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Computer model of acoustic link in a pipe with a flowing gas medium,
Part I: Perturbation of ultrasonic transducer directivity pattern

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ABSTRACT

The paper presents an aeroacoustic range equation which makes it possible to elaborate a computer model of ultrasonic link in air for static conditions. The model was made in form of a computer program called KML.

Extending the static model, a simulation of ultrasonic wave propagation in a pipeline with flowing air for different profiles of laminar and turbulent flow and different link parameters was performed. The simulation made it possible to work out a computer model of an acoustic link in flowing gas. The model as a computer program can be used to calculate disturbance of ultrasonic beam rays shape and directivity pattern perturbation. Directivity pattern of ultrasonic transmitting transducer, deformed by flowing medium, becomes non-axial symmetric. As a result of the flow, ultrasonic beam rays are curvilinear and the source directivity pattern depends on rectilinearly measured distance.

INTRODUCTION

A simple ultrasonic transmission system cosisting of a transmitting transducer, a gaseous medium and a receiving transducer can be described with an equation of an aeroacoustic link range. The range depends on ultrasonic transducer parameters and gaseous medium parameters.

Transducer parameters include: size, operational frequency, directivity pattern, supply voltage, transmitting transducer efficiency, receiving transducer sensitivity, noise and disturbances level in the receiving system.

The gaseous medium parameters are as follows: velocity of ultrasonic wave propagation, attenuation, temperature, humidity, pressure, chemical constitution of gas, medium motion, impurity, heterogeneity, noise and disturbance.

The range equation, formulated for hydroacoustic devices was adapted for the description and analysis of the aeroacoustic link (the paper is limited to the considerations concerning air) [3]:

$$\frac{P_T}{P_r} = \left(\frac{4\sqrt{2}\cdot c}{\pi D_T D_r f}\right)^2 d^2 e^{2\alpha d}, \qquad (1)$$

where: P_T - radiation power of transmitting transducer,
P_r - power received by the receiving transducer,
D_T - diameter of transmitting transducer,
D_r - diameter of receiving transducer,
c - velocity of ultrasonic wave propagation in air,
α - amplitude attenuation coefficient in air,
f - operational frequency of transmitting and receiving transducers,
d - distance between transducers in the link.

The equation was the basis for the elaboration of a computer model of the aeroacoustic link for static conditions [1,3,4] in the form of a program called KML. Extending the static model, a simulation of ultrasonic wave propagation in a pipeline with flowing air for different profiles of laminar and turbulent flow and different link parameters was performed. This enabled the authors to make a computer model of an aeroacoustic link in the flowing conditions.

COMPUTER SIMULATION OF THE FLOW

The velocity distribution in the axial-symmetric layers of air flowing through the pipeline can be mathematically modelled [2]. For a laminar and turbulent flow the flowing profile is described by the equation:

$$V(r,\phi) = V(r) = 1 - \left(\frac{r}{R}\right)^{2m} \quad , \qquad (2)$$

where: V(r) - velocity of the air flow in the definite point of the pipeline radius,
R - pipeline radius,
m - smoothing coefficient.

If coefficient m=1, the flow is laminar. For m>1, the coefficient m is the function of Reynolds number and the flow is turbulent. Multiplying the equation (2) by V_{max} (that is the maximum velocity of the medium flowing in the pipeline axis), we get the equation which makes the simulation of any medium flow profile possible. The equation was used in the computer model of the aeroacoustic link in the flowing conditions. Fig.1 presents the examples of flowing profile calculations on the basis of the equation (2) for m=1, R=0.25m and V_{max}=20m/s.

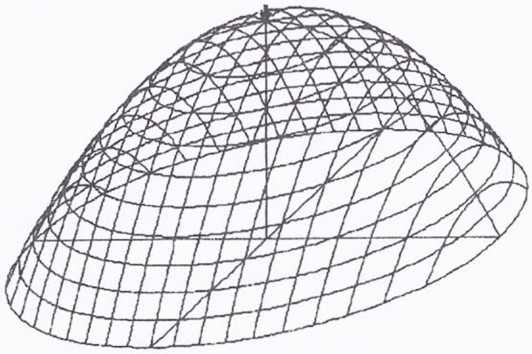


Fig.1. Simulation of laminar flowing profile of air in axial-symmetric pipeline.

