(D_{C,1}))$, $Tr(S^*)=V_{Tr}(S^*)$, $E_{Tr}(S^*))$ – input data: whirlpool related to cyclic steady state $D_{C,1}$, and the tree leadig to the deadlock S^*,

AC – set of pairs (S^a,S^b) of states possessing the same allocation.

The result of Algorithm 1 is the set AC of pairs of states (S^a,S^b) possessing the same allocation: $AC \subseteq V_W(D_{C,1}) \times V_{Tr}(S^*)$. If we assume, for simplicity reasons, that the considered digraphs ($W(D_{C,1})$, $Tr(S^*)$) have the same number of states (determined by ld), the computational complexity of the proposed Algorithm 1 is estimated by the quadratic function $f(ld)=\mathcal{O}(ld^2)$.

The presented algorithm makes it possible to estimate the reachability of only two selected digraphs $W(D_{C,1})$, $Tr(S^*) \in DG$ (DG – set of digraphs of the space $\mathcal{P}$). Investigating the reachability between all the digraphs of the set DG comes down to evaluating the reachability of each pair of this set. The computational complexity of a procedure of this kind amounts to: $f(ld,dg)=\frac{1}{2}(dg^2-dg)\cdot ld^2 (dg=|DG|)$.

Owing to the polynomial character of the computational complexity function, the problem of evaluating the reachability of whirlpools seen as subspaces of $\mathcal{P}$ is easy to handle.

It must be emphasized that all states of the set AC make it possible to return to the cyclic steady state as a result of changes in the values of semaphores. Therefore, using these states in the process of returning the system to the cyclic steady state enhances the system's robustness to the existing disruptions:

