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MEAN-SQUARE NON-LOCAL STABILITY OF SHIP IN STORM CONDITIONS OF OPERATION

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ABSTRACT

The purpose of the paper is to create a method for studying nonlocal stability in the mean and in the mean square of the ship, positioned on the beam of an intensive wind–waves mode, which is based on the use of the correlation theory of random functions close to continuous Markov processes. With the help of this method and the integral formula of event probability, a method for determining the reliability indicator of the ship in respect of the existing wind–waves excitations of the operating area is formed. An example of investigating the nonlinear motion of the ship, determining its local and nonlocal stability in the first approximation of the theory of considered random functions, is given. Such approximation uses correlation theory with models of acting excitations represented by the generalised derivatives of the Wiener process. Moreover, special attention is paid to reflecting the connection of the proposed methods for investigating the ship stability under constantly acting random excitations with the traditional methods of studying ship stability at small and large inclinations. The established connection defines the proposed methods as a development of the traditional methods of ship stability deterministic theory during the transition to its formation in the class of random functions, with the addition to these methods of the missing link of determining the level of reliability of ships towards the acting wind–waves excitations of the operation area.

Keywords: wind-waves mode; local and nonlocal stability; nonlocal stability in mean; nonlocal stability in mean square; reliability indicator

INTRODUCTION

This work is devoted to the presentation of methods for determining the probabilistic characteristics of ships' nonlinear motion processes and their stability based on the application of the methods of random functions close to continuous Markov random processes. First, the most complete consideration of the application of such methods to solving problems of determining the probabilistic characteristics of ship motion and the stability of motion has already been given in the monograph by the current paper's author [5]. In this monograph, the Fokker–Planck–Kolmogorov equation and the kinetic equations of a more complex structure are used to determine the probability densities and characteristic functions of nonlinear ship motion. The characteristics of the amplitudes and phases of nonlinear motion and parametrically excited random oscillations of the ships are studied as well. It is shown that a significant extension of the method of random amplitudes and phases is the method of statistical moments. Using this method, the investigation of ship nonlinear motion stability is effective not only at small, but also at large inclinations. Subsequent development of the proposed method is only partially described in [6–8].

In this work, the results obtained in [5–8] are combined into a united complex of the investigation of ship stability and reliability under constantly acting random wind–waves excitations. The first approximation of the developed theory, which operates with the first two statistical moments of random functions, is presented. For this, the concepts of a ship's local and nonlocal stability in the mean and in the mean square are introduced. The criteria of these types of stability and the relationship between them are given. Particular attention is paid to the connection of the proposed methods with traditional methods of studying the stability of ships and to determining their reliability indicators under the wind–waves action of the operation area.

FEATURES OF TRADITIONAL METHODS OF SHIP STABILITY THEORY

The study of ship stability under the action of stationary applied heeling moment is presented in Fig. 1. This figure shows that the local stability of the equilibrium position of the ship at the roll angle θ_S under the action of heeling moment M_h is determined by the method of additional small perturbations, and the nonlocal stability of the ship under the gradual increase in the heeling moment from the initial equilibrium position $\theta = 0$ in calm water is determined by the region of the existence of real solutions of the equation of acting moments realised in the interval $[-M_{hmax}, M_{hmax}]$.

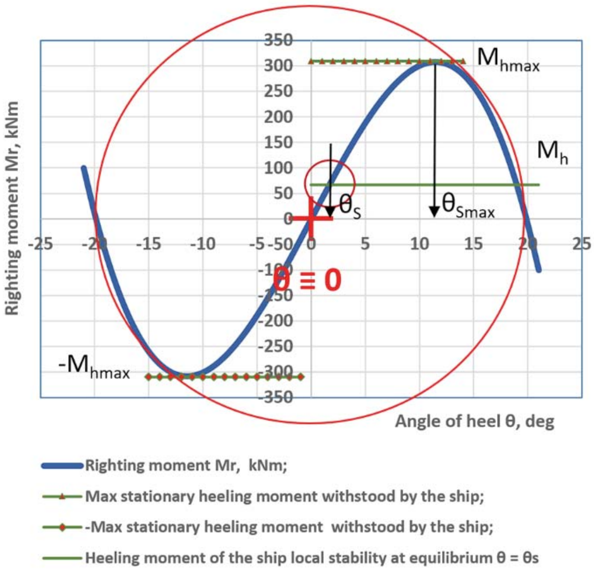

Fig. 1. Local and nonlocal (maintaining position of ship in the neighbourhood of equilibrium position in calm water θ ≡ 0) stability when stationary heeling moment is applied to the ship

The investigation of ship stability under the action of dynamically applied heeling moment is shown in Fig. 2. This figure determines that, under the action of dynamically applied heeling moment, the region of nonlocal stability of the ship (maintaining the position of the ship in the neighbourhood of the equilibrium position in calm water $\theta \equiv 0$) is determined by the region of the existence of real solutions of the equation of works (energies) of the operating moments.

