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SLOW EXTENSIONAL FLOW PAST A NON-HOMOGENEOUS POROUS SPHERICAL SHELL

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An analytical investigation of extensional flow past a porous spherical shell of finite thickness with velocity slip at the surface is presented. The permeability of the shell varies continuously as a function of the radial distance. The flow in the porous region is assumed to obey Darcy's Law. The drag has been calculated in terms of normal volume flux rate per unit area of the outer and inner surfaces. Particular cases of flow past a homogeneous sphere and no-slip boundary condition have been deduced.

Key words: porous medium, Stokes flow, permeable shell, extensional field, velocity slip.

1. Introduction

The theory of motion of fluid flow in porous media has been investigated extensively due to its vast applications. Many of these applications involve removal of impurities in integrated circuits in computers, study of landslides, nuclear waste disposal, study of micro organisms etc. This stimulates a detailed analysis of fluid flow past and through spherical geometries involving cavities, spherical shells, annulus etc. Nield and Bejan (2006) have shown that a porous medium can be considered as a solid matrix with an interconnected void. These voids can be of different shapes and sizes. The initial investigation on flow within porous media was performed by Henry Darcy. Darcy's law is a simple proportional relationship between the instantaneous discharge rate through a porous medium, the viscosity of the fluid and the pressure drop over a given distance

$$\boldsymbol{q} = -\frac{k}{\mu}\nabla P$$

where $\boldsymbol{q}$ is the filtration velocity, P is the pressure, k is the permeability of the porous material and μ is the viscosity. The study of the flow past the surface of a saturated porous material shows that there is a thin layer of streamwise moving fluid just beneath it. The fluid in this layer is pulled along by the main flow external to

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the porous medium. Thus, there is a net tangential drag due to the transfer of forward momentum across the permeable surface. This gave rise to the need and justification of a slip velocity condition. The slip-condition suggested by Beavers and Joseph (1967) is of the form

$$u - U = \frac{\sqrt{k}}{\alpha}\left(\frac{\partial u}{\partial n}\right)$$

where u is the fluid velocity at the permeable interface, U is the Darcy velocity inside the porous medium, $\left(\frac{\partial u}{\partial n}\right)$ is the gradient of the streamwise component of velocity along the normal to the surface drawn into the fluid, k is the permeability of the porous material and α is the empirical dimensionless constant depending on the nature of the porous material only. Happel and Brenner (1973) extensively studied the flow past a solid sphere at low Reynolds numbers. The slow flow of an Oldroyd fluid past a uniformly porous spherical shell of negligible thickness was investigated by Rajvanshi (1969). Prakash and Rajvanshi (1975) studied the slow flow of an incompressible viscous fluid past a porous spherical shell of finite thickness with velocity slip on the boundary. The shell is assumed to be non- homogeneous. The drag was calculated for some particular cases. In a subsequent paper Prakash and Rajvanshi (1978) evaluated the effective viscosity of a dilute suspension of porous spherical particles. They assumed that the small particles are homogeneous naturally permeable spheres and the tangential velocity slip applies at their saturated surfaces and that the flow inside is governed by Darcy's law. Pop and Ingham (1996) presented a closed form exact solution for a forced flow past a sphere which is embedded in a porous medium using the Brinkman model. Their analysis shows that there is no flow separation for this flow configuration. Petrie (1999) studied the behaviour of polymeric liquids and fibre suspensions under extensional flows. Bhatt (1993) studied slow extensional flows past a porous sphere, liquid sphere and solid smooth sphere with slip at the surface. Stokes approximation was used to obtain exact solution for slow motion. Mass transfer to the outer fluid due to these spheres was evaluated in each case. Bhatt and Shirley (2001) examined a flow past a sphere bounded by a spherical shell with a radially stretching surface. The effect of the inner sphere on the mass transfer to the fluid was obtained. Kawase and Moo-Young (1986) obtained the approximate solutions of the power law fluid flow past a solid sphere at small Reynolds number when the flow at infinity was given by the extensional field. Alazmi and Vafai (2001) analysed the conditions at the interface between a porous medium and a homogeneous fluid. Sahraoui and Kaviany (1992) considered a solid matrix constructed from regularly spaced cylinders and numerically compared the micro-solution with the macro solution using the Brinkman model. Srivastava and Srivastava (2005) discussed the steady flow of an incompressible viscous fluid streaming past a porous sphere at small Reynolds number with a uniform velocity using Brinkman equations. They used the boundary conditions suggested by Ochao-Tapia and Whitaker (1995a; 1995b) at the surface of the sphere. It is noted that the drag on the sphere reduces significantly when it is porous and it decreases with the increase in permeability of the medium. Padmavathi *et al*. (1993) used the Brinkman model to investigate slow flow past a porous sphere. Rudraiah and Chandrashekhar (2005) analysed a two-dimensional steady incompressible flow past an impermeable sphere embedded in a porous medium using the Brinkman model with a uniform shear away from the sphere. Grosan *et al*. (2010) presented the steady flow of a viscous fluid past a permeable porous sphere embedded in another porous medium using the Brinkman model.

