LIMITING FIBER EXTENSIBILITY MODEL FOR ARTERIAL WALL

Lukas Horny[1]*, Rudolf Zitny[2], Hynek Chlup[1], Svatava Konvickova[1],

CTU in Prague, Faculty of Mechanical Engineering,
4 Technicka str., 166 07 Prague Czech Republic
[1] Department of Mechanics, Biomechanics and Mechatronics
[2] Department of Process Engineering
* e-mail: Lukas.Horny@fs.cvut.cz

Arterial walls exhibit anisotropic, nonlinear and inelastic response to external loads. Moreover arterial wall is non–homogenous material with complicated internal structure. These facts make the question about the best material model for arterial wall still unanswered. Nowadays approach to building constitutive models is characterized by incorporating structural information when considering e.g. layers, fibers, fiber orientation or waviness. The most frequent method how to incorporate structural information is to regard arterial wall as a fiber reinforced composite. Considerations about preferred directions are subsequently implemented into the framework of continuum mechanics.

Constitutive models are usually based on the theory of hyperelastic materials. Thus mechanical response of an arterial wall is supposed to be governed by a strain energy (or free energy) density function like in (1). The theory of hyperelastic materials is widely applied and studied in details in polymer science. Due to some phenomenological and structural similarities between rubber–like materials and biological tissues, methods of polymer physics are frequently applied in biomechanics, see Holzapfel [1]. Gent [2] suggested the new isotropic model for strain energy density function which was based on an assumption of limiting chain extensibility in polymer materials. The Gent model expresses strain energy y as a function of first invariant I1 of the right Cauchy-Green strain tensor as follows

$$y = -\frac{1}{2} m J_m \ln\left(1 - \frac{I_1 - 3}{J_m}\right). \qquad (1)$$

In equation (1) μ denotes stress–like parameter, so–called infinitesimal shear modulus. J_m denotes limiting value of I_1-3. The domain of logarithm requires I_1-3<J_m. Thus, J_m can be interpreted as limiting value for macromolecular chains stretch. Horgan and Saccomandi in [3] suggested its anisotropic extension. They recently published modification based on usual concept of anisotropy related to fiber reinforcement, see paper [4]. Horgan and Saccomandi use rational approximations to relate the strain energy expression to Cauchy stress representation formula. We adopted this term with small modification as follows

$$y = -m J_m \ln\left(1 - \frac{(I_4 - 1)^2}{J_m^2}\right). \qquad (2)$$

In (2) μ denote shear modulus. J_m is the material parameter related to limiting extensibility of fibers. The similar definitional inequality like in (1) must be hold for logarithm in (2). Thus I_4 must satisfy (I_4-1)2<J_m^2. I_4 denotes so called fourth pseudo–invariant of the right Cauchy-Green strain tensor which arises from the existence of preferred direction in continuum. It is worth to note that total number of invariants of the strain tensor is five in the case of transversely isotropic material and nine in the case of orthotropy. Details can be found in e.g. Holzapfel [5]. Model (2) presumes two preferred directions in continuum which are mechanically equivalent. Due to cylindrical shape of an artery we can imagine it as helices with same helix angel but with antisymmetric rientation. This is illustrated in the FIG. 1.

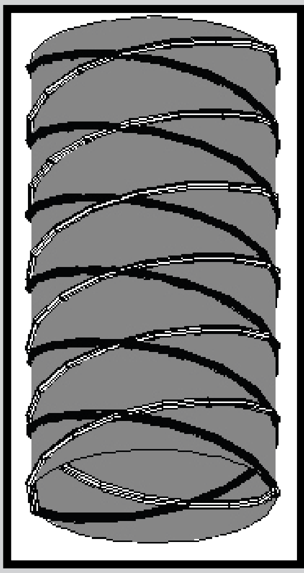

FIG. 1. Local orthotropy – the tube is reinforced by two families of mechanical equivalent fibers.

I_4 can be expressed in the form given in (3).

$$I_4 = l_t^2 \cos^2 b + l_z^2 \sin^2 b \qquad (3)$$

Stretched configuration of the tube is characterized by λ_t, what denotes circumferential stretch and λ_z what denotes axial stretch, respectively. Model (2) contains three material parameters. Above described μ, J_m and β. The third material parameter β has the meaning of angle between fiber direction and circumferential axis. There are two families of fibers with angle ±β, however, I_4 is symmetric with respect to ±β.