Bending of the axial-symmetric pipeline is one of the ways to disturb the flow. The disturbance can be described with the function [2]:

$$V(r,\phi) = \hat{v}\left(1-\frac{r}{R}\right)^{\frac{1}{n}} + m \cdot \frac{r}{R}\left(1-\frac{r}{R}\right)^{\frac{1}{p}} \cdot \phi^2 (2\pi - \phi)^2 \ , \qquad (3)$$

where $\hat{v}$, n, p, m - coefficients determining the disturbance quantity and type.

Having normalised and multiplied the function (3) by V_{max}, the equation was obtained which made it possible to perform a computer simulation of the air flowing profile in the pipeline after the bend. Fig.2 shows the picture of the disturbed flowing profile for $\hat{v}$=2, n=2, p=2, m=2.

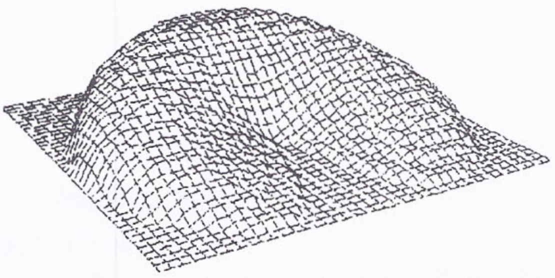


Fig.2. Simulation of the disturbed air flowing profile in the axial-symmetrical pipeline after the bend.

ULTRASONIC BEAM DEVIATION

The rays of ultrasonic beam in the flowing medium undergo deformations thus they are transformed from the rectilinear to curvilinear ones [5]. The trajectories of individual rays can be determined with the use of geometric theory of acoustic field (fig.3, remark: the shown shape of the velocity profile results from the need of a better presentation of geometric calculation method).

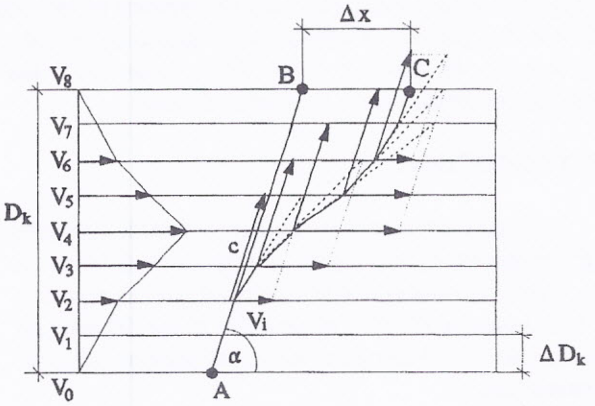


Fig.3. Determination method of the trajectory of one of the ultrasonic beam rays, deflected as a result of the medium flow (Nw=5).

The cross-section area of the axial-symmetric pipeline interior should be divided into Nw concentric axial-symmetric layers (rings). The smallest ring is a point situated in the pipeline axis. The assumptions is valid that in a whole single layer and on its edges, the velocity of the medium flow is a constant value. The layer structure makes that the diameter of the pipeline interior is divided into N=2·Nw-2 sections. Velocity of the flowing medium V_i (where $i=0,1,2,\ldots,N$), calculated from the equation (2) for the axial-symmetric flowing profile can be assigned to each of the sections.

A direct division of the inner diameter of the pipeline into N sections is made in the case of the disturbed profile, which does not show the axial symmetry (equation (3)). The division is justified due to the fact that in each case only the beam rays crossing the pipeline symmetry axis are considered. Marking the section size as ΔD_k it is possible to determined the length of the ray after the deflection from the points A to C (fig.3):

$$L=\sum_{i=0}^{N}\frac{\sqrt{V_i+c^2+2V_ic\cos\alpha}}{c\sin\alpha}\Delta D_k \quad . \tag{4}$$

