

# A Study of the Efficiency of Polish Foundries Using Data Envelopment Analysis

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## Abstract

The article presents a study on the effectiveness of the foundries using Data Envelopment Analysis (DEA) method. The aim of the article is to analyze the usefulness of DEA method in the study of the relative efficiency of the foundries. DEA is a benchmarking technique based on linear programming to evaluate the effectiveness of the analyzed objects. The research was conducted in four Polish and two foreign plants. Evaluated foundries work in similar markets and have similar production technology. We created a DEA model with two inputs (fixed assets and employment) and one output (operating profit). The model was produced and solved using Microsoft Excel together with its Solver add-in. Moreover, we wrote a short VBA script to perform automating calculations. The results of our study include a benchmark and foundries' ranking, and directions to improve the efficiency of inefficient units. Our research has shown that DEA can be a very valuable method for evaluating the efficiency of foundries.

**Keywords:** Benchmarking, Data Envelopment Analysis, Application of information technology to the foundry industry

## 1. Introduction

Evaluation of the effectiveness of manufacturing or service system is usually done by means of a ratio which is the quotient of the results of the object by its inputs. Such calculation is not simple, if we are dealing with multiple inputs and outputs, especially of various kinds. To avoid an arbitrary choice of weights for each inputs and outputs, Charnes, Cooper, and Rhodes [1] introduced a non-parametric analysis method known as Data Envelopment Analysis (DEA). DEA was primarily designed to evaluate the efficiency of service institutions, as many of their inputs and effects have no price expression. Since that classic article, DEA has proved its usefulness in the analysis of many industries and services, including banks, power plants, hospitals, food production [4, 6]. DEA is a benchmarking technique based on linear programming to evaluate the effectiveness of the analyzed Decision Making Units (DMUs). A measure of this

assessment determines the position of the DMU in the ranking, and the effective DMUs determine efficient production frontier. DEA assesses the relative efficiency of units, which transform multiple inputs into multiple outputs.

In this paper we focus on DEA approach for efficiency evaluation of foundries. Section 2 presents the conception of DEA method and the formulation of DEA as a linear programming model. In Section 3 the details of spreadsheet model for determining the efficiency of examined foundries are given. Section 3 provides also results of benchmarking and objects ranking, as well as directions to improve the efficiency and the structure of technology for inefficient objects. The conclusions are drawn in Section 4.

## 2. Data Envelopment Analysis

DEA is a popular method for comparative efficiency analysis that allows to find the most efficient objects in a test set without determining the functional relationship between inputs and outputs. The basic assumption is that each DMU uses the same kind of inputs to produce the same kind of outputs. It is also assumed that the costs and results are non-negative, and at least one result and one edition of a DMU is positive. The amounts of inputs and outputs vary DMUs and provide a basis for the evaluation of the comparative efficiency.

Input-output relations of most efficient DMUs form a so-called "envelopment surface", often described as the "efficient frontier". By measuring the distance of a particular DMU's relation to this efficient frontier, we obtain an assessment of the efficiency of considered DMU. The result is a single number, but taking into account all inputs and all outputs. The basic idea allowing the DEA to combine multiple inputs and outputs in a single score is determining properly scaled weights for all inputs and outputs. Weights are treated as variables in the Linear Programming (LP) model, and its solution gives the numerical values of all weights (and thus - efficiency ratio).

DEA offers many models differing in assumptions and destination. The most popular one is CCR (from the names of its authors – A. Charnes, W. Cooper and E. Rhodes) model, which can occur in two basic variants: as input-oriented or as output-oriented. In the next part input-oriented CCR model will be discussed. The aim of the input oriented model is to find how to reduce the inputs of non-efficient units in order to reach the efficient frontier.