Thus, the traditional study of ship stability and determination of the boundaries $\boldsymbol{\Gamma}_s$ and $\boldsymbol{\Gamma}_d$ of regions $\boldsymbol{\Omega}_s$ and $\boldsymbol{\Omega}_d$ of its nonlocal stability under the action of stationary and nonstationary forces uses:

– nonlinear equation of roll inclinations of ship under the action of heeling moment

$$(J_x + \mu_{44})\ddot{\theta} + M_r(\theta) = M_h(t) \tag{1}$$

– equation of ultimate ship position under the stationary action of external forces

$$M_r(\theta)_{max} - M_h = 0; \tag{2}$$

– equation of the maximal possible equality of works of heeling and righting moments (energies of external forces and potential resources of ship) under the dynamic action of heeling moment

$$A_r(\theta)\big|_0^{\theta_{Dmax}} - A_h(\theta)\big|_0^{\theta_{Dmax}} = 0. \tag{3}$$

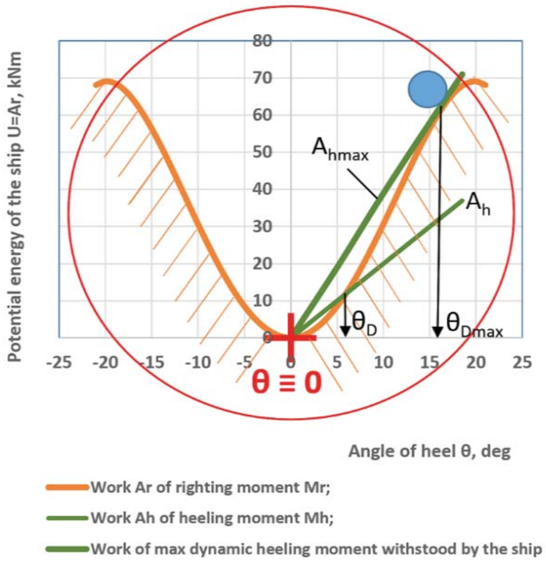

Fig. 2. Nonlocal stability of ship under the action of dynamically applied heeling moment

METHOD FOR SOLVING THE PROBLEM OF SHIP STABILITY IN STORM AS PROBLEM OF LOCAL AND NONLOCAL STABILITY IN THE MEAN SQUARE

GENERAL POSITIONS

The method uses:

– stochastic nonlinear differential equations simulating the processes of wind–waves actions and ship motion, which are built on the basis of representing the acting forces by

segments of multi-dimensional Taylor series in degrees of the displacements and velocities of the ship and in degrees of the kinematic characteristics of the wind–waves fields in the neighbourhood of the ship's equilibrium position in calm water [5];
- presentation of initial acting excitations in stochastic models of external wind–waves actions in the form of "white noise" in the sense of Stratonovich [9], i.e., in the form of random processes with finite power values;
- conditions of the Germaidze–Krasovsky theorem on the stability of dynamic systems under constantly acting excitations that are bounded in the mean [3].

EQUATIONS FOR THE PROBABILISTIC CHARACTERISTICS OF THE SHIP MOTION

Under the stated preconditions, the system of nonlinear differential equations of ship motion under the action of wind and waves is written in the following general form [5]:

$$\frac{d\boldsymbol{Y}}{dt} = \boldsymbol{F}(\boldsymbol{Y}, \boldsymbol{X}, \boldsymbol{\psi}, t)$$

where $\psi(t)$ is the vector of generalised derivatives of the Wiener process ("white noise"); $\boldsymbol{X}(t)$ is the vector of the processes of wind–waves actions; $\boldsymbol{Y}(t)$ is the vector of the ship's motion processes.

To determine the probabilistic characteristics of ship motion processes $\boldsymbol{Y}(t)$ based on the stochastic equations (4) of the motion of the considered dynamic system "ship-excited media", an infinite system of equations for statistical moments is constructed. The construction of the system is carried out either using the equation for the characteristic function, or using the Fokker–Planck–Kolmogorov equation [5]. The presence of nonlinearities in equations of motion (4) leads to the appearance in the subsystems of equations for statistical moments of lower orders 1 ... m statistical moments of higher orders $m + 1$ ... n. If we restrict ourselves to considering statistical moments no higher than the order of p, this circumstance will require the closure of the considered subsystems of equations. The closure is carried out on the basis of introducing the hypothesis about the nature of the distribution law of the processes under consideration. This can be the normal distribution when the problem is considered within the framework of the correlation theory. Under fuller use of the mentioned positions of the Germaidze–Krasovsky theorem, it is the beta distribution, the Pearson distribution of type I, etc. [8].

In a first approximation, the approximation of the correlation theory of random functions, the closure of the considered subsystems is carried out by expressing the statistical moments of higher orders through the statistical moments of lower orders using relations between the moments of the normal distribution. As a result, the following system of nonlinear differential equations is formed:

$$\frac{d\boldsymbol{\alpha}_1}{dt} = \boldsymbol{Q}_1(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2, \boldsymbol{G}_1, \boldsymbol{G}_2, t); \tag{5}$$

$$\frac{d\boldsymbol{\mu}_2}{dt} = \boldsymbol{Q}_2(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2, \boldsymbol{G}_1, \boldsymbol{G}_2, t), \tag{6}$$

where $\boldsymbol{\alpha}_1(t)$ is the vector of statistical moments of the first order, i.e., the vector of the expected values of the processes of motion of the dynamic system "ship–environment"; $\boldsymbol{\mu}_2(t)$ is the vector of second-order central moments, which includes variances (dispersions) and mutual correlation moments of the dynamic system processes; $\boldsymbol{G}_1(t)$ and $\boldsymbol{G}_2(t)$ are the vectors of the first- and second-order intensity coefficients characterising the average values and average powers of the acting excitations (in limited cases, these vectors are represented by random functions with non-differentiable realisations, by "white noise" $\boldsymbol{\psi}(t)$).