In order to verify capability of (2) to govern multi–axial mechanical response of an artery regression analysis based on previously published experimental

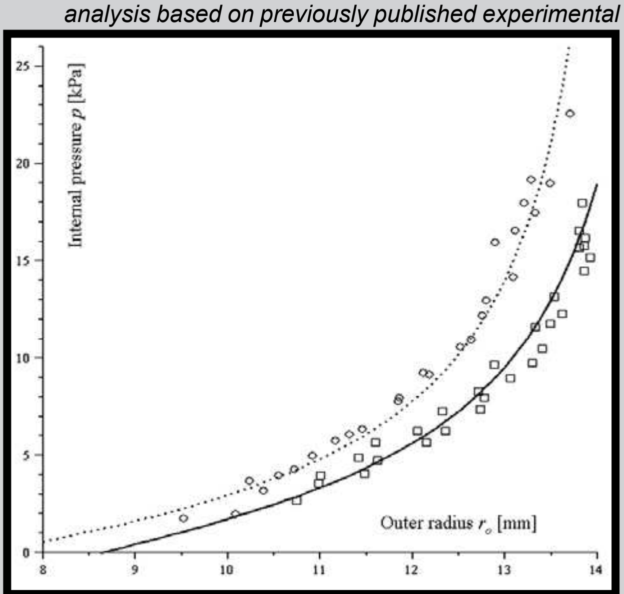

FIG. 2. Inflation test and model prediction: red/square – λ_z=1.3; blue/circle – λ_z=1.42.

data was performed. Details of experimental method and specimen can be found in Horny et al. [6]. Briefly we resume basic facts. Male 54–year–old sample of thoracic aorta underwent inflation test under constant axial stretch. The tubular sample was 6 times pressurized in the range 0kPa–18kPa–0kPa under axial pre–stretch λ_z=1.3 and 3 times in the pressure range 0kPa–20kPa–0kPa under λ_z=1.42, respectively. The opening angle was measured in order to account residual strains. Radial displacements were photographed and evaluated by image analysis.

Regression analysis based on least square method gave the estimations for material parameters μ, Jm and β. The vessel was modeled as thick–walled tube with residual strains. The material was supposed to be hyperelastic and incompressible. No shear strains were considered. Fitting of material model was based on comparison of model predicted and measured values of internal pressure. Results are illustrated in FIG. 2. We can conclude that proposed material model fits experimental data successfully. Thus strain energy given in (2) is suitable to govern arterial response during its inflation and extension. Estimated values of parameters for material model (2) are as follows: μ =26kPa; J_m=1.044; β=37.2°.

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AN ULTRASONIC METHOD FOR ESTIMATION OF ELASTIC PROPERTIES OF R-BONE CEMENT AFTER IMMERSION IN RINGER'S SOLUTION

Izabella Rajzer[1,3]*, Jan Piekarczyk[2], Oscar Castano[3], Elisabeth Engel[3], Josep A. Planell[3]

[1] ATH-University of Bielsko-Biala,
Faculty of Materials and Environmental Sciences,
Institute of Textile Engineering and Polymer Science,
Department of Polymer Materials,
2 Willowa street, 43-309 Bielsko-Biała, Poland
[2] AGH-University of Science and Technology,
Faculty of Materials Science and Ceramics,
Department of Advanced Ceramics,
Al. Mickiewicza 30, 30-059 Cracow, Poland
[3] IBEC, Institute for Bioengineering of Catalonia,
13 Baldiri Reixac, 08028 Barcelona, Spain
* e-mail: ipiekarc@yahoo.es

Abstract

The aim of this investigation is to establish an improved non-destructive ultrasonic through-transmission technique to monitor the setting behaviour of calcium phosphate cement samples.

On the basis of ultrasound techniques the elastic properties of cement paste after different soaking time in Ringer´s solution were measured. Young's modulus, the rigidity modulus, and Poisson's ratio were calculated from measurements of density and ultrasonic longitudinal and shear wave velocities.

Keywords: *calcium phosphate cement, elastic properties, injectability, ultrasonic method*

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Introduction

Calcium-phosphate based ceramics (such as hydroxyapatite) are promising materials for orthopedic and dental surgery. They closely resemble the mineral phase of the bone extracellular matrix, so they are expected to be osteoinductive and osteoconductive, promoting regeneration of the damaged bone tissue [1-2]. A calcium phosphate cement (CPC) was developed with the advantage of being moldable (the CPC paste intimately adapts to the bone cavity) and capable of in situ setting to form hydroxyapatite [3]. The ability of obtaining biologic apatite under physiologic conditions (in an aqueous environment at body temperature), opening a new strategy to traditional ceramic sintered at high temperatures, is one of the most important factors in the creation of a new class of bone substitute implants. Calcium phosphate cements generally consist of a powder and an aqueous liquid which are mixed to form a paste. The paste is placed into a defect as a substitute for the damaged part of the bone. Most conventional CPC are mixed with an aqueous solution immediately before application. In the clinical situation, the ability of the surgeon to properly mix the cement and then place the cement paste into the defect within the prescribed time is a crucial factor in achieving optimum results. Therefore, it is desirable