The time of flight the ultrasonic wave along the distance is the same as the time of flight on a straight line from the point A to the point B with the constant velocity c (fig.3) and it can be calculated from the equation:

$$t=\frac{D_k}{c\sin\alpha} \quad . \tag{5}$$

The shift of the ray Δx (section BC in fig.3) can be determined from the relationship:

$$\Delta x=\sum_{i=0}^{N}\sqrt{\Delta L_i^2-\Delta D_k^2}-D_k\cdot|ctg\alpha| \quad , \tag{6}$$

where ΔL_i is the deflected ray way in i-th layer, which is expressed with the equation:

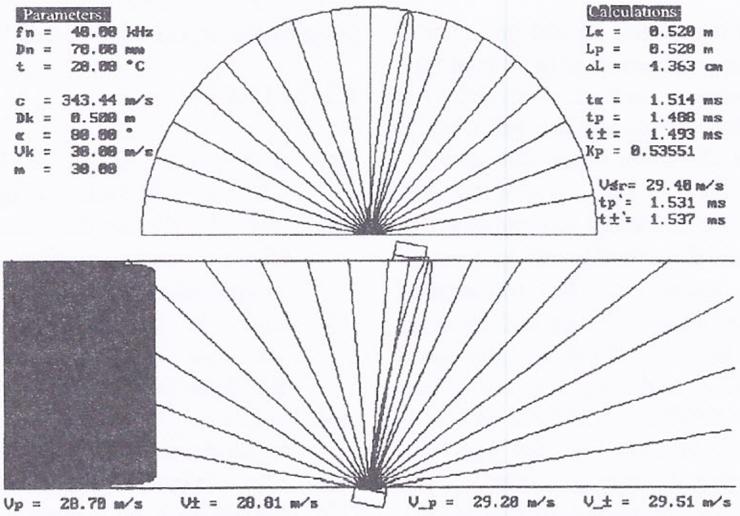
$$\Delta L_i=\frac{\sqrt{V_i+c^2+2V_ic\cos\alpha}}{c\sin\alpha}\Delta D_k \quad . \tag{7}$$

The algorithms presented here were used in the computer model of the aeroacoustic link taking into account the flow.

DISTORTION OF THE DIRECTIVITY PATTERN

If the ultrasonic source is placed in the point A (fig.3) and the pattern of all rays of the ultrasonic beam radiating in the direction of the point at the α angle (fig.3) are determined taking into consideration the flowing medium, the directivity pattern will undergo transformation. The source directivity pattern, distorted as a result of the flowing medium becomes non axial symmetric and unfortunately it depends on the distance from the source, which is rectilinearly measured. One of the ways to normalise the directivity pattern to assess in a visual way the acoustic pressure changes in the receiving point B (fig.3) is to adopt the length of the curve AC as a standard of an equal unity. To determine a curvilinear beam ray linking points A and B (fig.3) consists in finding such a rectilinear beam ray before the shift that after the shift we will get $\Delta x=0$ (equation (6)). The pressure drop K_p (figs.4 and 5), defined for such a rectilinear ray for the directivity pattern from the source directivity pattern, will determine an actual pressure change in the receiving point B (fig.3).

The procedure presented here, was used in the computer model applying appropriate relationship to determine source directivity pattern [1]. The examples of the program print-out are shown in figs.4 and 5. The flowing profile, presented in fig.5, is only of a demonstrative character due to the velocity of the air flow. However for such a high velocity it is possible to observe the curvature of the ultrasonic beam rays and the way of directivity pattern deformation.



```
Parameters:
fn =  40.00 kHz
Dn =  70.00 mm
t  =  20.00 °C

c  = 343.44 m/s
Dk =  0.500 m
α  =  80.00 °
Vk =  30.00 m/s
m  =  30.00

Calculations:
Lα =   0.520 m
Lp =   0.520 m
ΔL =   4.363 cm

tα =   1.514 ms
tp =   1.400 ms
t± =   1.493 ms
Kp = 0.53551

Vśr= 29.40 m/s
tp`=  1.531 ms
t±`=  1.537 ms

Vp =  20.70 m/s    V± =  20.81 m/s    V_p =  29.20 m/s   V_± =  29.51 m/s
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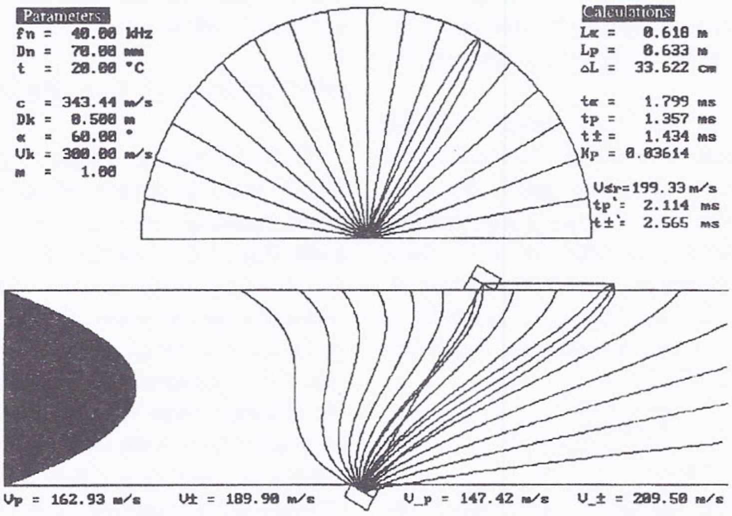
Fig.4. Calculation results obtained by means of the computer model (V_k=30m/s, α=80°).