The aim of the present work is to study the extensional flow past a porous spherical shell of finite thickness with velocity slip at the surface. The permeability of the shell is taken to be variable. The Darcy law is applicable in the porous region and using the slip condition given by Jones (1973) the solutions are obtained. Limits on the filter velocities imposed by considerations of physical possibility of the flow are indicated. Values are calculated for the flow past a thin spherical shell under the conditions of no-slip and for flow past a homogeneous spherical shell of negligible thickness. The drag is evaluated for the shell under consideration and some particular cases are obtained.

2. Formulation

The slow flow past a porous non homogeneous spherical shell of finite thickness is investigated. The inner radius of the spherical shell is taken as b and the outer radius as a.

The flow at infinity is given by an extensional field with velocity components as

$$u_x = -\frac{\gamma}{2}x\,, \qquad u_y = -\frac{\gamma}{2}y\,, \qquad u_z = \gamma z \tag{2.1}$$

where γ is a constant.

The flow domain is divided into three regions which are labelled as I, II, III where
region I $(r > a)$ is the outer free fluid region,
region II $(b < r < a)$ is the permeable Darcy region, and
region III $(r < b)$ is the region inside the porous sphere.

Spherical polar coordinates (r, θ, φ) are considered with the pole at the centre of the shell. Axial symmetry is assumed and the derivatives with respect to φ vanish identically. The velocity components at any point are denoted by u^* and v^* in the radial and tangential directions, respectively. The pressure is denoted by P^* .The density ρ and viscosity μ of the fluid are taken as constant. The velocity components corresponding to regions I, II and III are indicated by subscripts 1, 2, 3, respectively.

The equations of motion using Stokes approximation for regions I and III are given by

$$\nabla^2 u^* - \frac{2}{r^{*2}}(u^* + \frac{\partial v^*}{\partial\theta} + v^*\cot\theta) = \frac{1}{\mu}\left(\frac{\partial P^*}{\partial r^*}\right), \tag{2.2}$$

$$\nabla^2 v^* - \frac{2}{r^{*2}}\left(2\frac{\partial u^*}{\partial\theta} - v^*\operatorname{cosec}^2\theta\right) = \frac{1}{\mu r^*}\left(\frac{\partial P^*}{\partial\theta}\right), \tag{2.3}$$

where
$$\nabla^{*2} = \frac{\partial^2}{\partial r^{*2}} + \frac{2}{r^*}\frac{\partial}{\partial r^*} + \frac{1}{r^*}\frac{\partial^2}{\partial\theta^2} + \frac{\cot\theta}{r^{*2}}\frac{\partial}{\partial\theta}\,. \tag{2.4}$$

The porous region II is governed by Darcy's Law

$$u_2^* = -\frac{k}{\mu}\left(\frac{\partial P_2^*}{\partial r^*}\right), \tag{2.5}$$

$$v_2^* = -\frac{k}{\mu r^*}\left(\frac{\partial P_2^*}{\partial\theta}\right). \tag{2.6}$$

We introduce S_1^* and S_2^* as the filter velocities of the fluid in the radial direction across the outer and inner surfaces of the shell. The boundary conditions are now prescribed as
Region I

$$u_1^* = -S_1^* \quad \text{at} \quad r^* = a\,, \tag{2.7}$$

$$u_1^* = \frac{U\gamma r^*}{2a}\left(2\cos^2\theta - \sin^2\theta\right), \quad v_1^* = -\frac{U3\gamma r^*}{2a}\sin\theta\cos\theta \quad \text{as} \quad r^* \to \infty . \tag{2.8}$$

Region II

$$u_2^* = -S_1^* \quad \text{at} \quad r^* = a , \quad \text{and} \quad u_2^* = -S_2^* \quad \text{at} \quad r^* = b . \tag{2.9}$$

