Article citation info:
JODEJKO-PIETRUCZUK A, WERBIŃSKA-WOJCIECHOWSKA S. Analysis of maintenance models' parameters estimation for technical systems with delay time. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2014; 16 (2): 288–294.

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ANALYSIS OF MAINTENANCE MODELS' PARAMETERS ESTIMATION FOR TECHNICAL SYSTEMS WITH DELAY TIME

ANALIZA PARAMETRÓW MODELI OBSŁUGIWANIA SYSTEMÓW TECHNICZNYCH Z OPÓŹNIENIEM CZASOWYM*

In the article authors are interested in BIP performance for three-element system ("k-out-of-n" reliability structure), the maintenance policy which is one of most commonly used in practice. The BIP may be implemented in technical systems when some information about reliability characteristics is known. The basic reliability parameters that have to be specified in such systems are: an estimation of system components' time to failure and some delay time characteristics. In order to determine the effects of possible errors and to specify sufficient accuracy of the estimation, the analysis of system costs was done for various values of the expected delay time, assuming three different probability distributions of the delay time (Weibull, Uniform, and Normal). The modelling process was based on the use of GNU Octave software. Test analysis of delay time parameter, assuming different types of probability distributions is the base to conclude: if the form of the distribution has any meaning for economic results of the system, and what kind of consequences may result from improper mean delay time estimation E(h).

Keywords: *delay-time, maintenance model, parameters estimation methods.*

W pracy analizie poddano system trzyelementowy (struktura niezawodnościowa progowa), którego procesy obsługiwania realizowane są zgodnie z założeniami Polityki Przeglądów Blokowych (BIP). Strategia ta może być zastosowana w procesie utrzymania systemów technicznych, gdy znane są pewne jego charakterystyki niezawodnościowe, bazujące m.in. na informacjach o czasach pomiędzy uszkodzeniami elementów systemu. W badaniach skupiono się na trzech rozkładach prawdopodobieństwa tej zmiennej losowej (normalny, Weibull, prostokątny). Model symulacyjny opracowano przy wykorzystaniu oprogramowania GNU Octave. Analiza okresu opóźnienia czasowego, przy założeniu różnych postaci rozkładów prawdopodobieństwa tej zmiennej losowej, pozwoliła na ocenę: czy znajomość typu rozkładu prawdopodobieństwa zmiennej losowej h ma istotne znaczenie dla wyników ekonomicznych funkcjonowania systemu, oraz jakie konsekwencje mogą wystąpić w wyniku niewłaściwej estymacji wartości średniej E(h).

Słowa kluczowe: *opóźnienie czasowe, model obsługiwania, metody estymacji parametrów.*

1. Introduction

In the case of complex systems, in which an important issue is a problem of modeling the relationship between two separate subsystems that have an impact on the overall system availability, a lot of works draw attention to the delay times occurrence during operational processes performance [8].

In the year 1976 Christer (following [16]) proposed the delay-time (DT) concept, used to this day in the renewal theory in order to optimize the technical system downtime due to not detected failures occurrence (e.g. [9, 10, 14, 16, 20]). The basic idea rests on an observation that a failure does not usually occur suddenly, but is preceded by a detectable fault for some time prior to actual failure, called a delay time and is denoted by h (Fig. 1) [8, 12]. This research area has been widely studied in the literature, e.g. in [2, 3, 6, 7, 8, 12, 13, 24, 25, 26, 27, 28], where reviews of delay-time models from the application point of view are provided, and in [22, 23, 25, 32] which focus on the possibilities of DT models use for multi-unit systems maintenance performance.

The correctness of maintenance model selection for technical systems with time delay is directly dependent on the accuracy of model parameters estimation process. Generally, there is no possibility to measure directly either the delay time associated with a defect, or the initial point u. There can be proven a possibility to estimate the delay time for a set of specific faults and failures, and from this deduce the location of the initial point and estimate the delay-time and initial-point distributions [12, 25].