Compared to the characteristic temporary constants of the ship, such as the period of its free oscillations and the time of their relaxation, the storm develops very slowly, in a quasi-stationary way. Due to the quasi-stationary nature of the random wind–waves processes acting on the ship in the storm, the system of differential equations for statistical moments (5) is transformed into the following system of nonlinear algebraic equations:

$$\boldsymbol{Q}_1(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2, \boldsymbol{G}_1, \boldsymbol{G}_2) = \boldsymbol{0}; \tag{7}$$

$$\boldsymbol{Q}_2(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2, \boldsymbol{G}_1, \boldsymbol{G}_2) = \boldsymbol{0}, \tag{8}$$

The solution of the system of algebraic equations (7)–(8) for given intensities of external excitations $\boldsymbol{G}_1^{(0)}$, $\boldsymbol{G}_2^{(0)}$ determines the quasi-stationary mode of the ship motion with expected values $\boldsymbol{\alpha}_1^{(0)}$ and statistical moments of second-order $\boldsymbol{\mu}_2^{(0)}$.

The solution to the system of equations (7)–(8) is found using nonlinear programming methods [1], which are applied in the process of solving the following problem of unconstrained minimisation of the objective function:

$$\boldsymbol{OF_1}(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2) = \boldsymbol{Q}_1^2(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2) + \boldsymbol{Q}_2^2(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2) \to \min \tag{9}$$

for fixed values of $\boldsymbol{G}_1^{(0)}$, $\boldsymbol{G}_2^{(0)}$ and variations of the independent variables $\boldsymbol{\alpha}_1^{(0)}$, $\boldsymbol{\mu}_2^{(0)}$.

RESEARCHING SHIP MOTION MODES STABILITY

If, in the formation of subsystems of equations for statistical moments, we restrict ourselves to considering statistical moments of order no more than p, then the corresponding stability investigation will be called the investigation of p-stability [2]. In this case, local and nonlocal p-stability should be distinguished.

In the first approximation, the study of p-stability is carried out in the framework of the correlation theory of random functions, which operates with two types of statistical moments: the first and the second orders. In this case $p = 2$.

In the investigation of local stability of this type, small perturbations are introduced into the quasi-stationary mode of ship motion under consideration, and statistical moments $\boldsymbol{\alpha}_1^{(0)}$, $\boldsymbol{\mu}_2^{(0)}$ acquire additional increments $\Delta\boldsymbol{\alpha}_1(t)$ and $\Delta\boldsymbol{\mu}_2(t)$.

The values of the additional increments of the moments $\Delta\boldsymbol{\alpha}_1(t)$ and $\Delta\boldsymbol{\mu}_2(t)$ are determined by the solution of the following system of equations:

$$\frac{d\Delta\boldsymbol{\alpha}_1}{dt} = \frac{d\boldsymbol{Q}_1}{d\boldsymbol{\alpha}_1}\Delta\boldsymbol{\alpha}_1 + \frac{d\boldsymbol{Q}_1}{d\boldsymbol{\mu}_2}\Delta\boldsymbol{\mu}_2; \quad (10)$$

$$\frac{d\Delta\boldsymbol{\mu}_2}{dt} = \frac{d\boldsymbol{Q}_2}{d\boldsymbol{\alpha}_1}\Delta\boldsymbol{\alpha}_1 + \frac{d\boldsymbol{Q}_2}{d\boldsymbol{\mu}_2}\Delta\boldsymbol{\mu}_2 \quad (11)$$

For this system of differential equations, the roots λ_{il}, $i = 1,2$; $l = 1, 2, 3, ..., N$ of the characteristic equation are determined:

$$Ch(\boldsymbol{\lambda}) = 0. \quad (12)$$

In accordance with Lyapunov theory [4], the quasi-stationary motion of the ship, determined by the probabilistic characteristics $\boldsymbol{\alpha}_1^{(0)}$, $\boldsymbol{\mu}_2^{(0)}$, is stable if the real parts of the roots of the characteristic equation (12) are negative, i.e. Re $[\lambda_{il}] < 0$.

According to the accepted terminology [2], if the solution of the subsystem of equations for expected values (5) satisfies the noted Lyapunov stability requirements, then the solution of the original differential equations (4) is called stable in the mean. Correspondingly stable in the mean is the process of the ship's motion. If the noted stability requirements according to Lyapunov are satisfied for the solution of the equation of the second-order statistical moments (6), then the solution of the original differential equations (4) is called stable in the mean square. From stability in the mean square follows stability in the mean [2]. Obviously, in all the considered cases, the local stability of the quasi-stationary modes of ship motion is meant.

For investigation of the nonlocal stability, a ship that is locally stable in the equilibrium position in calm water at $\theta = 0$ is considered. The nonlocal stability of this equilibrium position of the ship under storm conditions can be investigated by gradually increasing the intensities of the existing wind–waves excitations in the ship motion equations (4). Obviously, such increase of excitations without the loss of ship stability, as in the traditional theory of ship stability represented by equations (1)–(3), can be realised only in the region of the existence of the real solutions of equations (7)–(8) for statistical moments. In this region, the kinematic characteristics of the ship movement are represented by surfaces having extremes. Therefore, the limiting values of real solutions and, correspondingly, the boundary $\boldsymbol{\Gamma}$ of the region $\boldsymbol{\Omega}$ of ship nonlocal stability will be at the extrema of these surfaces.

