

COMPARISON OF CHARACTERISTICS OF FOCUSED AND NON-FOCUSED BEAM

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The paper presents results of comparison of characteristic features of wave of finite amplitude propagating in form of focused or non-focused beam. Some resemblances in both types of beams are pointed out. It occurs mainly due to nonlinear wave distortion during the propagation. In addition, certain differences are indicated, this is the consequence of impact of diffraction in the beam forming process. Findings are illustrated with results of experimental and numerical investigations.

INTRODUCTION

Propagation of finite amplitude wave in nonlinear medium involves wave distortion, which manifests itself in higher harmonic presence. The finite radiating area of real source caused diffraction effects. In consequence, each system transmitting the finite amplitude wave is strongly affected by the combined influence of nonlinearity, diffraction and attenuation. All of these factors are accounted for as separate terms in the KZK equation[7]:

$$\frac{\partial^2 p}{\partial z \partial \tau} = \frac{c_o}{2} \nabla_{\perp}^2 p + \frac{b}{2\rho_o c_o^3} \frac{\partial^3 p}{\partial \tau^3} + \frac{\varepsilon}{2\rho_o c_o^3} \frac{\partial^2 p^2}{\partial \tau^2} \quad (1)$$

where p denotes the sound pressure, c_o is the small - signal sound speed, ρ_o is the ambient density, b is the attenuation factor, ε is coefficient of nonlinearity, z is the coordinate along the axis of beam, $\tau = t - z/c_o$ is retarded time, the operator ∇_{\perp}^2 is the Laplacian in the (x,y) plane, perpendicular to the axis of beam. The KZK equation is often used in numerical modelling of the finite wave propagation with arbitrary source conditions [1, 5]. It means it is used for plane and focusing sources. Chiefly diffraction effects bring out the main difference between focused and non-focused beam of finite amplitude wave.

The paper contains results of experimental and numerical investigations of plane and focusing sources. Measurements of the wave field emitted from the source transducer were made using arrangement in which a PVdF needle hydrophone with a nominal diameter of 1 mm was scanned along the beam axis and across the beam at various distances from the radiating surface. The investigation was conducted with the use of high precision device

controlling the movement of the receiver. To simulate CW operation the tone burst with 50 cycles was used. The numerical investigations were carried out using the model based on the solution of the KZK equation.

1. NONLINEAR DISTORTION

Harmonic generation in a bounded beam appears as a result of varying phase speed. The distortion produced by varying propagation speed is cumulative. The growth of nonlinear distortion could be evaluated by showing the changes in the amplitude of the pressure harmonics. Examples of growing nonlinear distortion determined numerically and experimentally for plane and focusing source are shown in Fig. 1 and Fig. 2.

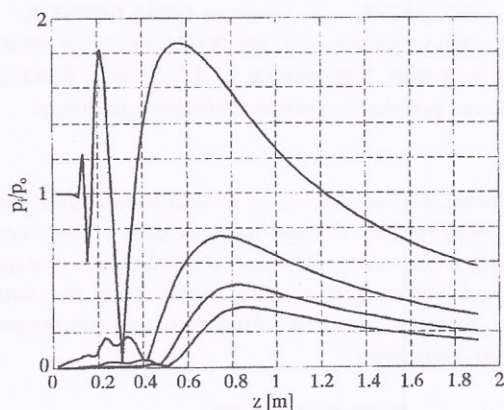


Fig. 1. The pressure distribution of the first four harmonic components along the axis of plane piston source; $f=1\text{ MHz}$, $a=23\text{ mm}$, $p_0=127\text{ kPa}$

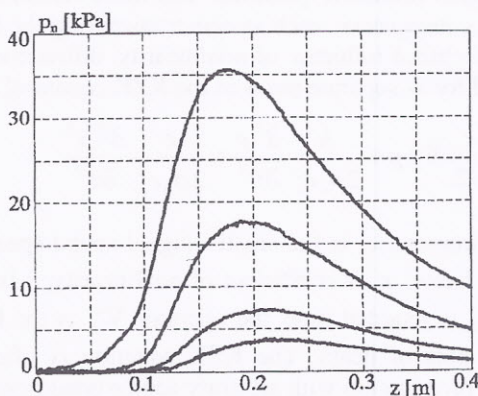


Fig. 2. The pressure distribution of higher harmonic components in axis source; composed of plane circular piston and focusing lens; $f=1\text{ MHz}$, $a=19\text{ mm}$, $d=145\text{ mm}$

At first sight both phenomena run similar: the nonlinear distortion grows up, higher harmonics arise, reach their maxima and gradually fade down. In both cases maxima in on-axis pressure distributions appear in succession according to the number of harmonics. In

addition, in focused and non-focused beam the near-field oscillations in the distributions of higher harmonic components are visible. They are associated with rapid phase variations, which create poor conditions for efficient harmonic generation. When a plane circular source is used it appears prior the last axial null ($z = R_o/2\pi$), where $R_o = ka^2/2$ denotes the Rayleigh distance [4].