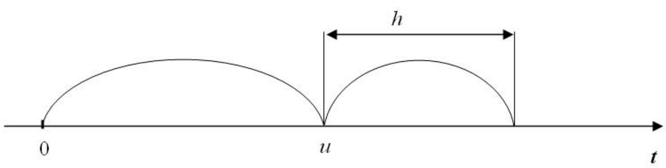

Fig. 1. Delay-time concept [15]

There are two methods to estimate delay-time h and initial point u parameters in the literature. When the operating data are available and reliable, it is possible to estimate the model parameters using the maximum likelihood method (e.g. [30]). Otherwise, there is applied a subjective estimation of parameters based on expert opinion (e.g. [29]). One of the first works in which authors have developed and investigated the method of subjective opinion of experts use in the parameter h estimation process are [10, 14, 15, 16, 20]. These works regarded to maintenance processes performed in civil engineering [10], industrial systems [16], manufacturing [15, 20] and transport maintenance [14].

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

In the next paper [13], authors continued their research on the delay time models parameter estimation issues. The authors presented an overview of the literature in the study area and discussed the revision delay-time models. Moreover, they investigated the issue of biased estimates determination in the analysed research area. This issue was later continued in [12], where e.g. the parameter estimation methods criteria used in maintenance decision processes have been analyzed. A literature review of the fields of subjective probability and the experts' opinions application in maintenance decision-making processes performance with the use of DT concept was presented in [7, 29].

In the next works, authors presented the implementation of subjective method of delay time parameters estimation in maintenance issues of industrial systems [19], manufacturing systems [1], and city bus transport companies [21].

On the other hand, for the first time, authors in [5] used an objective method in the process of delay time parameters estimation for the repairable object performance. Model parameters were estimated using maximum likelihood method and Akaike Information Criterion (AIC). The proposed model was developed in [4], where the authors investigated a technical system consisting of multiple components.

In the next work [17] authors considered a multi-element system, in which inspection actions are carried out systematically at time T and at failure. The issue of an objective estimation of the model parameters for complex system were analyzed e.g. in [18].

The problem of model parameters estimation, when there are available only data about the moments of failure occurrence (lack of information about system maintenance process performance), was analyzed in [11]. In other work [31], there is presented an example of time between maintenance actions performance for a manufacturing company.

Particularly noteworthy is the work [30], in which authors developed a model of delay time parameter estimation using both estimation methods.

Literature studies show that the problem of the proper estimation of the parameters of the random variable delay time h is extremely important and there are developed methods for continuous improvement of these estimates. In practice, there is not always the possibility of a correct and accurate approximation of all the delay time parameters for maintained system. In many cases, the available data allow only for estimation of the expected value and standard deviation of the delay time variable. Following this, there can be asked a question about possible consequences of incorrect estimation of the model parameters and the legitimacy of estimation improvement process performance. Therefore, the aim of this article is to assess:

- consequences - economic and reliability ones - which should be taken into account for maintained system, if only selected parameters of delay time will be possible to estimate in practice,
- the necessary delay time parameters estimation accuracy, allowing the selection of the correct time between inspection actions performance in multi-component system.

Achievement of such defined objectives has been performed by carrying out a simulation analysis of a technical system operation process with implementation of different delay time characteristics. The system is maintained according to Block Inspection Policy - BIP. Research results analysis allowed assessing the influence of selected delay time parameters changes on the obtained system maintenance costs and availability ratio. This process enables to assess the consequences connected with parameters under- and overestimation or with absence of delay time parameter data. Moreover, there is a possibility to define the influence of such consequences on performance outputs of multi-unit system, maintained according to BIP strategy. The article bases on simulation studies implementation because of the lack of analytical models development which could be used to define maintenance costs and system availability, depending on the multi-unit system characteristics and BIP strategy requirements and conditions.

In the next Sections, there are discussed the main assumptions used in BIP model, used simulation algorithm, and there are presented the obtained research results.

2. Block Inspection Policy Model

The study investigates system comprised of *3* identical elements, in a *k-out-of-n* reliability structure, working independently under the same conditions. The used maintenance policy is a BIP which assumes that the diagnosis operations of the state of a system are carried out at regular intervals of T time units. This maintenance strategy can be used in the maintenance process of technical system, where some of its reliability characteristics are known, based on e.g. the information about the time between failures of system components. It is assumed, that the system components are independent, as well as the first signs of forthcoming failures (defects occurrence). The inspections are assumed to be perfect. Thus, any component's defect, which occurred in the system till the moment of inspection, will be identified. All elements with identified defects will be replaced within the inspection period. The performance of the investigated system being illustrated in Fig. 2 is also defined by the additional assumptions:

- maintenance actions restores system to as good as new condition,
- failures of the system are identified immediately, and repairs or replacements are made as soon as possible,
- system incurs costs of: new elements, when they are replaced, inspection costs, and some additional, consequence costs, when system fails,
- elements' lifetime, repair time, replacement time and the length of the delay time before element's failure are random and their probability distributions are known.