In this regard, the boundary $\boldsymbol{\Gamma}_1$ of the region $\boldsymbol{\Omega}_1$ of nonlocal stability in the mean of ship equilibrium position $\theta \equiv 0$ as the boundary of the stability region of the average roll angle $\boldsymbol{\alpha}_1$ is determined using the necessary condition for the extremum of the surface $\boldsymbol{Q}_1$, presented in the form

$$\frac{\partial Q_1(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2, \boldsymbol{G}_1, \boldsymbol{G}_2)}{\partial \alpha_1} = 0. \quad (13)$$

The second main characteristic of the ship roll motion is the variance of the roll oscillations $\boldsymbol{\mu}_2$. Therefore, the boundary $\boldsymbol{\Gamma}_2$ of the region $\boldsymbol{\Omega}_2$ of nonlocal stability in the mean square of the ship is found using the necessary condition for the extremum of the surface $\boldsymbol{Q}_2$:

$$\frac{\partial Q_2(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2, \boldsymbol{G}_1, \boldsymbol{G}_2)}{\partial \mu_2} = 0. \quad (14)$$

In the sequence of increasing intensities of the acting wind excitations $\boldsymbol{G}_1^{(0)}$, the boundary $\boldsymbol{\Gamma}_2$ is determined by solving the corresponding sequence of nonlinear programming tasks with the objective function

$$\begin{aligned}&\boldsymbol{OF}_2(\alpha_1, \mu_2, \boldsymbol{G}_1, \boldsymbol{G}_2) = \\ &= \boldsymbol{Q}_1^2(\alpha_1, \mu_2, \boldsymbol{G}_1, \boldsymbol{G}_2) + \boldsymbol{Q}_2^2(\alpha_1, \mu_2, \boldsymbol{G}_1, \boldsymbol{G}_2) + \left(\frac{\partial Q_2(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2, \boldsymbol{G}_1, \boldsymbol{G}_2)}{\partial \mu_2}\right)^2\end{aligned} \quad (15)$$

under variations of the independent variables $\boldsymbol{\alpha}_1^{(0)}$, $\boldsymbol{\mu}_2^{(0)}$ and $\boldsymbol{G}_2^{(0)}$.

It was shown in [5,6] that the regions of nonlocal stability $\boldsymbol{\Omega}_1$, $\boldsymbol{\Omega}_2$ and their boundaries $\boldsymbol{\Gamma}_1$, $\boldsymbol{\Gamma}_2$ should be considered not only in the space of kinematic characteristics $\boldsymbol{\alpha}_1$, $\boldsymbol{\mu}_2$, but also in the space of the parameters of the excitations $\boldsymbol{G}_1$, $\boldsymbol{G}_2$.

If the main parameters of the acting wind–waves excitations are the average wind velocity V, significant wave height $H_{1/3}$ and average period of sea T, then the nonlocal stability regions $\boldsymbol{\Omega}_1$, $\boldsymbol{\Omega}_2$ and their boundaries $\boldsymbol{\Gamma}_1$, $\boldsymbol{\Gamma}_2$ should be considered as functions of the variables V, $H_{1/3}$, and T. Such functions are determined as a result of solving a sequence of nonlinear programming tasks for the objective function (15).

In the general case, depending on the structure of the initial nonlinear ship motion equations, in the regions $\boldsymbol{\Omega}_1$, $\boldsymbol{\Omega}_2$ of nonlocal stability defined by the boundaries $\boldsymbol{\Gamma}_1$, $\boldsymbol{\Gamma}_2$ of real solutions of the equations for statistical moments, locally unstable ship motion modes may appear. Therefore, a generalised approach to the investigation of the stability of ship nonlinear motion processes should include not only a search for the boundaries of real solutions of the equations for statistical moments, but also a check of the local stability of solutions within these regions. This approach is implemented through a joint investigation of the local and nonlocal stability of the ship at the stages of increasing the intensity of constantly acting wind–wave excitations.

METHOD OF DETERMINING THE RELIABILITY OF SHIP WITH LOSSES OF NONLOCAL STABILITY IN THE MEAN SQUARE

Consideration is given to ship failures associated with capsizing under the action of wind and waves, i.e. failures which are caused by the losses of nonlocal stability of the ship equilibrium position $\theta \equiv 0$.

In this case, for the kth state of the weight load and the heading angle φ of the ship meeting with the wind and waves in the lth geographical area of its operation, the weather conditions in which are determined by the joint probability density of the long-term (regime) distribution of wind and wave characteristics $f(V, H_{1/3}, T)$, the probability of the ship nonlocal stability remaining in the mean square is determined by the expression [5,6]

$$P_{kl}(\varphi) == \iiint_{0}^{\Gamma_2(\bar{V}, H_{1/3}, T)} f(\bar{V}, H_{1/3}, T)\, d\bar{V}\, dH_{1/3}, dT, \quad \textbf{(16)}$$

where $\boldsymbol{\Gamma}_2[V, H_{1/3}, T]$ is the boundary surface of the region $\boldsymbol{\Omega}_2\{V, H_{1/3}, T\}$ of the values of parameters of the wind–waves modes withstood by the ship.