Linear programming model for input-oriented CCR can be formulated as follows [3]:

$$\begin{aligned} &\text{Minimize } \theta_o & (1) \\ &\text{subject to} \end{aligned}$$

$$\sum_{j=1}^J \lambda_{oj} x_{ij} \leq \theta x_{io}, \quad i = 1, \dots, I \quad (2)$$

$$\sum_{j=1}^J \lambda_{oj} y_{rj} \geq y_{ro}, \quad r = 1, \dots, R \quad (3)$$

$$\theta_o \leq 1 \quad (4)$$

$$\lambda_{oj} \geq 0, \quad j = 1, \dots, J \quad (5)$$

Where:

$\theta_o$  an efficiency score of analysed  $o$ -th DMU, i.e. variable expressing the reduction rate of inputs in order to reach the efficient frontier,

$\lambda_{oj}$  weight assigned to unit  $j$ ,

$x_{ij}$  amount of input  $i$  for unit  $j$ ,

$y_{rj}$  amount of output  $r$  for unit  $j$ .

The decision variables are  $\theta_o$  and the set of  $\lambda_{oj}$ ; it should be emphasized that this model must be solved for each unit separately.

The goal (1) is to find the efficiency of DMU being evaluated. Constraint (2) provides that the weighted sum of DMUs' inputs are no greater than  $\theta$ -part of the inputs incurred by  $o$ -th unit. According to constraint (3) the weighted sum of DMUs' outputs are not less than the results achieved by  $o$ -th object. Constraint (4)

ensures that the efficiency score cannot be greater than 1. Constraint (5) provides that all weights are non-negative.

The DMU is efficient if the following two conditions hold:

1. The optimum value of the variable  $\theta_o^*$  is equal to 1.
2. The values of  $\lambda$  for all other units are equal to zero.

$\theta_o < 1$  means that the optimal inputs needed to obtain such results, which were observed in the examined object, are less than the inputs actually incurred by this object. In consequence, the object is not fully effective, and the degree of its inefficiency is equal to  $1 - \theta_o$ .

A solution of CCR model can determine [3]:

- efficient and inefficient DMUs,
- ranking of inefficient units,
- optimal technologies<sup>1</sup> and benchmarking formula for inefficient objects,
- excess inputs and deficits result in inefficient units,
- the type of economies of scale,
- target (optimal) technologies for inefficient objects,
- the structure of the optimal technology,
- sensitivity of the problem to changes in inputs and outputs.

## 3. Example of DEA analysis for foundry industry

The review of the literature concerning DEA method, has shown that there are no reports on the practical applications of DEA for the foundry industry.

### 3.1. Data for analysis

Due to the limited availability of data we could examined only six foundry units: four Polish and two foreign ones. Evaluated foundries work in similar markets and have similar production technology. The data were taken from 2012 year. The 2012 was chosen, as we had the full data from this year for all units and all units conducted only casting activities (in the following years Polska Grupa Odlewnicza SA, Automotive Components Europe SA and Componenta Corp. have extended their activities to other sectors). We considered two inputs: fixed assets and employment, and one result – operating profit. The names of the foundries and the values of the variables are given in Table 1. Employment is measured in workers, the other variables – in thousand PLN (for foreign entities these values have been calculated according to average annual exchange rate of EUR/PLN in 2012).

<sup>1</sup> In terms of DEA method, technology is understood as DMU's vector of empirical inputs and results

Table 1.  
Data characteristics

Company	Fixed Assets	Employees	Operating Profit
Zetkama SA	53,200.0	353	12,091.0
Odlewnie Polskie SA	35,531.9	323	6,644.1
Odlewnia Żeliwa SA	58,257.8	466	93.6
PGO SA	143,375.0	921	27,254.0
ACE SA (Luxemburg)	197,277.8	777	12,794.2
Componenta Corp. (Finland)	1,246,352.6	4,642	41,852.0

### 3.2. Spreadsheet model

Although specialized DEA software, both free and commercial, are available, solution for the presented model can be easily achieved using a spreadsheet such as Microsoft Excel together with its Solver add-in [5]. Figure 1 shows a screen capture of model created by the authors and formulated as described below.

The identification number of analysed DMU is entered in cell E10. This value, as a parameter of INDEX function, is used to read the inputs and outputs of the corresponding unit. The worksheet then uses that data to insert in rows 11-13 the values of the unit input variables multiplied by the efficiency score, and the output variables (the right-hand sides of Equations 2 and 3). The composite units' inputs and outputs consist of the sum of the weight on each unit multiplied by its inputs and outputs (the left-hand sides of Equations 2 and 3). For example (DMU index=1), the formula in cell C11 is =SUMPRODUCT(C2:C7;\$F\$2:\$F\$7), and in E11 = \$C\$14\*INDEX(\$C\$2:\$D\$7;\$E\$10;1). The efficiency factor (cell C14) and the weights of each DMUs (F2:F7) are the decision variables.