The performance of the chosen system is modeled with the use of simulation processes. The modeling process was based on the use of *GNU* Octave software. The list of tested system parameters, which were used in the simulation models, is given in Table 1. The scheme of the simulation algorithm is given in Fig. 3.

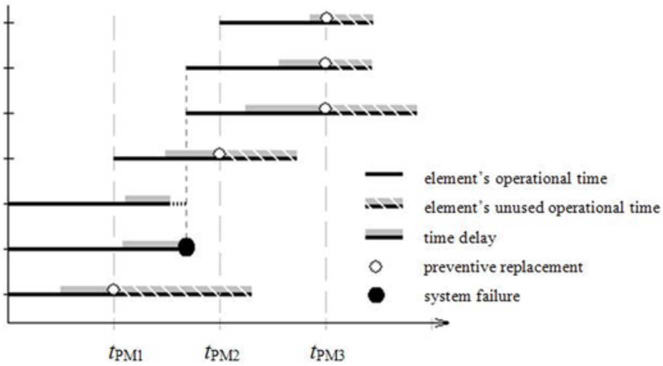

Fig. 2. Idea of the Block Inspection Policy for investigated system

The research analysis regards to the assessment of delay time parameters influence on the investigated system economic results. As a result, there is a necessity to define the expected costs per unit of operating time of the system resulting from BI maintenance policy performance $C_{BI}(T)$:

$$C_{BI}(T) = CE_{BI}(T) + CC_{BI}(T) + CI_{BI}(T) \quad (1)$$

where: C_{BI} – the expected cost resulting from *BI* maintenance policy performance, T – the period between two consecutive inspection actions performance, CE_{BI} – the expected costs of new elements per unit time for *BI* maintenance policy, CC_{BI} – the expected cost of the con-

Table1. Modelled system parameters

Symbol	Description	Tested values
n	Number of system elements	3
k	Minimal number of up-stated elements keeping the system in upstate	1,3
c_e	The cost of new element	1
c_i	The cost of an inspection	1
c_c	The cost of a system failure	10 000
T_i	The time required for inspection	0
T	Constant time between maintenance actions performance	–
$F(t=u+h)$	C.d.f. of elements' lifetime	$F(t)=1-e^{-(\frac{t}{100})^{3,5}}$
$G_r(t)$	C.d.f. of single elements' repair time (resulting from a system failure)	$G_r(t)=1-e^{-(\frac{t}{100})^{2,3}}$
$G_p(t)$	C.d.f of single element's replacement time (resulting from preventive replacement)	$G_p(t)=1-e^{-(\frac{t}{10})^{2,3}}$
$f_h(h)$	C.d.f. of elements' delay time	$f_h(h)=\frac{\alpha}{\beta^{\alpha_h}}h^{\alpha_h-1}e^{-(\frac{h}{\beta_h})^{\alpha h}}$
		$f_h(h)=\begin{cases}\frac{1}{b-a}, \text{ if } a\le h\le b\\ 0, \text{ if } h\le a \text{ or } b\le h\end{cases}$
		$f_h(h)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\left(\frac{(h-\mu)^2}{2\sigma^2}\right)}$

