The solutions similarity of the similar conflicts

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The work deals with the examination of solutions similarity of similar conflicts, presented in the form of multiperson cooperative games. There is examined the similarity of the two most well-known in the literature concepts of the cooperative game solutions: Shapley solution and *nucleolus* (Schmeidler solution [9]). The work presents an idea of using a solution of the pattern conflict most similar to the considered conflict as a its solution.

Keywords: model of conflict, cooperative game, similarity of conflicts, ideal game, pattern recognition.

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1. Introduction

The genesis of the considerations set out in the work is the thesis that similar conflicts have similar solutions. Area of research is the class conflicts that can be modeled as a multiperson cooperative game [2, 4, 5]. Multiperson cooperative game, very often can be presented as follows:

$$\Gamma = \left(\mathcal{N}, v\right) \tag{1}$$

Where $\mathcal{N} = \left(1, 2, ..., n, ..., N\right)$ a set of numbers of conflict sides (players).

$v : 2^{\mathcal{N}} \to \mathcal{R}$ is the characteristic function of the game [2, 3], which assume that is *essential*, and at least *super-additive* [2, 4, 5]. The characteristic function (the mathematical model of a conflict situation) is a function of the type:

$$v\left(S\right) \in \mathcal{R}, S \in 2^{\mathcal{N}} \tag{2}$$

The value $v\left(S\right)$ is interpreted as the maximum, aggregate, guaranteed payout (profit, effect) for members of the coalition $S \subset \mathcal{N}$ [2, 5]. For $S = \varnothing$ we assumed $v\left(\varnothing\right) = 0$. Very often, conflict model as a characteristic function (2) is presented in the convention of the graphic model class, for example a typical *web model* (*radar model*) [1, 3, 5]. In the work, because of the simplicity of interpretation as well as the simplicity of the necessary calculations was adopted simplest model of the similarity of objects so-called the similarity metric model.

2. The similarity of conflicts

About two conflicts K_1 i K_2 we say that they are similar, if their models $M\left(K_1\right)$ and $M\left(K_2\right)$ are similar in a certain sense [1, 9]. If models of conflicts K_1 and K_2 are cooperative games $\Gamma_1 = \left(\mathcal{N}, v_1\right)$, $\Gamma_2 = \left(\mathcal{N}, v_2\right)$, we say that they are similar if v_1 is similar to v_2. The models v_1 i v_2 can be treated as elements Euclidean space (see (2) and the assumption that $v\left(\varnothing\right) = 0$).

We therefore save $v_1, v_2 \in \mathcal{R}^{2^N - 1}$, so $v_1, v_2 \in \mathcal{R}^7$. To determine the similarity of conflicts v_1, v_2, to utilize many other models of similarity [1, 2, 9]. The simplest but very "non--exact" model of similarity is the metric similarity model. We say that objects v_1, v_2 are metrically similar if $d\left(v_1, v_2\right) \le \varepsilon$.

Where $d\left(v_1, v_2\right)$ – distance from v_1 to v_2 (for instance the Euclidean distance or Minkowski distance) [1, 3, 6, 9].

$\varepsilon > 0$ – the conventional threshold of similarity.

The classic similarity model of objects is a Tversky model [1, 9]. The objects a and b we say that they are similar in the Tversky sense if

$$p\left(a,b\right) =$$
$$= \gamma f\left(A \cap B\right) - \alpha f\left(A - B\right) - \beta f\left(B - A\right) \ge t$$

where $p(a,b)$ – an indicator of the Tversky similarity [1, 9]

A, B – sets of attributes of objects a and b

$t \geq 0$ – limit parameter of similarities

α, β, γ – coefficients of features, importance for common features of objects and for different features of these objects [9]. The function $f(S)$ determines the degree of similarity resulting from the set $S \subset A \cup B$ of features. A more general model of similarity is the multi-criteria model, which in short can be represented as follows. The assessment of the similarity of objects a and b is a vector: $F(a,b) = (F_1(a,b), F_2(a,b), F_3(a,b))$ for $(a,b) \in X \times X$ ((X – a set of objects) where

$$F_1(a,b) = \gamma f(A \cap B)$$

$$F_2(a,b) = \alpha f(A - B)$$

$$F_3(a,b) = \beta f(B - A)$$

About the objects a and b we say that they are similar if

$$F_1(a,b) \geq t_1$$

$$F_2(a,b) \leq t_2$$

$$F_3(a,b) \leq t_3$$

where

t_1, t_2, t_3 – the limit thresholds of similarity resulting from the weights of common features and distinctive features of the objects a and b. A special case of this model is the classic Tversky model [1, 3, 9]:

$p(a,b) = F_1(a,b) - F_2(a,b) - F_3(a,b)$. Such formulation of the problem allows you to easily define the relation of similarity detection R :A pair of objects (a,b) we say that it is more similar (to each other) than a pair (c,d) of what we write $((a,b),(c,d)) \in R$ when:

$$F_1(a,b) \geq F_1(c,d)$$

$$F_2(a,b) \leq F_2(c,d)$$

$$F_3(a,b) \leq F_3(c,d)$$

If we assume that $b = d = x$, this notation will mean $((a,x),(c,x)) \in R$ that the object a is more similar to the pattern x than the object *c*. This is the task of object recognition (classification according to the pattern [1, 3, 4]).

An alternative formulation is a formulation of the task of pattern recognition [1, 3, 4]. Make it certain objects b, d from set of patterns, and $a = c = x$ (*x* the recognized object, in terms of similarities to the patterns). This notation $((x,b),(x,d)) \in R$ will mean that the respondent ("unidentified"), object x is more similar to the pattern b than to the pattern *d*. It will so when

$$F_1(x,b) \geq F_1(x,d)$$

$$F_2(x,b) \leq F_2(x,d)$$

$$F_3(x,b) \leq F_3(x,d)$$

Below, is a simple example which allows to compare the similarity of three games, defined on the basis of various types of similarity between objects [1, 2, 5]. All games will be converted to the (0,1) standard equivalent game [4, 5, 7, 10].

Example 1

There are three 3-person games : $\Gamma_1 = (\mathcal{N}, v_1)$, $\Gamma_2 = (\mathcal{N}, v_2)$, $\Gamma_3 = (\mathcal{N}, v_3)$,

where

$\mathcal{N} = \{1,2,3\}$ and $v_k : 2^{\mathcal{N}} \to \mathcal{R}$

and the characteristic functions are presented in the form of Table 1.

Tab. 1

No.	0	1	2	3	4	5	6	7
S	$\varnothing$	1	2	3	1,2	1,3	2,3	1,2,3
$v_1(S)$	0	3	2	2	6	7	6	10
$v_2(S)$	0	1	3	1	7	5	5	8
$v_3(S)$	0	2	4	1	7	5	6	9

The similarity of conflicts v_k, v_l, we can define on the basis of their distance $d(v_k, v_l)$. After completion of the calculations (Euclidean distance) was obtained:

1. $d(v_1, v_2)$ = 3,87
2. $d(v_1, v_3)$ = 3,46
3. $d(v_2, v_3)$ = 2,00

We can therefore say that the conflict v_1 is more similar to the conflict v_3 than the conflict v_2, and that conflict v_3 is more similar to the conflict v_2 than to the conflict v_1. The most similar to each other there are the v_2 and v_3

conflicts, then ν_1 and ν_3 and the least – conflicts ν_1 and ν_2. For further calculations because of the need for standardization of similarity indices they will be used for equivalent games normalized to the form (0,1) [2, 3], presented in Table 2.

Tab. 2

No.	0	1	2	3	4	5	6	7
S	$\varnothing$	1	2	3	1,2	1,3	2,3	1,2,3
ν_1	0	0	0	0	1/3	2/3	2/3	1
ν_2	0	0	0	0	1	1	1/3	1
ν_3	0	0	0	0	1/2	1	1/2	1

Metric similarity indicator of conflicts ν_k, ν_l it will be calculated by formula [1, 6, 9]:

$$p(\nu_k, \nu_l) = 1 - \frac{1}{\sqrt{N}} \left\| \nu_k - \nu_l \right\|_2 \qquad (3)$$

The rest of the work will be devoted to research the solution similarity of similar conflicts in terms of the possibilities of using conflict resolution model (solution "model") [2, 5] as a solution to the conflict considered.

3. Analysis of the solutions similarity of similar conflict

In theory, cooperative multiplayer game known many very different concepts of conflict resolution [1, 3, 6, 7]. The best known and most commonly used is a solution Shapley [3, 7] and the so-called nucleolus (solution Schmeidler) [4, 6, 9]. Let – person rooms, critical and super-additive cooperative game [2, 5].