Region III

$$u_3^* = -S_2^* \quad \text{at} \quad r^* = b , \quad \text{and} \quad u_3^*, \nu_3^* \quad \text{are finite at} \quad r^* = 0 . \tag{2.10}$$

The second boundary condition is the slip condition at the spherical surface adopted by Jones (1973) which is

$$2a\beta_1 e^*_{r^*\theta} = \left(\nu_1^* - \lambda\nu_2^*\right) \quad \text{at} \quad r^* = a , \tag{2.11}$$

$$2a\beta_2 e^*_{r^*\theta} = \left(\nu_3^* - \lambda\nu_2^*\right) \quad \text{at} \quad r^* = b \tag{2.12}$$

where β_1 and β_2 are the values of the dimensionless slip parameter $\frac{\sqrt{k}}{a\alpha}$ at the outer and inner surfaces of the shell, respectively. $e^*_{r^*\theta}$ is the shear strain rate. α is the empirical dimensionless constant depending on the nature of the porous material only. λ is a constant which has been introduced so that the no slip condition may be recovered by setting $\lambda = \beta_1 = \beta_2 = 0$.

The above equations are rendered dimensionless by defining the following variables

$$r = \frac{r^*}{a}, \quad u = \frac{u^*}{U}, \quad \nu = \frac{\nu^*}{U}, \quad P = \frac{aP^*}{\mu U}, \quad K = \frac{k}{a^2}, \quad S_1 = \frac{S_1^*}{U}, \quad S_2 = \frac{S_2^*}{U}, \quad c = \frac{b}{a}$$

where U is the characteristic velocity.

Therefore Eqs (2.2) and (2.3) are written as

$$\nabla^2 u - \frac{2}{r^2}\left(u + \frac{\partial v}{\partial\theta} + v\cot\theta\right) = \frac{1}{\mu}\frac{\partial P}{\partial r}, \tag{2.13}$$

$$\nabla^2 v - \frac{1}{r^2}\left(2\frac{\partial u}{\partial\theta} - v\operatorname{cosec}^2\theta\right) = \frac{1}{\mu r}\frac{\partial P}{\partial\theta} \tag{2.14}$$

where $$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta} .$$

The equations applicable in region II are

$$u_2 = -K\left(\frac{\partial P_2}{\partial r}\right), \tag{2.15}$$

$$v_2 = -\frac{K}{r}\left(\frac{\partial P_2}{\partial \theta}\right). \tag{2.16}$$

By the equation of continuity, the divergence of the velocity vector is zero in each of the three regions.

The modified boundary conditions are

$$u_1 = \frac{\gamma r}{2}\left(2\cos^2\theta - \sin^2\theta\right), \qquad v_1 = \frac{-3\gamma r}{2}\sin\theta\cos\theta \qquad \text{as} \qquad r \to \infty, \tag{2.17}$$

$$u_1 = u_2 = -S_1 \qquad \text{at} \qquad r=1, \tag{2.18}$$

$$u_2 = u_3 = -S_2 \qquad \text{at} \qquad r=c, \tag{2.19}$$

u_3 and v_3 are finite at $r=0$ (2.20)

$$\beta_1 e_{r\theta} = \left(v_1 - \lambda v_2\right) \qquad \text{at} \qquad r=1, \tag{2.21}$$

$$\beta_2 e_{r\theta} = \left(v_3 - \lambda v_2\right) \qquad \text{at} \qquad r=c. \tag{2.22}$$

Upon substitution of the expressions for u_2 and v_2 from above, the equation of continuity yields

$$\nabla\cdot\left(K\nabla P_2\right) = 0\,. \tag{2.23}$$

Further, we assume the permeability of the shell to be given by a particular harmonic function of r such that

$$K^{\frac{1}{2}}\left(r\right) = A + \frac{B}{r}\,. \tag{2.24}$$

The pressure may be expressed in terms of a function $G\left(r,\theta\right)$ as

$$P_2\left(r,\theta\right) = K^{-\frac{1}{2}}\left(r\right)G(r,\theta)\,. \tag{2.25}$$

3. Solution

Stokes stream function ψ is introduced to satisfy the equation of continuity such that

$$u = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} \qquad \text{and} \qquad v = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}\,. \tag{3.1}$$

Region – I

The Stokes stream function ψ is assumed as

$$\psi^1 = \left(\frac{A_1}{r} + A_2 r + A_3 r^3 + A_4 r^4\right)\sin^2\theta\cos\theta \tag{3.2}$$

where A_1, A_2, A_3 and A_4 are constants required to be determined by boundary conditions.