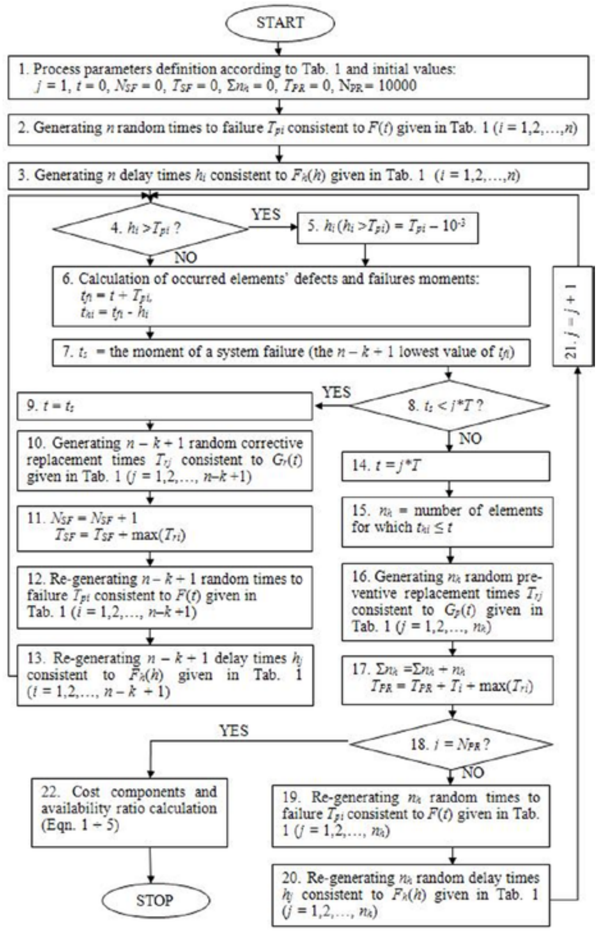

Fig. 3. Simulation algorithm

sequences resulting from a system failure for BI maintenance policy, CI_{BI} – the expected costs of performed inspections.

The expected costs of new elements per unit time can be obtained from the following formula:

$$CE_{BI}(T)=\frac{[(n-k+1)\cdot N_{SF}(T)+\sum n_{PR}(T)]\cdot c_e}{OT} \tag{2}$$

where: n – number of elements of the system, k – minimal number of up-stated elements for having system in an operational state, N_{SF} – the number of system failures during assumed time horizon, Σn_{PR} – the expected number of elements that were preventively replaced after inspection, c_e – the cost of new element, OT – the operating time of the system.

Moreover, the expected cost of the consequences resulting from a system failure for the investigated maintenance policy can be calculated from the following formula:

$$CC_{BI}(T)=\frac{N_{SF}(T)\cdot c_c}{OT} \tag{3}$$

where: c_c – the cost of the consequence resulting from a system failure occurrence.

There is also a necessity to define the expected costs of performed inspections:

$$CI_{BI}(T)=\frac{N_{PR}(T)\cdot c_i}{OT} \tag{4}$$

where: N_{PR} – the number of preventive replacements of system elements, c_i – the cost of an inspection performance.

The next step is connected with availability analysis. Thus, the availability ratio is calculated with the use of the following formula:

$$A_{BI}(T)=\frac{OT(T)}{OT(T)+T_{PR}(T)+T_{SF}(T)} \tag{5}$$

where: T_{PR} – the expected time of preventive replacements of system elements, T_{SF} – the expected time of corrective replacements of system elements.

3. Estimation of delay time parameter *h* – analysis of obtained simulation results

While performing the simulation analysis, it was assumed that there are tested three systems, built and operated in the same way and differing only in the area of delay time characteristics of its components. It was assumed, that the systems delay time differs in the form of the probability distribution of the variable h (Weibull, rectangular, normal distributions), but the expected value of the variable is the same ($E(h) = 35$). Analysis of the delay time period, assuming the different forms of probability distributions of variable h, allows assessing:

- if the form of the probability distribution of the random variable h *is important for* system performance in terms of obtained economic and availability results, and hence - if the knowledge of the form of its probability distribution function is necessary for the proper selection of time between inspections T,

- what kind of the economical and reliability consequences can arise in a maintained system in the case of incorrect estimation of parameters describing the variable *h*.

The Figures 4 – 9 show the economic results and availability ratio level of three-element system performing according to BIP policy depending on the expected value E(*h*), the length of the period *T*, and the type of the probability distribution of the random variable *h*. Moreover, the presented analysis results refer to the technical system performing in two extreme reliability structures "*1-out-of-3*" (parallel reliability structure – bright markers) and "*3-out-of-3*" (serial reliability structure - dark markers).