AN EXAMPLE OF SOLVING THE PROBLEM OF SHIP NONLOCAL STABILITY IN THE MEAN SQUARE

We consider a ship in beam seas (φ = 90 deg).

It is assumed that the frequency bandwidth of this ship is relatively small, therefore its roll motion is implemented in accordance with the solution of the following equation:

$$(J_x + \mu_{44})\ddot{\theta} + \lambda_{44}^{(1)}\dot{\theta} + \lambda_{44}^{(3)}\dot{\theta^3} + Dh_1\theta + Dh_3\theta^3 = M_h^{wind}(t) + M_h^{wave}(t), \quad \textbf{(17)}$$

where the heeling moment from the wind action $M_h^{wind}(t)$ is a function of the wind velocity $V = \bar{V} + \tilde{V}(t)$, and its component $\tilde{M}_h^{wind}(t)$, caused by the pulsations of the wind velocity $\tilde{V}(t)$, and the heeling moment from the waves action $\tilde{M}_h^{wave}(t)$ are represented by "white noise" in the sense of Stratonovich [9].

After introducing the notation $\theta = Y_1$; $\dot{\theta} = Y_2$, equation (17) is written in the form:

$$\frac{dY_1}{dt} = a_{1,1_2} Y_2;$$
$$\frac{dY_2}{dt} = a_{2,1_1} Y_1 + a_{2,3_1} Y_1^3 + a_{2,1_2} Y_2 + a_{2,3_2} Y_2^3 + X_2(t), \quad \textbf{(18)}$$

where

$$a_{1,1_2} = 1;$$
$$a_{2,1_1} = \frac{Dh_1}{J_x + \mu_{44}}; \quad a_{2,3_1} = \frac{Dh_3}{J_x + \mu_{44}};$$
$$a_{2,1_2} = \frac{\lambda_{44}^{(1)}}{J_x + \mu_{44}}; \quad a_{2,3_2} = \frac{\lambda_{44}^{(3)}}{J_x + \mu_{44}};$$
$$X_2(t) = X_h^{wind}(t) + X_h^{wave}(t);$$
$$X_h^{wind}(t) = \frac{M_h^{wind}(t)}{J_x + \mu_{44}}; \quad X_h^{wave}(t) = \frac{M_h^{wave}(t)}{J_x + \mu_{44}}.$$

The probabilistic characteristics of the nonlinear roll motion of the ship are considered as:

– initial statistical moments of the first order or average values of roll angles and angular velocities

$$\mathbf{M}[\theta] = \alpha_{1_1}, \ \mathbf{M}[\dot{\theta}] = \alpha_{1_2};$$

– initial statistical moments of the second order

$$\mathbf{M}[\theta^2] = \alpha_{2_1}, \mathbf{M}[\dot{\theta}^2] = \alpha_{2_2}, \mathbf{M}[\theta\dot{\theta}] = \alpha_{1_1 1_2};$$

– central moments of the second order or roll angles and angular velocities dispersions

$$\mathbf{M}[\tilde{\theta}^2] = \mu_{2_1}, \ \mathbf{M}[\tilde{\dot{\theta}}^2] = \mu_{2_2}.$$

Based on the system of stochastic differential equations (18), an appropriate system of equations for the statistical moments of the first and second orders is constructed. After the closure of this system of equations for statistical moments, using the relations for the moments of the normal distribution, we have [5]

$$\frac{d\alpha_{1_1}}{dt} = a_{1,1_2}\alpha_{1_2};$$
$$\frac{d\alpha_{1_2}}{dt} = a_{2,1_1}\alpha_{1_1} + a_{2,3_1}(3\alpha_{1_1}\alpha_{2_1} - 2\alpha_{1_1}^3)$$
$$+ a_{2,1_2}\alpha_{1_2} + a_{2,3_2}(3\alpha_{1_2}\alpha_{2_2} - 2\alpha_{1_2}^3) + G_{1_2}(t);$$
$$\frac{1}{2}\frac{d\alpha_{2_1}}{dt} = a_{1,1_2}\alpha_{1_1 1_2};$$
$$\frac{d\alpha_{1_1 1_2}}{dt} = a_{1,1_2}\alpha_{2_2} + a_{2,1_1}\alpha_{2_1}$$
$$+ a_{2,3_1}(3\alpha_{2_1}^2 - 2\alpha_{1_1}^4) + a_{2,1_2}\alpha_{1_1 1_2}$$
$$+ a_{2,3_2}(3\alpha_{1_1 1_2}\alpha_{2_2} - 2\alpha_{1_1}\alpha_{1_2}^3) + \alpha_{1_1} G_{1_2}(t);$$
$$\frac{1}{2}\frac{d\alpha_{2_2}}{dt} = a_{2,1_1}\alpha_{1_1 1_2} + a_{2,3_1}(3\alpha_{1_1 1_2}\alpha_{2_1} - 2\alpha_{1_1}^2\alpha_{1_2}) + a_{2,1_2}\alpha_{2_2} + a_{2,3_2}(3\alpha_{2_2}^2 - 2\alpha_{1_2}^4) + \alpha_{1_2} G_{1_2}(t) + \frac{1}{2} G_{2_2}(t), \quad \textbf{(19)}$$

where $G_{1_2}(t)$ and $G_{2_2}(t)$ are the coefficients of the intensity of the first and the second orders [9].