The velocity components are found as

$$u_1 = \left(\frac{A_1}{r^3} + \frac{A_2}{r} + A_3 r + A_4 r^2\right)(2\cos^2\theta - \sin^2\theta), \tag{3.3}$$

$$v_1 = \left(\frac{A_1}{r^3} - \frac{A_2}{r} + 3A_3 r - 4A_4 r^2\right)(\sin\theta\cos\theta). \tag{3.4}$$

The boundary condition (2.17) gives

$$A_3 = \frac{\gamma}{2}, \qquad A_4 = 0. \tag{3.5}$$

Therefore

$$u_1 = \left(\frac{A_1}{r^3} + \frac{A_2}{r} + \frac{\gamma}{2} r\right)\left(2\cos^2\theta - \sin^2\theta\right), \tag{3.6}$$

$$v_1 = \left(\frac{A_1}{r^3} - \frac{A_2}{r} + \frac{3\gamma}{2} r\right)\left(\sin\theta\cos\theta\right). \tag{3.7}$$

Using Eq.(2.13) the pressure is found as

$$P_1 = \left(\frac{A_1}{r^4} + 3\frac{A_2}{r^2}\right)\left(2\cos^2\theta - \sin^2\theta\right) + \text{const.} \tag{3.8}$$

Region –III

The Stokes stream function ψ is assumed as

$$\psi^3 = \left(\frac{C_1}{r} + C_2 r + C_3 r^2 + C_4 r^4\right)\sin^2\theta\cos\theta \tag{3.9}$$

where C_1, C_2, C_3 and C_4 are constants required to be determined by boundary conditions.

The velocity components are found as

$$u_3 = \left(\frac{C_1}{r^3} + \frac{C_2}{r} + C_3 + C_4 r^2\right)(2\cos^2\theta - \sin^2\theta), \tag{3.10}$$

$$v_3 = \left(\frac{C_1}{r^3} - \frac{C_2}{r} - 2C_3 - 4C_4 r^2\right)(\sin\theta\cos\theta). \tag{3.11}$$

Using condition (2.20)

$$C_1 = C_2 = 0\,. \tag{3.12}$$

Therefore

$$u_3 = \left(C_3 + C_4 r^2\right)\left(2\cos^2\theta - \sin^2\theta\right), \tag{3.13}$$

$$v_3 = \left(-2C_3 - 4C_4 r^2\right)(\sin\theta\cos\theta)\,. \tag{3.14}$$

The pressure is obtained as

$$P_3 = \left(6C_4 + 4\frac{C_3}{r}\right)\left(2\cos^2\theta - \sin^2\theta\right) + \text{const.} \tag{3.15}$$

Region –II

Comparison of Eqs (2.25) and (3.8) with Eq.(3.15) suggests that $G(r,\theta)$ is of the form $F(r)\left(2\cos^2\theta - \sin^2\theta\right)$. Since G is a harmonic function of r and θ, the desired form is assumed as

$$G(r,\theta) = \left(B_1 r + B_2 r^{-2}\right)\left(2\cos^2\theta - \sin^2\theta\right) \tag{3.16}$$

where B_1 and B_2 are constants required to be determined by boundary conditions.

Using Eqs (2.24) and (2.25) the pressure in Region-II is obtained as

$$P_2 = \frac{\left(B_1 r + B_2 r^{-2}\right)}{\left(A + B r^{-1}\right)}\left(2\cos^2\theta - \sin^2\theta\right). \tag{3.17}$$

Substituting the above in Eqs (2.15) and (2.16) gives the following components

$$u_2 = -\left[A\left(B_1 - \frac{2B_2}{r^3}\right) + B\left(\frac{2B_1}{r} - \frac{B_2}{r^4}\right)\right]\left(2\cos^2\theta - \sin^2\theta\right), \tag{3.18}$$

$$v_2 = \left(A + \frac{B}{r}\right)\left(B_1 + \frac{B_2}{r^3}\right)6\sin\theta\cos\theta\,. \tag{3.19}$$

By using the boundary conditions (2.18) and (2.19) the following expressions are obtained

$$S_1 = s_1\left(2\cos^2\theta - \sin^2\theta\right) \quad \text{and} \quad S_2 = s_2\left(2\cos^2\theta - \sin^2\theta\right) \tag{3.20}$$

where s_1 and s_2 are positive constants depending upon the normal filter velocity across the surfaces of the shell.