Definition 1

Nucleolus $n(\nu)$ (*N-kernel* game or Schmeidler solution) is called solution multiperson cooperative, resulting solution sequences properly formulated [7, 8] tasks to minimize so-called the *excess* $e(S, x)$.

Now it is not known analytical form of the solution (attempt to present analytical form of the solution for games 3 persons is contained in [7]). The solution $n(\nu)$ obtained as a result of a very complex and tedious calculations, however, has very good properties [4, 8]. It exists for each cooperative game, it is the unique and it is stable in coalition sense by the fact that it belongs to the C-core game unless it is empty [7, 8]. Below, will be considered an example of examining the similarity of Shapley and Schmeidler solutions for ten randomly selected 3-person games in a standardized form [3, 4].

Let $V(3)$ – a set of all the 3-person game normalized to (0,1) form [4, 5, 7]. $V_C(3) \subset V(3)$ – a subset of games with non-empty C-core.

$V_\varnothing(3) \subset V(3)$ – a subset of the game with an empty C-core.

$V_*(3) \subset V(3)$ – a subset of games with one-element C-core (set of ideal games (patterns) [1, 3, 5]).

$\Gamma_k \in V(3)$, $k = 1, 2, \ldots, 10$ – game number *k*, drawn from the $V(3)$ set.

Tables 3 and 4 contain data about the characteristic functions of randomly selected games and the designated the Shapley and Schmeidler solutions [3, 7, 8]. Table 3 presents the values of the normalized characteristic functions of the tested games (columns: the second, third and fourth) and the characteristic functions of the nearest (most similar) pattern game (ideal game) [2, 3, 4], designated in accordance with the task of minimizing the distance – the columns: the fifth, sixth and seventh.

Figure 1 shows the similarity of the chosen conflict to the other conflicts.

The game $\Gamma_\circ = \left(\mathcal{N}, \overset{\circ}{\nu} \right)$ is the chosen game, which will be similar compared to other games (as well as the similarity of its solutions). In Table 4 were stored value of Shapley solutions, (see Definition 1 and Schmeidler solutions (*nucleoluses*) (see Definition 2) considered games.

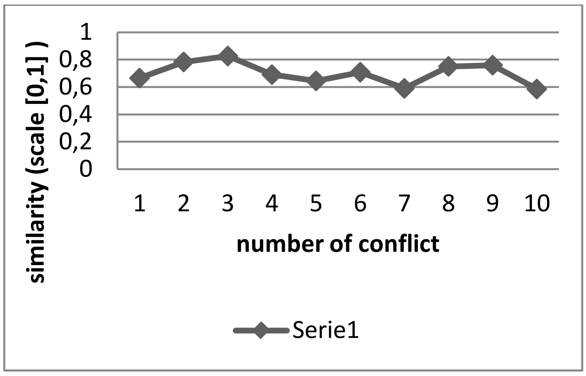

Fig. 1. The similarity of the chosen conflict to the other conflicts

Tab. 3

Γ_k	1,2	1,3	2,3	1,2	1,3	2,3
1	0,17	0,67	0,50	0,39	0,89	0,72
2	0,25	0,33	0,17	0,67	0,75	0,58
3	0,33	0,25	0,00	0,81	0,72	0,47
4	0,25	0,13	0,33	0,68	0,56	0,76
5	0,75	0,50	0,75	0,75	0,50	0,75
6	0,56	0,67	0,78	0,55	0,66	0,77
7	0,75	0,50	0,88	0,71	0,46	0,83
8	1,00	0,75	1,00	0,75	0,50	0,75
9	0,50	1,00	0,25	0,63	1,00	0,37
10	0,33	0,25	0,92	0,54	0,46	1,00
$\overset{\circ}{\nu}$	0,33	0,66	0,33	0,78	0,66	0,56

Tab. 4

Γ_k	Φ_1	Φ_2	Φ_3	n_1	n_2	n_3
1	0,31	0,23	0,46	0,33	0,17	0,50
2	0,37	0,30	0,33	0,33	0,33	0,34
3	0,43	0,31	0,26	0,33	0,33	0,34
4	0,29	0,38	0,33	0,33	0,33	0,34
5	0,29	0,41	0,30	0,25	0,50	0,25
6	0,28	0,34	0,38	0,22	0,34	0,44
7	0,25	0,44	0,31	0,17	0,54	0,29
8	0,30	0,40	0,30	0,25	0,50	0,25
9	0,50	0,12	0,38	0,75	0,00	0,25
10	0,12	0,46	0,42	0,33	0,33	0,34
$\overset{\circ}{\nu}$	0,38	0,24	0,38	0,51	0,16	0,33