The assumption that the storm is developing slowly allows us to transform the system of differential equations (19) into the following system of nonlinear algebraic equations to determine the probabilistic characteristics of the quasi-stationary modes of ship roll motion [5]:

$$
\begin{gathered}
\mathbf{M}[\dot{\theta}] = \alpha_{1_2} = 0; \quad \mathbf{M}[\theta\dot{\theta}] = \alpha_{1_1 1_2} = 0; \\
-Dh_1\, \mathbf{M}[\theta] - Dh_3(\mathbf{M}[\theta])^3 \\
-3\, Dh_3 \mathbf{M}[\theta]\mathbf{M}[\tilde{\theta}^2] + G_{1_2}(\bar{V})(J_x + \mu_{44}) = 0; \\
\frac{1}{2}\, (J_x + \mu_{44})\mathbf{M}\left[\dot{\tilde{\theta}}^2\right] \\
-\frac{1}{2}\left\{ \begin{matrix} Dh_1 \mathbf{M}[\tilde{\theta}^2] + 3\, Dh_3\left(\mathbf{M}[\tilde{\theta}^2]\right)^2 \\ +3\, Dh_3(\mathbf{M}[\theta])^2 \mathbf{M}[\tilde{\theta}^2] \end{matrix} \right\} = 0; \\
-\lambda_{44}^{(1)} \mathbf{M}[\dot{\tilde{\theta}}^2] - 3\lambda_{44}^{(3)}(\mathbf{M}[\dot{\tilde{\theta}}^2])^2 + \frac{1}{2}\, [G_{2_2}(\bar{V}) + G_{2_2}(H_{\frac{1}{3}})](J_x + \mu_{44}) = 0,
\end{gathered} \tag{20}
$$

where

$$G_{1_2}(\bar{V})(J_x + \mu_{44}) = \mathbf{M}\left[M_h^{wind}(\bar{V})\right] = (\bar{V})^2/2c(z_s)(z_s - z_h)A_s$$

is the average heeling moment of the wind action:
$c(z_s)$ is the coefficient of air flow resistance, A_s and z_s are the sail area and the applicate of the sail area centre, z_h is the applicate of the line of the drift force action;

$$G_{2_2}(\tilde{V}) = [\rho_A\, (\bar{V})^2/2c(z_s)(z_s - z_h)A_s]^2 \times (\omega_\theta)(J_x + \mu_{44})^{-2}$$

is the average power of the pulsation component of the wind exciting moment;
$S_v(\omega_\theta)$ is the spectral density of the wind velocity;
ω_θ is the frequency of small free roll oscillations of the ship;

$$G_{2_2}(H_{1/3}) = [Dh_1 \kappa(\omega_\theta)(\omega_\theta)^2/g]^2 \times S_\zeta(\omega_\theta)(J_x + \mu_{44})^{-2} \times$$

is the average power of the wave exciting moment;
$\kappa(\omega_\theta)$ is the reduction coefficient of the main part of the wave exciting moment;
$S_\zeta(\omega_\theta)$ is the wave spectral density.

In equations (20), the second-order intensity factor G_{2_2} is written without taking into account the correlation between wind and wave heeling moments.

The physical interpretation of the last three equations of the system of equations for statistical moments (20) is determined by the following expressions:

$$-\bar{M}_r + \bar{M}_h = 0; \tag{21}$$

$$-\bar{E}_k + \bar{E}_p = 0; \tag{22}$$

$$-\bar{P}_d + \bar{P}_w = 0, \tag{23}$$

where $\bar{M}_r$ is the average value of the righting moment of the ship; $\bar{M}_h$ is the average value of the heeling moment from the wind action; $\bar{E}_k$ is the average kinetic energy of the ship roll oscillations; $\bar{E}_p$ is the average potential energy of the ship position during roll motion; $\bar{P}_d$ is the average power of the energy dissipation of the oscillating ship; $\bar{P}_w$ is the average power of the exciting moment due to the action of excitations.

Equations (21)–(23) will be called, respectively, the equations of average moments, average energies and average powers of the quasi-stationary ship motion mode. The first two of these equations are similar to the relations of the traditional theory of ship stability at large angles of inclination – the equation of moments (2) and the equation of works (energies) (3).

The boundary $\boldsymbol{\Gamma}_2$ of the region $\boldsymbol{\Omega}_2$ of nonlocal stability in the mean square in the considered case is determined by the maximum of the average potential energy $\bar{E}_p$, the position of which, as shown by the system of equations (20), depends on the average angle of heel. The value of this maximum is determined by the formula

$$
\begin{gathered}
\frac{\partial Q_2(\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2, \boldsymbol{G}_1, \boldsymbol{G}_2)}{\partial \mu_2} = \frac{\partial \bar{E}_p}{\partial \mu_{2_1}} = \frac{1}{2}\left\{Dh_1 + 6\, Dh_3 \boldsymbol{M}[\tilde{\theta}^2] + \right. \\
\left. 3\, Dh_3(\mathbf{M}[\theta])^2 \right\} = 0.
\end{gathered} \tag{24}
$$

In accordance with the method for investigating the nonlocal stability of the ship under constant action of random wind–waves excitations, an analysis of the capsizing of ship No 6010 with a displacement of Δ = D = 4905 kN (L = 38.7 m, B = 7.0 m, d = 2.98 m) in beam position to the wind and waves was performed. The diagrams of the static and dynamic stability of this ship are presented in Figs. 1 and 2. The accident of the ship took place with wind of ~ 7 state under the conditions of developing waves with the intensity of ~ 4 state. The dependence of the average wave period T on the wave intensity $H_{1/3}$ was estimated by the relation $T = 3.217\sqrt{H_{1/3}}$.