Using the slip-conditions (2.21) and (2.22), the following equations hold true

$$A_1 - A_2 - \frac{3\gamma}{2} - 6\lambda(A+B)(B_1+B_2) = \beta_1(-10A_1 - 4A_2 - 3\gamma), \tag{3.21}$$

$$-2C_3 - 4C_4c^2 - 6\lambda\left(A + \frac{B}{c}\right)\left(B_1 + \frac{B_2}{c^3}\right) = -\beta_2\left(\frac{-4C_3}{c} - 10C_4c\right). \tag{3.22}$$

Using the Eqs (3.20), (3.21), (3.22) the constants are obtained as

$$A_1 = \frac{s_1(1-4\beta_1) + \gamma(1-\beta_1) + 6\lambda(A+B)(B_1+B_2)}{2(1+3\beta_1)},$$

$$A_2 = \frac{s_1(1+10\beta_1) - 2\gamma(1-\beta_1) - 6\lambda(A+B)(B_1+B_2)}{2(1+3\beta_1)},$$

$$M = (2A+B)\left(Ac^4 + 2Bc^3\right) - (A+2B)(2Ac+B),$$

$$B_1 = \frac{c^4(2A+B)s_2 - (2Ac+B)s_1}{M}, \qquad B_2 = \frac{-c^4(A+2B)s_2 + \left(Ac^4 + 2Bc^3\right)s_1}{M},$$

$$C_3 = \frac{3\lambda(Ac+B)\left(B_1c^3 + B_2\right) + c^3 s_2(5\beta_2 + 2c)}{c^3(3\beta_2 + c)},$$

$$C_4 = \frac{-3\lambda(Ac+B)\left(B_1c^3 + B_2\right) + c^3 s_2(-2\beta_2 - c)}{c^5(3\beta_2 + c)}.$$

3.1. Pressure jump

The pressure jump at the surface may be expressed in terms of the dimensionless parameters e_1 and e_2 defined by the relations

$$P_1 = P_2 + 2e_1\left(\frac{\partial u_1}{\partial r}\right) \quad \text{at} \quad r=0, \tag{3.23}$$

$$P_3 = P_2 + 2e_2\left(\frac{\partial u_3}{\partial r}\right) \quad \text{at} \quad r=c. \tag{3.24}$$

These equations on simplification yield

$$\frac{(B_1+B_2)}{A+B} = A_1 + 3A_2 + e_1(6A_1 + 2A_2 - \gamma), \tag{3.25}$$

$$\frac{\left(B_1c^3 + B_2\right)}{Ac+B} = 2C_4c^2(3 - 2e_2) + 4C_3. \tag{3.26}$$

At the surfaces of a naturally permeable material, the pressure is continuous that means $e_1 = e_2 = 0$. The normal filter velocity is determined by this fact and cannot be prescribed arbitrarily.

3.2. Discussion

For physically possible flows, s_1 and s_2 are positive or zero, since the normal flow at points on front hemisphere must be in the direction of the impinging stream. The other conditions are that

$$P_1 \geq P_2 \quad \text{and} \quad P_2 \geq P_3 \quad \text{for} \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}. \tag{3.27}$$

Using these conditions the limits on filtration velocity parameters may be prescribed by the inequalities

$$0 \leq s_2 c^4 \leq s_1 \left(\frac{G_2}{G_1} \right) - \frac{5\gamma}{2} \left(\frac{G_3}{G_1} \right), \tag{3.28}$$

$$0 \leq s_1 \leq s_2 \left(\frac{G_5}{G_4} \right) \tag{3.29}$$

where the constants are given by

$$G_1 = (1+3\beta_1)(A-B), \quad G_2 = M(2+3\beta_1)(A+B) - F(1+3\beta_1), \quad G_3 = M(1-\beta_1)(A+B),$$

$$G_4 = \frac{-6\lambda(Ac+B)^2 - c^3(3\beta_2 + c)}{c^3(Ac+B)}, \quad G_5 = \frac{-2\mathrm{M}(4\beta_2 + c) - c^7(2A+B) + c^4(A+2B)}{c^3(B-Ac)},$$

$$F = Ac^4 + 2Bc^3 - 2Ac - B.$$

3.3. Some particular cases

Taking the limit as $c \to 1$, the limits on s_1 for a physically possible flow past a thin spherical shell under the condition of no-slip are

$$0 \leq s_2 \leq s_1 \left(\frac{2M(A+B) + (A-B)}{(A-B)} \right) - \frac{5\gamma}{2} \left(\frac{M\ (A+B)}{(A-B)} \right). \tag{3.30}$$