For all the three investigated cases, the obtained economic results are very similar regardless of the type of the probability distribution of the variable *h*. Both parameters: the time between maintenance action performance (*T*) and the expected value of time delay (E(*h*)) have a significant impact on the level of expected maintenance costs. However, obtained results rather do not depend on the type of the probability distribution of the variable *h*. Areas of maintenance costs for both systems, performing in serial and parallel reliability structures, have the same characteristics, e.g. for the cases of the longest delay time (E(*h*) = *MTTF* = 100) maintenance costs for serial system range 0–200 depending on the desired length of time between inspections *T*, but independently of the type of the probability distribution of the elements delay time *h* (Fig. 4, 6, 8). Exactly the same effect can be observed for the other tested values of E(*h*) in both system's reliability structures – the same maintenance costs boundaries, as well as the same curvature of the maintenance cost surfaces. This fact leads to the conclusion that the optimization process of BIP policy parameters for the analysed system requires no knowledge about the type of the probability distribution of the random variable *h*. Hence, the wrong assumption of the type of probability distribution of the random delay time *h* does not cause significant differences in the system maintenance cost assessment. A similar effect can be observed in the analysis of system availability ratio level (Fig. 5, 7, 9).

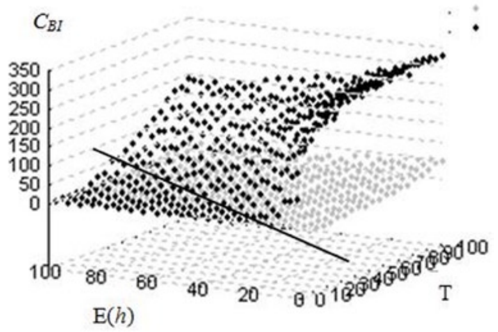

Fig. 4. The expected costs C_{BI} when probability distribution of delay time variable is Weibull distribution

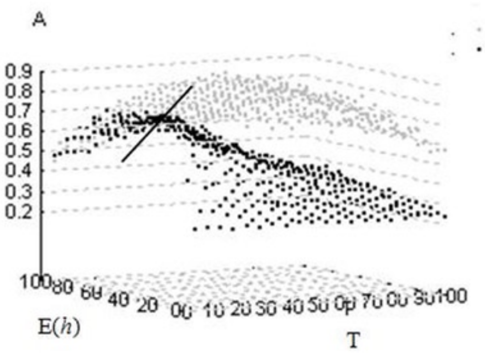

Fig. 5. System availability ratio (A) when probability distribution of delay time variable is Weibull distribution

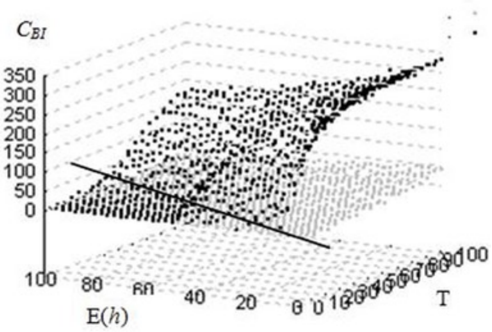

Fig. 6. The expected costs C_{BI} when probability distribution of delay time variable is rectangular distribution

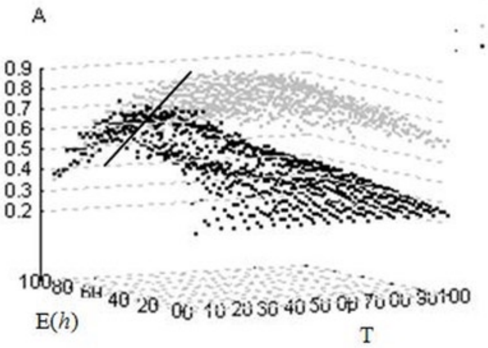

Fig. 7. System availability ratio (A) when probability distribution of delay time variable is rectangular distribution

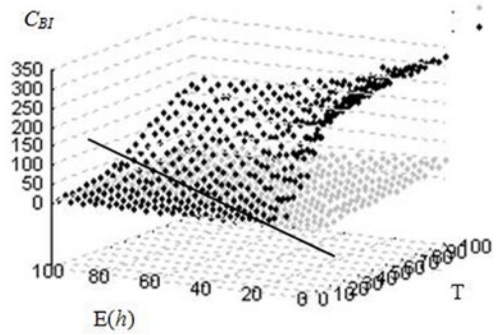

Fig. 8. The expected costs C_{BI} when probability distribution of delay time variable is normal distribution