```
Parameters:
fn =  40.00 kHz
Dn =  70.00 mm
t  =  20.00 °C

c  = 343.44 m/s
Dk =  0.500 m
α  =  60.00 °
Vk = 300.00 m/s
m  =   1.00
```

```
Calculations:
Lα =   0.618 m
Lp =   0.633 m
ΔL =  33.622 cm

tα =   1.799 ms
tp =   1.357 ms
t± =   1.434 ms
Kp = 0.03614

Vśr=199.33 m/s
tp`=  2.114 ms
t±`=  2.565 ms
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Vp = 162.93 m/s     V± = 189.90 m/s          V_p = 147.42 m/s    V_± = 209.50 m/s
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Fig.5. Calculation results obtained by means of the computer model (V_k=300m/s, α=60°).

CONCLUSIONS

Due to the fact that there is no a coherent mathematical model which describes turbulence it was assumed that they are a medium flow disturbance of a random character and they bring about accidental oscillation of ultrasonic beam shift around a certain value (fig.6) [5].

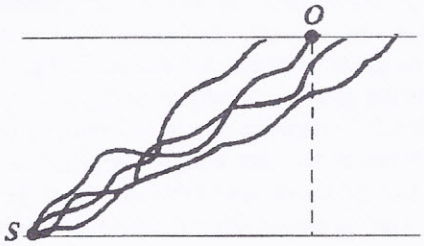

Fig.6. Random shift of ultrasonic beam brought about by turbulence.

The errors, caused by turbulence should be minor if the link parameter values are measured in an appropriately long time interval and then are averaged. The assumption is valid that the turbulence influence is little in the computer elaboration of the link model.

The computer model of the aeroacoustic link including the flow makes it possible to present undisturbed and disturbed air flow profile and graphically determine directivity pattern of the transmitting transducer placed in the connector pipe and radiating the ultrasonic wave to the axial-symmetric pipeline at given angle α with regard to the pipeline axis. There is an assumed air flow profile, established earlier, in the pipeline. Moreover in each case the following magnitudes are determined, among others, sound velocity in air (c), distance between transducers (L_α), length of the ultrasonic wave way from the centre of the transmitting transducer to the receiving transducer calculated on the curvature of an appriopriate ray (L_p), dislocation of the central ray of the beam due to the flow (ΔL), the acoustic pressure drop on the ray reaching the receiver taking into consideration the flow (K_p), the mean air flow velocity in the pipeline calculated by averaging the flow profile ($V_{śr}$) for the given pipeline parameters (diameter of the pipeline D_k), air (temperature t), flow (maximum flow velocity V_k, smoothing coefficient m) and parameters of ultrasonic transducers (operational frequency of the transducers f_n, diameter of the transducers D_n). The computer model of the aeroacoustic link, which takes into account the flow, can be extended in future by appropriate options making it possible to calculate the pressure level on the receiving transducer surface at an assumed value of the transmitting transducer efficiency, taking into consideration attenuation of ultrasonic waves in the medium and losses which occur at crossing the layers of different flow velocities.

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