	A	B	C	D	E	F
1	No.	Company	Fixed Assets	Employees	Operating Profit	$\lambda$
2	1	Zetkama SA	53,200.0	353	12,091.0	1.000
3	2	Odlewnie Polskie SA	35,531.9	323	6,644.1	0.000
4	3	Odlewnia Żeliwa SA	58,257.8	466	93.6	0.000
5	4	PGO SA	143,375.0	921	27,254.0	0.000
6	5	ACE SA (Luxemburg)	197,277.8	777	12,794.2	0.000
7	6	Componenta Corp. (Finland)	1,246,352.6	4,642	41,852.0	0.000
8						
9						
10		Constraints	Evaluated DMU		1	
11		Fixed Assets	53,200.0	<=	53,200.0	
12		Employees	353.0	<=	353.0	
13		Operating Profit	12,091.0	>=	12,091.0	
14		Goal - efficiency $\theta$	1.000	<=	1.000	

Fig. 1. Spreadsheet model for CCR model

Window containing the Solver Parameters dialog box is shown in Figure 2.

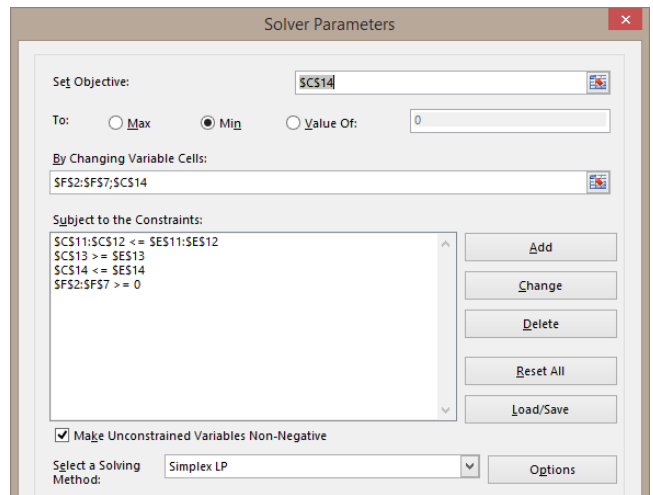


Fig. 2. Solver Parameters for CCR model

Since a ranking of the DEA efficiency scores of  $J$  DMUs requires the solution of  $J$  LP problems, it is much more convenient to use VBA script. We wrote a simple script that:

- enters index of the next unit into cell E10,
- gives initial values to the decision variables,
- calls Solver to perform optimization,
- writes in the worksheet obtained efficiency factor and corresponding DMUs' weights.

With this automation the process of solving the CCR model is fast and user-friendly.

### 3.3. Results of the analysis

The results of formulated CCR model are given in Table 2.

Table 2.  
Results of input-oriented CCR model

DMU	$\theta$	Weights $\lambda_{ij}$					
		DMU1	DMU2	DMU3	DMU4	DMU5	DMU6
1	1.000	1	0	0	0	0	0
2	0.823	0.550	0	0	0	0	0
3	0.007	0.008	0	0	0	0	0
4	0.864	2.254	0	0	0	0	0
5	0.481	1.058	0	0	0	0	0
6	0.263	3.461	0	0	0	0	0

The conclusions coming from the achieved results are as follows:

- Among evaluated foundries, fully effective is only Zetkama SA. The least effective is Odlewnie Żeliwa SA, whose efficiency is only 0.7% of this, the unit could reach, if its technology has been based on the best foundry.
- Visible are large disparities between foundries in terms of effectiveness in transformation of inputs onto results.
- Polish foundries are more effective than foreign ones.

The situation, if only one DMU is effective, severely limits the interpretive power of the DEA. In particular, the construction of the efficient frontier is not possible. Nevertheless, some analysis can be valuable, and they are presented later in this section.