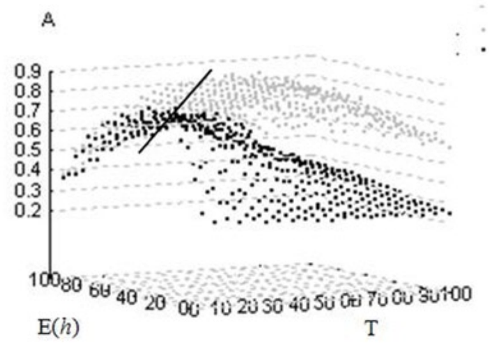

Fig. 9. System availability ratio (A) when probability distribution of delay time variable is normal distribution

The research works [22, 23], which investigate the BIP policy in multi-unit systems performance implementation, determine the relationship between time *T* and the expected value of the variable *h* in systems with serial reliability structure, which is obtained for the lowest maintenance cost and highest system availability:

$$\frac{E(h)}{T} \approx 2 \qquad (6)$$

This relationship is shown in the Figures 4 – 9 as an additional line, near which are the best solutions for the economic and availability criteria and analysed expected values E(h). Expression (6) and the results presented in the Figures show that "nearly optimal" period between inspections of the series system performed according to BIP strategy should be determined on the basis of information about the expected value of the delay time.

In order to confirm the fact that the type of the probability distribution of the random variable h has no significant effect on the obtained economical results of BIP use, the relationship described by equation (6) was subjected to further analysis (Figures 10 – 15). For this purpose, all maintenance cost results and system availability ratios, obtained during the simulation experiments are shown in the form of the relationship E(h)/T. The ratio of E(h)/$T \approx 0$ means systems in which:

- the expected value of the elements delay time is very low (short time of defect occurrence before failure),
- the time between inspections T is very overestimated in the relation to the length of the expected elements delay time (wrongly defined time T).

As it can be seen, in both the investigated cases the system economical results, as well as the availability ratio level are very unfavorable. With the increase of the relation E(h)/T, the economic results seem to improve (maintenance costs decrease), there is also observable an increasing system's availability ratio level. The expected maintenance costs of system C_{BI} reaches a minimum at the values of the E(h)/$T \approx 2$ for technical system performing in series reliability structure. The optimal values of costs C_{BI} for the system operating in parallel reliability structure are observed for the smaller value of the relationship E(h)/T, regardless of the type of the probability distribution of the random variable h. In the case, where E(h)/T >> 2, maintenance costs are still low what is connected with the "safe" maintenance variant including frequent system elements inspections performance. However, the availability ratio decreases due to the existence of unnecessary, redundant maintenance actions performance. The discussed analysis results are the same for all the studied probability distributions characterizing the elements delay times.

To sum up, the obtained results (Figures 10 – 15) confirm the conclusions reached in the works [22, 23] and lead to the conclusion that the optimal length of time between every inspections performance T can be determined even when we do not have complete information about the form of the probability distribution of the random variable h. The basic parameter of BIP model, which must be evaluated as precisely as possible, is the expected value of the delay time E(h). This is due to its significant impact on all of the system analysed results. On the other hand, for the system in which the random variable h is

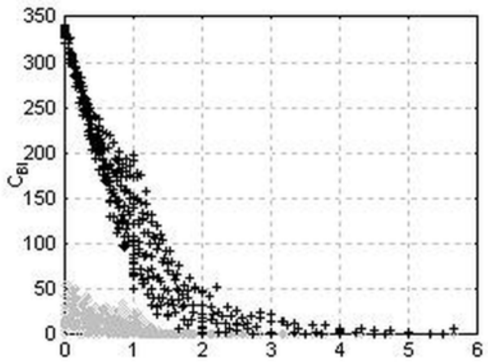

Fig. 10. The expected costs C_{BI} when probability distribution of delay time variable is Weibull distribution