$$Rob(dist)=\frac{|VW|+|AC|+|AD|}{NT(dist)} \tag{6}$$

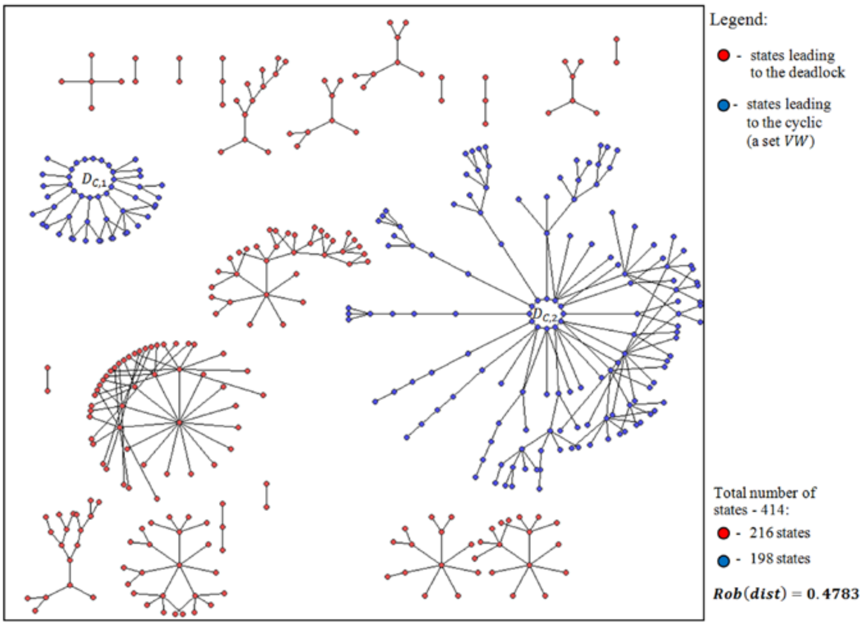

Fig. 9. State space of the system from Fig. 1

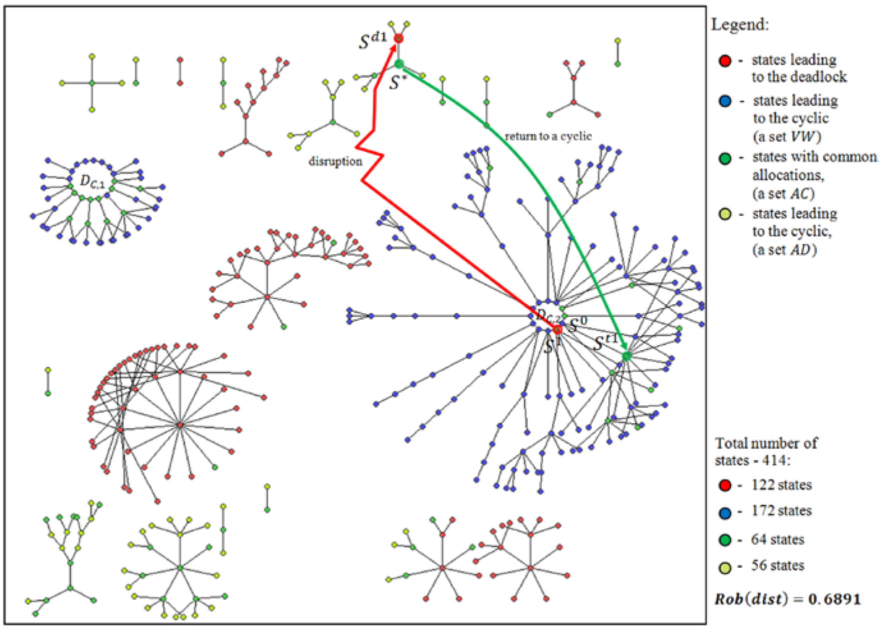

Fig. 10. State space of the system from Fig. 1 with highlighted states possessing the same allocation (set AC) and states leading to them (set AD)

where: VW – set of states constituting whirlpools of the state $\mathcal{P}$, AC - set of states possessing the same allocations, AD – set of states leading to the states of the set AC.

Contrary to (5), with the expression (6), the states that make it possible to return to cyclic steady state include not only states of the set VW, but also states with the same allocation (set AC) and all the states leading to them (set AD).

5. Numerical Example

The developed approach to determining states possessing the same allocation was used to evaluate the robustness of the system from Fig. 1 to disruptions in operation control. For this purpose, the state space $\mathcal{P}$ (see Fig. 9) was determined with use of the method presented in [3]. The space $\mathcal{P}$ includes 414 states, out which 198 states are elements of whirlpools (in the space there are two whirlpools leading to two cyclic steady states). The robustness (5) of such systems to disruptions in operation control (disruptions which do not lead to changes in state space) amounts to $Rob(dist)$=0.4783. In practice, it means that over a half of disruptions (if we assume that all disruption are equally probable) lead to the stoppage of the system (deadlock).

The robustness of the system can be enhanced by taking into account states possessing the same allocation in the process of returning to cyclic steady state. These states were determined on the basis of algorithm 1. Figure 10 shows the space $\mathcal{P}$ of the system from Fig. 1 along with states with the same allocation (set AC) and states leading to them (set AD). Owing to the knowledge about these states it is possible to return to the cyclic steady state even when the system passes to a state belonging to a tree. An example of such a transition is presented in Fig. 10 – the transition between the states S^1, S^{d1}, S^*, S^{r1}, S^0. By taking into consideration states of the sets AC and AD the robustness of the system (6) can be increased to the value $Rob(dist)$=0.6891.

6. Conclusions

The article has discussed the major disruptions in the labor of systems with cyclic multimodal processes and focused mainly on disruptions in operation control. In order to evaluate the robustness of NTN to this kind of disruptions the measure $Rob(dist)$ has been introduced: it determines the system's ability to return to the cyclic steady state.

In order to enhance the robustness of NTN, an approach based on a property which states that the return to the cyclic steady state is possible from states possessing the same allocation, is proposed.

In this case, the return is possible as a result of change in access rights (semaphores) to the shared resources. The analyzed example shows that owing to the fact that these states are classified as the so-called "safe" states the robustness of system can be increased by as much as 44% (the value $Rob(dist)$ has risen from z 0.4783 up to 0.6891).

The proposed algorithm of determining states possessing the same allocation is characterized by polynomial computational complexity. Therefore, it is possible to use this approach in networks with a scale that is met in practice.

The use of the proposed conditions (Property 1) is restrained to disruptions in operation control. That is why, further investigations will focus on an attempt to expand the developed conditions to the area of disruptions including, among others, the structural disruption.

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