#### DMUs' ranking

The solution of CCR model allows to create the ranking only in relation to not fully effective objects. In the case of objects which are all effective their scores  $\theta = 1$  and they form a group of objects classified in the first place. In our example we have only one fully effective DMU, hence we can easily carry out a full ranking. Classification of foundries is as follows: 1<sup>th</sup> place – Zetkama SA, 2<sup>nd</sup> – PGO SA, 3<sup>rd</sup> – Odlewnie Polskie SA, 4<sup>th</sup> – ACE SA, 5<sup>th</sup> – Components Corporation, 6<sup>th</sup> – Odlewnia Żeliwa SA.

#### Benchmarking and optimal technologies

In this case, benchmarking means to follow the example of the best units. The benchmarking formula for inefficient  $o$ -th DMU is described by the optimal coefficients  $\lambda_{oj}$  ( $j = 1, \dots, J$ ), which are shown in Table 2. These weights show the value by which you must multiply the  $j$ -th object technology to obtain an optimal technology for  $o$ -th object. As we have only one fully effective unit, all benchmarking formulas for the rest of DMUs are constructed in relation to Zetkama SA. It can be observed in Table 2 that only first column of weights (i.e. for Zetkama SA) contains non-zero values.

For example, the optimal technology for DMU2 is 0.55 of DMU1 technology. This means that DMU2 at its optimum technology could get results of DMU1 object (Operating Profit = 12,091 PLN) with much less effort, employing no more than  $\theta_o = 55\%$  of inputs incurred by DMU1.

Relevant calculations for inefficient foundries are presented in Table 3.

Table 3.  
Optimal technologies for DMU2-DMU6

DMU	$\lambda_{o1}$	Fixed Assets	Employees	Operating Profit
2	0.550	29,233.6	194	6,644.1
3	0.008	412.0	3	93.6
4	2.254	119,916.7	796	27,254.0
5	1.058	56,293.9	374	12,794.2
6	3.461	184,147.4	1,222	41,852.0

The data show the size of inefficiency of examined foundries in relation to the reference object. For example, Zetkama SA would need only PLN 412,000.0 its fixed assets and three employees to achieve the result it has been achieved by Odlewnia Żeliwa SA.

#### Target technology

Target technology for inefficient object is the technology that guarantees 100% efficiency. This can be achieved by a proportional reduction of all inputs to the level of  $\theta$  times the

current. Relevant calculations for inefficient foundries are presented in Table 4.

Table 4.  
Target technologies for DMU2-DMU6

DMU	$\theta_o$	Fixed Assets	Employees	Operating Profit
2	0.823	29,233.6	266	6,644.1
3	0.007	412.0	3	93.6
4	0.864	123,867.3	796	27,254.0
5	0.481	94,837.8	374	12,794.2
6	0.263	328,068.5	1,222	41,852.0

For example, Odlewnie Polskie SA should use PLN 29,233.6 fixed assets and 266 employers (current levels are 35,531.9 and 323, respectively) to be fully effective.

## 4. Conclusions

The DEA method discussed in this paper has the potential to provide crucial information about foundries' conditions and management performance for top managers and stock investors. The DEA method is very general, permitting multiple criteria for evaluation purposes. Moreover, DEA requires only data on the quantity of inputs and outputs; price data are not necessary. This could be very appealing in the analysis of foundries because some foundries' inputs and outputs have no price equivalent.

The main advantages of DEA method, compared with traditional approaches, are [2]:

1. DEA can handle multiple inputs and multiple outputs simultaneously.
2. DEA does not require relating inputs to outputs.
3. Comparisons are directly against compeers.
4. Inputs and outputs can have very different units.

On the other hand, main weaknesses of DEA method are:

1. DEA does not measure "absolute" efficiency; it is intended to measure the relative effectiveness.
2. Statistical tests are not applicable or very complicated to perform.
3. Large problems can be computationally intensive.

Zetkama SA proved to be the most effective of the foundries examined under 2012 year conditions. The study by the DEA method revealed that examined Polish foundries operate in general more efficiently than comparable foreign plants.

In order to obtain more reliable information, we will need to extend the study over a larger number of foundries with a larger number and variety of inputs and outputs. Furthermore, it would be interesting to take into account the results that are negative, for example: amount of scraps, waste water or dusts emitted into the atmosphere.

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