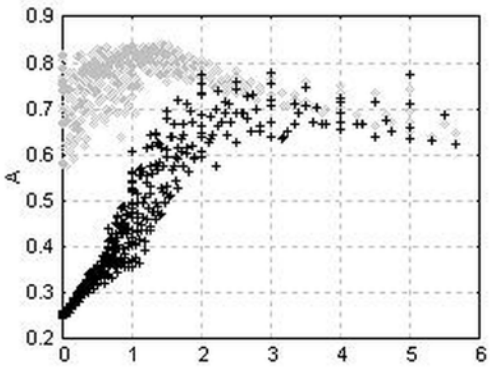

Fig. 11. System availability ratio (A) when probability distribution of delay time variable is Weibull distribution

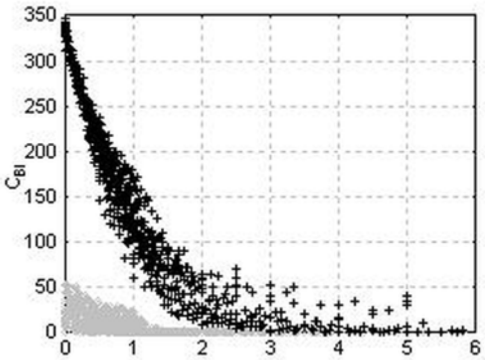

Fig. 12. The expected costs C_{BI} when probability distribution of delay time variable is rectangular distribution

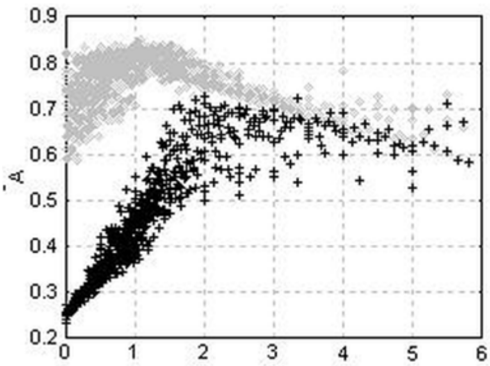

Fig. 13. System availability ratio (A) when probability distribution of delay time variable is rectangular distribution

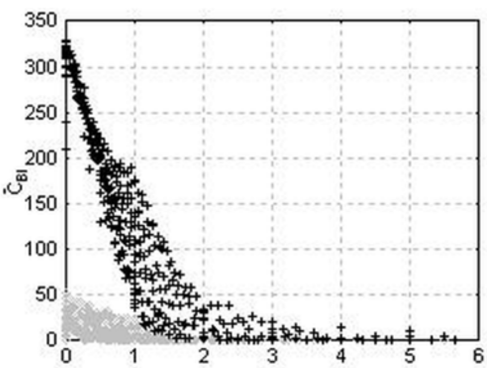

Fig. 14. The expected costs C_{BI} when probability distribution of delay time variable is normal distribution

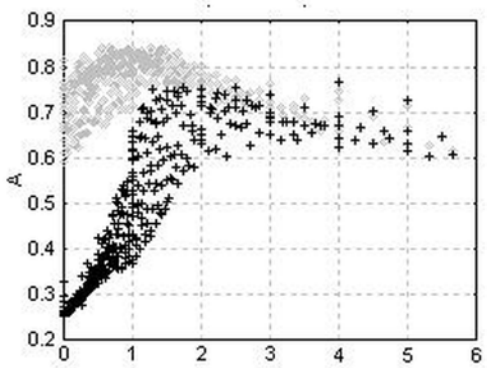

Fig. 15. System availability ratio (A) when probability distribution of delay time variable is normal distribution

described by the rectangular probability distribution, one can detect an economic results variability in the area of their obtained minimum point (Figures 12, 13, E(h)/$T \approx 2$). Unlike the results obtained for other types of probability distributions of the random variable h, even for the relation E(h)/$T > 3$ there are cases where the costs $C_{BI} > 30$. These results confirm that greater dispersion of the values of the variable h (and hence its less predictability) reduces the effectiveness of the BIP strategy implementation. It also means, that if there is a practical possibility, there should be estimated the standard deviation of the random variable – delay time. The exemplary results, obtained for different values of variation coefficient (v) and identical expected value delay time (E(h) = 35) for two probability distributions of the variable h: rectangular and Weibull ones, are presented in the Fig. 16 – 17. These results give us the possibility to conclude, that:

- despite various forms of the probability distributions of random variable h, the economical results are very similar for a similar range of variation coefficient v and time period T,
- when $v = 0$, the system components should be inspected when T = E(h),
- the increasing variation coefficient of the variable h causes, that the "optimum" time between inspections T should be reduced in relation to the expression (6),
- in order to choose the best period between inspections T, one should try to estimate the variability range of the delay time real values, such as the variation coefficient.