Analysing the on-axis distribution of higher harmonics components normalised to the fundamental component virtually non-existent nonlinear interaction beyond the focus can be noticed. It was reported by Hurt and Hamilton [3]. The relative absence of nonlinear interaction is explained as the result of the phase mismatch in focal area introduced by diffraction. In focal area, a 180° phase shift of the fundamental component is observed, whereas the on-axis phases of higher harmonics exhibit phase shifts that are at least 90° greater than those of the previous harmonics. The area of virtual absence of nonlinear effects extends approximately up the distance of $3d$, where d is the focal length, as it is shown in Fig. 3.

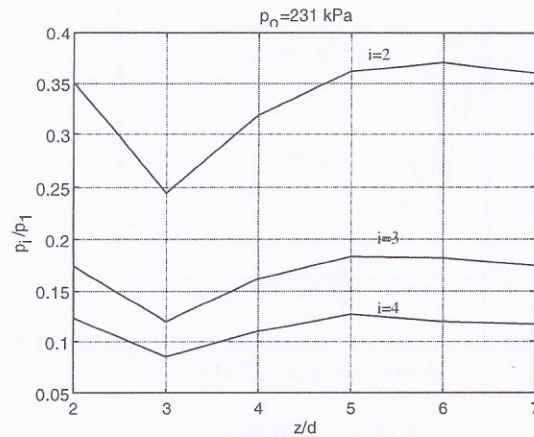


Fig. 3. On-axis changes in higher harmonic components normalised to the fundamental one

In the post focal region at the distance above $3d$ the nonlinear distortion of the wave grows again and in certain area, the wave propagates as a wave of steady shape.

2. BEAM PATTERNS

Directivity of the harmonic components in the field of plane circular source grows with the number of harmonic. In the far field, the width of the main lobe of n -th harmonic component could be approximately assessed as [6]:

$$D_n(\Theta) = D_1^n(\Theta) \quad (2)$$

where $D_1(\Theta)$ denotes the beam pattern of the fundamental wave described by function $2J_1(x)/x$, J_1 is the Bessel function of the first kind of order one. An asymptotic, quasilinear solution to the KZK equation presented by Bernstein et al. [2] predicted the additional sidelobes in higher harmonic beam patterns. The additional sidelobes, dubbed *fingers* by Bernstein et al. were shown to be generated in the nearfield and were later shown to decay asymptotically as $1/z$ rather than $(\ln z)/z$, which is typical for the other sidelobes.

The form of the linear solution for the amplitude distribution in the focal plane is expressed in similar form as for the plane circular source in the farfield, $2J_1(x)/x$. As in the farfield of unfocused circular sources, the second harmonic component has twice as many sidelobes as the fundamental component, the third harmonic has three times as many, and so on. The field structure near the focal plane is thus qualitatively similar to the structure observed in the farfield of unfocused sources. One occurring difference concerns the position of the nonlinearly generated sidelobes relative to the structure of the fundamental component. In focused sound fields, the nonlinearly generated sidelobes appear slightly closer to the axis than do their counterparts in the farfield. The phenomenon illustrates Fig. 4, in which the structure of the field in the focal plane is shown.

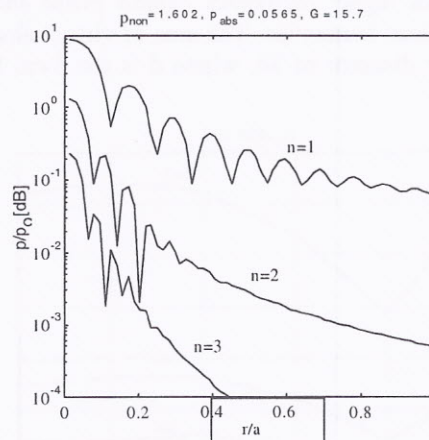


Fig. 4. Beam patterns for first three harmonic components in focal plane: $f=1.2$ MHz, $a=25$ mm, $d=100$ mm, $p_0=127$ kPa

CONCLUSIONS

The combined effects of nonlinearity, diffraction and absorption influenced the phenomena occurring in bounded beams of finite amplitude wave. The phase effects play significant role in development of nonlinear distortion in both kinds of considered beams.

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