On taking the limit as $c \to 0$ for the flow past a non-homogeneous sphere with velocity slip

$$\frac{5\gamma}{2} \left(\frac{(-A-2B)B(A+B)(1-\beta_1)}{(A-B)(1+3\beta_1)} \right) \leq s_1 \left(\frac{B - (A+2B)B(A+B)(2+3\beta_1)}{(A-B)(1+3\beta_1)} \right). \tag{3.31}$$

For the case of flow past a thin homogeneous spherical shell with $B=0$ and $c \to 1$

$$0 \leq s_2 \leq s_1. \tag{3.32}$$

4. Drag

The general expression for drag on a porous sphere is

$$D = \iint_{\sum_1 + \sum_2} (1-m)\overrightarrow{\varphi_n} d\sigma - \iiint_V m \, grad \, P \, dV \tag{4.1}$$

where $\overrightarrow{\varphi_n}$ is the stress vector on an element $d\sigma$ of the surface, m is the porosity of the material occupying the volume V bounded by the surfaces $\sum_1$,and $\sum_2$.

The drag on the spherical shell in the negative direction of the polar axis is

$$D = 8\pi \left\{ (1-m)(A_1 + 3A_2) - m\left(\frac{(B_1 + B_2)}{A+B} - \frac{(B_1c^3 + B_2)}{c(Ac+B)} \right) \right\}. \tag{4.2}$$

The drag in the case of a homogeneous permeable shell is

$$D = 8\pi \left\{ \frac{\left\{ (1-m)2s_1(2+13s_1\beta_1) - 5\gamma(1-\beta_1) - 12\lambda k^2 \left(\left(s_2c^4 + s_1c^4 - s_1c \right) / M \right) \right\}}{2(1+3\beta_1)} + \right.$$
$$\left. -m\left(\frac{s_2c^4 + s_1c^4 - s_1c}{M} \right) - \frac{(B_1c^3 + B_2)}{c^2k} \right\}. \tag{4.3}$$

In the case of flow past a homogeneous sphere of vanishingly small porosity, the drag is found as

$$D = 8\pi \left\{ \frac{2s_1(2+13s_1\beta_1) - 5\gamma(1-\beta_1) - 12\lambda k^2 \left(\left(s_2c^4 + s_1c^4 - s_1c \right) / M \right)}{2(1+3\beta_1)} \right\}. \tag{4.4}$$

On the assumption of no-slip condition in the above case

$$D = 4\pi(4s_1 - 5\gamma). \tag{4.5}$$

5. Conclusion

The extensional flow past a porous spherical shell of finite thickness with velocity slip and variable permeability has been investigated. Limits on the filter velocities imposed by considerations of physical possibility of the flow have been indicated and, in particular, values have been calculated for the flow past a thin spherical shell under the conditions of no-slip and for flow past a homogeneous spherical shell of negligible thickness. The drag has been evaluated for the spherical shell under consideration. From the present investigation the drag for the flow past a homogeneous sphere of vanishingly small porosity and no-slip has been found as $4\pi(4s_1 - 5\gamma)$. The problem undertaken may enhance research in the field of study of landslides, micro-organisms and other practical applications.

Nomenclature

a, b – outer and inner radii of the spherical shell
e_1, e_2 – dimensionless parameters
$e_{r\theta}$ – non-dimensional shear rate strain
$e^*_{r^*\theta}$ – shear rate strain
k – permeability of the porous material
m – porosity of the material
P – non-dimensional pressure
P^* – pressure
$\boldsymbol{q}$ – filtration velocity
r, θ, φ – spherical polar coordinates
U – Darcy velocity inside the porous medium
U – characteristic velocity
u – fluid velocity at the permeable interface
u, ν – non-dimensional velocity components in radial and transverse directions u^*
u^*, ν^* – velocity components in radial direction and transverse directions
V – volume
α – non-dimensional constant
β – non-dimensional slip parameter
γ – constant
λ – constant
μ – viscosity of the fluid
ρ – density of the fluid
ψ – non-dimensional stream function

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