To analyse this accident, a number of solutions of the system of quasi-stationary equations for statistical moments (18) were obtained with a gradual increase in the intensity of wind–waves excitations from the equilibrium position of the ship in calm water at $\theta = 0$ to the loss of nonlocal stability in the mean square. The solution of equations (20) and calculation of the criterion of nonlocal stability in the mean square (24) were obtained by using the generalised reduced gradient method in nonlinear programming problems (9) and (15).

The characteristics of the considered stages of increasing wind and wave intensity and the results of solving the equations for statistical moments (20) are presented in Table 1. Table 2 summarises the main characteristics of the corresponding roll motion modes under conditions of increasing storm intensity.

Tab. 1. Results of solving the system of equations for statistical moments (20)

Stages of storm	$\bar{V}$, m/s	$H_{1/3}$, m	α_{1_1}, rad	μ_{2_1}, rad^2	μ_{2_2}, (rad/s)^2	$G_1(\bar{V})$, s^{-2}	0,5 $G_2(H_{1/3})$, s^{-3}
1	1.00	0.0043	0.00009	0.00000	0.00000	0.00000	0.00000
2	5.00	0.1076	0.00234	0.00000	0.00000	0.00204	0.00000
3	9.00	0.3485	0.00761	0.00003	0.00003	0.00662	0.00000
4	14.00	0.8433	0.01866	0.00048	0.00041	0.01603	0.00002
5	15.00	0.9680	0.02163	0.00081	0.00069	0.01840	0.00004
6	16.00	1.1014	0.02533	0.00187	0.00153	0.02093	0.00009
7	17.00	1.2434	0.03199	0.00577	0.00418	0.02363	0.00016
8	17.50	1.3176	0.04140	0.01169	0.00678	0.02504	0.00040
9	17.65	1.3403	0.05690	0.01840	0.00737	0.02547	0.00044

Tab. 2. Main characteristics of the ship roll motion modes

Stages of storm	$\bar{V}$, m/s	$\bar{M}_h$, kNm	$\bar{P}_w$, kW	α_{1_1}, deg	$\sqrt{\mu_{2_1}}$, deg	$2\bar{E}_k$, kNm	$2\bar{E}_p$, kNm
1	1.00	0.00000	0.00000	0.00537	0.00000	0.00000	0.00000
2	5.00	5.41510	0.00005	0.13437	0.01812	0.00001	0.00001
3	9.00	17.54494	0.00414	0.43586	0.31352	0.03450	0.03450
4	14.00	42.45441	0.06346	1.06932	1.25622	0.54350	0.54350
5	15.00	48.73593	0.10620	1.23922	1.63369	0.90860	0.90860
6	16.00	55.45066	0.23718	1.45137	2.47762	2.02344	2.02344
7	17.00	62.59860	0.42076	1.83292	4.35219	5.53135	5.53135
8	17.50	66.33502	1.07138	2.37244	6.19519	8.97896	8.97896
9	17.65	67.46685	1.16741	3.26011	7.77259	9.76429	9.76429

Tab. 3. Results of the investigation of local stability of ship roll motion modes in developing storm, i.e. the roots of the characteristic equation (12) corresponding to the system of differential equations for statistical moments (19)

Stages of storm	$\bar{V}$, m/s	α_{1_1}	α_{1_2}	μ_{2_1}	$\alpha_{1_1 1_2}$	μ_{2_2}
1	1.00	-0.028 +0.964i	-0.028 -0.964i	-0.028 +0.906i	-0.028 -0.906i	-0.062
2	5.00	-0.028 +0.964i	-0.028 -0.964i	-0.028 +0.905i	-0.028 -0.905i	-0.062
3	9.00	-0.028 +0.964i	-0.028 -0.964i	-0.028 +0.897i	-0.028 -0.897i	-0.062
4	14.00	-0.028 +0.964i	-0.028 -0.964i	-0.028 +0.867i	-0.028 -0.867i	-0.062
5	15.00	-0.028 +0.964i	-0.028 -0.964i	-0.028 +0.854i	-0.028 -0.854i	-0.062
6	16.00	-0.028 +0.958i	-0.028 -0.958i	-0.028 +0.825i	-0.028 -0.825i	-0.063
7	17.00	-0.029 +0.919i	-0.029 -0.919i	-0.027 +0.722i	-0.027 -0.722i	-0.065
8	17.50	-0.029 +0.858i	-0.029 -0.858i	-0.026 +0.529i	-0.026 -0.529i	-0.068
9	17.65	-0.030 +0.800i	-0.030 -0.800i	-0.000 +0.000i	-0.000 -0.000i	-0.093

For each stage of the developing storm, the local stability of the quasi-stationary modes of the ship roll motion was also investigated. The roots of the characteristic equation (12) corresponding to the system of differential equations for statistical moments (19) are given in Table 3. This table shows the fulfillment of the criterion of nonlocal stability in the mean square (24) on the boundary $\boldsymbol{\Gamma}_2$ of the domain $\boldsymbol{\Omega}_2$.