The results shown in the Figures 4 – 17 are the basis for the response to the second question defined in the initial part of the chapter relating to the potential consequences of improper estimation of the parameters of the variable h. The consequences level (e.g. financial ones) depends on the level of committed error. Basic observations of obtained results indicate, that the bad estimation of the parameter

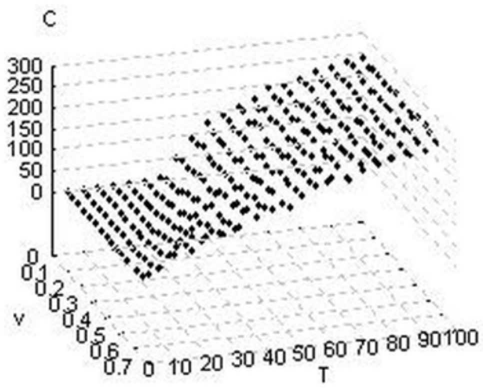

Fig. 16. The expected costs C_{BI} for various values of variation coefficient (v) of delay time parameter when probability distribution of delay time variable is rectangular distribution

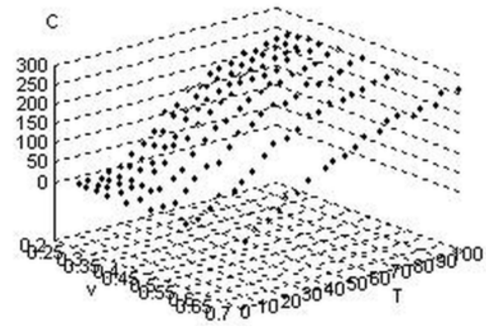

Fig. 17. The expected costs C_{BI} for various values of variation coefficient (v) of delay time parameter when probability distribution of delay time variable is Weibull distribution

E(h) value causes a significant difference between the value of estimated time between inspections T and the value obtained from equation (6). As a result, it can impair the efficiency of the results of BIP strategy implementation. This effect is particularly easy to observe when the expected value of the variable h is overvalued relative to its actual value. Following this, the selected time between inspections T (too long) will cause, that the ratio satisfies the following inequality: E(h)/$T < 2$. Thus, this can cause significant financial and reliability consequences.

4. Summary

The obtained research results of the BIP model analysis allow obtaining a preliminary answer to the question of how the level of delay time parameter estimation may affect the performance of the technical system. The analysis involved observation of the impact of the expected value of the variable h and the forms of the three selected probability distributions of this random variable on the performance level of multi-element technical system in "*k-out-of-n*" reliability structure. There were also analysed certain rules of BIP policy implementation, defined by the authors in [22, 23].

On the basis of this carried out research study, it can be concluded that:

- the main parameter that must be estimated as accurately as it is possible, based on the available statistical data, is the expected duration of the delay time; this parameter unequivocally influences the analyzed cost and reliability results,
- the knowledge about the form of probability distribution of random variable h is important only from the point of view of its dispersion and need not be estimated on the basis of statistical data,
- when there is a possibility to estimate the dispersion of random variable h results, it should be assessed to properly define the time between inspections T,
- there should be conducted further research to determine type of influence of the variation coefficient on optimization formula, described in equation (6).

In the article, authors continue their research related to the DT modelling for multi-unit systems developed in [22, 23, 25, 32]. The next step of our research analysis should be connected with its complementation for assumption of a constant mean value and standard deviation of a variable h. Moreover, authors focus on definition of the DT models implementation possibilities for real technical systems performance optimization (e.g. imperfect maintenance assumption). This will allow defining the basic principles of preventive maintenance policy selection process from the point of view of the person who manages the operational processes of technical system.

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