In the Table 4 the columns mean: the second, third and fourth this is the Shapley solution. Columns: fifth, sixth and seventh this is the Schmeidler solution. Using the Euclidean distance function $d(\nu_k, \nu_l)$ has been designated appropriate metric similarity of the formula (3). $p(\nu_k, \nu_l) = 1 - \frac{1}{\sqrt{N}} \|\nu_k - \nu_l\|_2$ and the similarity of their solutions response in the Shapley and Schmeidler sense.

Figure 2 shows the similarity of compromise solution [1, 4] of the chosen game to the others. Figure 3 and Figure 4 show similarity of the Shapley and Schmeidler solutions.

Figure 5 shows comparison similarity all of solutions and conflicts.

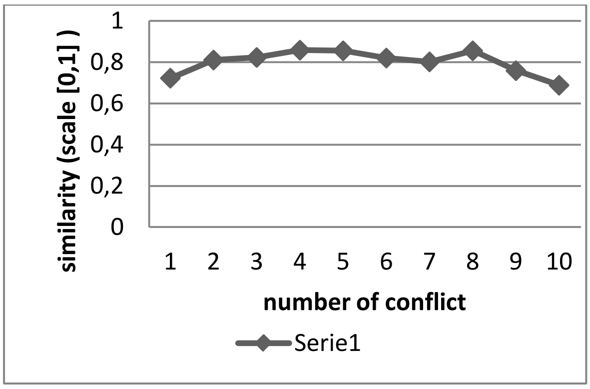

Fig. 2. The similarity of the compromise solution [4] of the chosen conflict to the compromise solution of the other conflicts

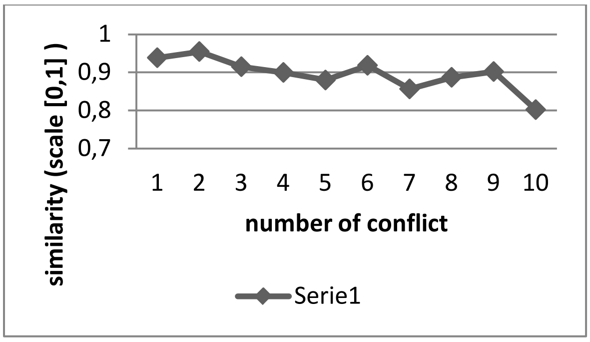

Fig. 3. The similarity of the Shapley solution of the chosen conflict to the Shapley solution of the other conflicts

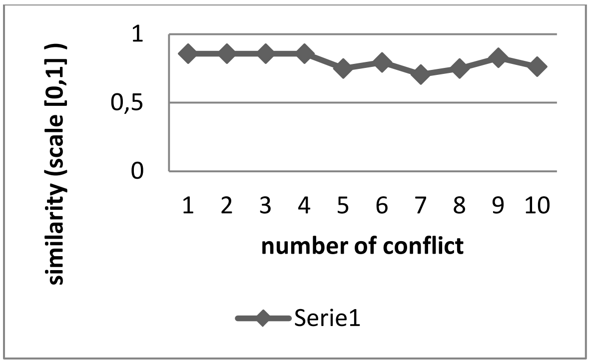

Fig. 4. The similarity of the nucleolus of the chosen conflict to the nucleolus of the other conflicts

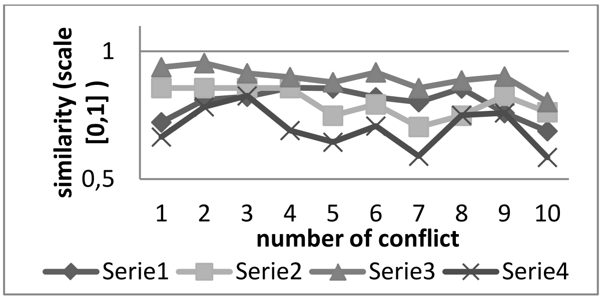

Fig. 5. The comparison of the similarities of conflicts and their solutions

4. Pattern recognition methodology as a method the games solving

The idea of the method use solution of conflict most similar to the considered conflict is solved as follows. A set of the best games (ideal games $V_*(N)$ [1, 4]) is a set of special conflict models. This "specificity" lies in the fact that each game has a solution that is unique and stable in the sense of belonging to the C-core game, unless they not empty. An additional very important feature (as opposed to the Shapley and Schmeidler solutions) is that, it is determined by a very simple method. For 3-person games it is enough to solve simple equations:

$$x_1 + x_2 = \overset{*}{\nu}(1,2)$$

$$x_1 + x_3 = \overset{*}{\nu}(1,3)$$

$$x_2 + x_3 = \overset{*}{\nu}(2,3)$$

where

$x_n \in [0,1], n \in \{1,2,3\}$ – game solution [2, 4], provided that the game $\Gamma_* = \left(\mathcal{N}, \overset{*}{\nu}\right)$ is most similar metrically game from the patterns set $V_*(N)$ for the dissolved game $\Gamma = (\mathcal{N}, \nu)$.

Optimization problem can be formulated as follows: appoint such a ideal game $\overset{\circ}{\nu} \in V_*(N)$, that

$$\left\| \nu - \overset{\circ}{\nu} \right\|_2 = \min_{\overset{*}{\nu} \in V_*(N)} \left\| \nu - \overset{*}{\nu} \right\|_2 \qquad (4)$$

The method of solution of the problem (4) for the 3-person game is very simple (uses the theorem about orthogonal projection of ν function to the set $V_*(N)$ [1, 4]. For game 3-personal information as described in Table 3 will receive the appropriate ideal game $\overset{*}{\nu}_k, k = 1, 2, \ldots, 10$ (the data in Table 3).

Table 5 presents the solutions of most similar games of respondents $\Gamma_k = (\mathcal{N}, \nu_k), k = 1, 2, \ldots, 10$ which may be adopted as a solution to pending conflicts. They are the compromise solutions [1, 4]. Figure 2 shows the similarity of the compromise solution of the chosen game to the analogous compromise solutions other games.

Tab. 5

Γ_k	1,2	1,3	2,3	x_1^p	x_2^p	x_3^p
1	0,17	0,67	0,50	0,28	0,11	0,61
2	0,25	0,33	0,17	0,42	0,25	0,33
3	0,33	0,25	0,00	0,53	0,28	0,19
4	0,25	0,13	0,33	0,24	0,44	0,32
5	0,75	0,50	0,75	0,25	0,50	0,25
6	0,56	0,67	0,78	0,22	0,34	0,44
7	0,75	0,50	0,88	0,17	0,54	0,29
8	1,00	0,75	1,00	0,25	0,50	0,25
9	0,50	1,00	0,25	0,63	0,00	0,37
10	0,33	0,25	0,92	0,00	0,54	0,46
$\overset{\circ}{\nu}$	0,33	0,66	0,33	0,44	0,34	0,22

5. Summary

The considerations as well as the results obtained confirm the thesis about the possibility of using solutions similar games as a method of solving games. The study analyzed compounds the similarity metric models of gaming presented in the form of so-called characteristic function. The results, especially the results obtained using pattern recognition are extremely interesting because of the properties of solutions of games model (ideal) coming of games solved. Properties of these solutions is devoted among other works [2, 5, 7]. Analogical tests could be performed using the more advanced models of similarity based on the three-step similarity Tversky model or similarity multi-criteria models [1, 3, 9]. Received results based on a simple model of the metric similarities entitle to optimistic expectations regarding the application of more advanced similarity models.

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Podobieństwo rozwiązań konfliktów podobnych

A. AMELJAŃCZYK

Artykuł dotyczy badania podobieństwa konfliktów podobnych, przedstawianych w postaci wieloosobowych gier kooperacyjnych. Zbadano podobieństwo dwóch najbardziej znanych w literaturze koncepcji rozwiązań wieloosobowych gier kooperacyjnych: rozwiązania Shapleya oraz *nucleolusa* (rozwiązania Schmeidlera) [9]. Przedstawiono ideę wykorzystania rozwiązania najbardziej podobnego konfliktu wzorcowego do rozwiązania konfliktu badanego.

Słowa kluczowe: model konfliktu, podobieństwo konfliktów, gra idealna, rozpoznawanie wzorców konfliktów.