The data of Tables 1 and 2 are presented in Figs. 3–6.

Fig. 3 shows the diagram of the average righting moments $\bar{M}_r = f_1(\alpha_{1_1}, \sqrt{\mu_{2_1}})$ as a function of the average heel angle $m_\theta = \alpha_{1_1}$ and the mean square deviation of the ship's roll oscillations $SD_\theta = \sqrt{\mu_{2_1}}$. At $SD_\theta = \sqrt{\mu_{2_1}}$, the function $\bar{M}_r = f_1(\alpha_{1_1}, \sqrt{\mu_{2_1}})$ represents the diagram of ship static stability shown in Fig. 1.

Fig. 4 represents the dependence of the ship average potential energy $2\bar{E}_p = 2f_2(\alpha_{1_1}, \sqrt{\mu_{2_1}})$ on the kinematic characteristics of the ship roll motion and .

Figs. 3 and 4 respectively show: the boundary $\boldsymbol{\Gamma}_1$ (Boundary 1) of the region $\boldsymbol{\Omega}_1$ of nonlocal stability in the mean and the boundary $\boldsymbol{\Gamma}_2$ (Boundary 2) of the region $\boldsymbol{\Omega}_2$ of nonlocal stability in the mean square of the ship equilibrium position in calm water $\theta \equiv 0$.

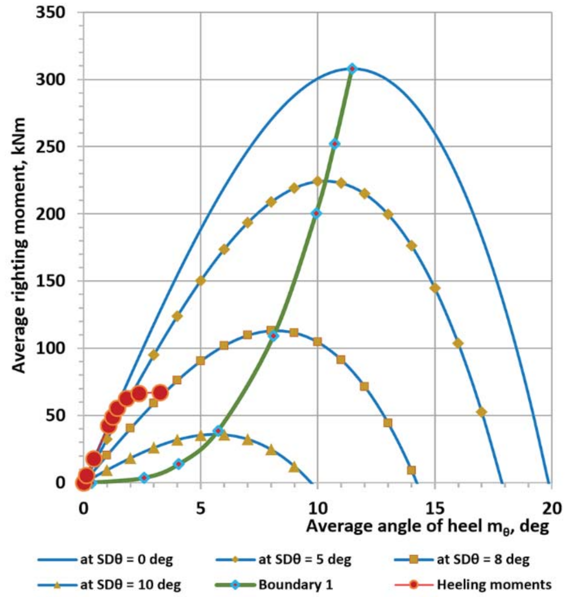

Fig. 3. Diagram of average righting moments with boundary $\boldsymbol{\Gamma}_1[\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2]$ *(Boundary 1) of the domain* $\boldsymbol{\Omega}_1\{\boldsymbol{\alpha}_1, \boldsymbol{\mu}_2\}$ ***nonlocal stability in the mean*** *of the ship equilibrium position in calm water* $\theta \equiv 0$ *and values of average heeling moments* $\bar{M}_h$ *of the* ***wind action*** *considered in Tables 1, 2 on the stages of storm growth*

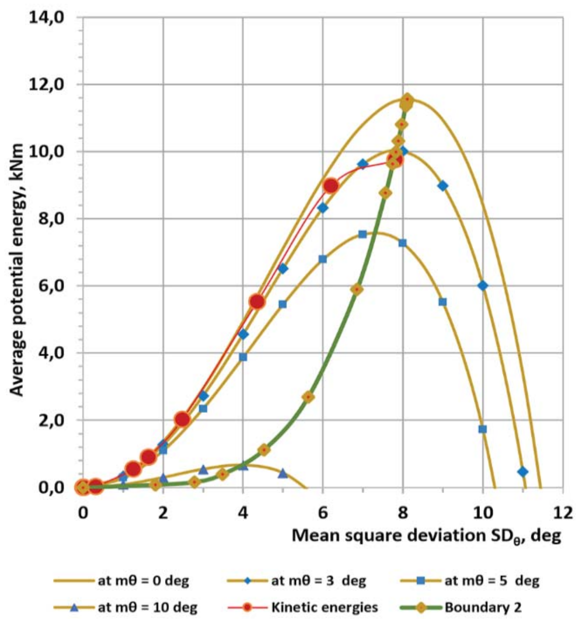

Fig. 4. Diagram of the average potential energies $2\bar{E}_p$ *with the boundary* ***Γ2 [α1,μ2]*** *(Boundary 2) of the region* $\boldsymbol{\Omega}_2$***{α1,μ2}*** *of* ***nonlocal stability in the mean square*** *of the ship equilibrium position in calm water* $\theta \equiv 0$ *and the average kinetic energies* $2\bar{E}_k$ *of the* ***wind and waves action*** *considered in Tables 1, 2 on the stages of storm growth*

In Figs. 5 and 6, the surfaces $\bar{M}_r = f_1(\alpha_{1_1}, \sqrt{\mu_{2_1}})$ and $2\bar{E}_p = 2f_2(\alpha_{1_1}, \sqrt{\mu_{2_1}})$ are presented in the coordinates $m_\theta = \alpha_{1_1}, SD_\theta = \sqrt{\mu_{2_1}}$ together with the solutions of the equations for statistical moments (20) as the stages of storm intensity increase.

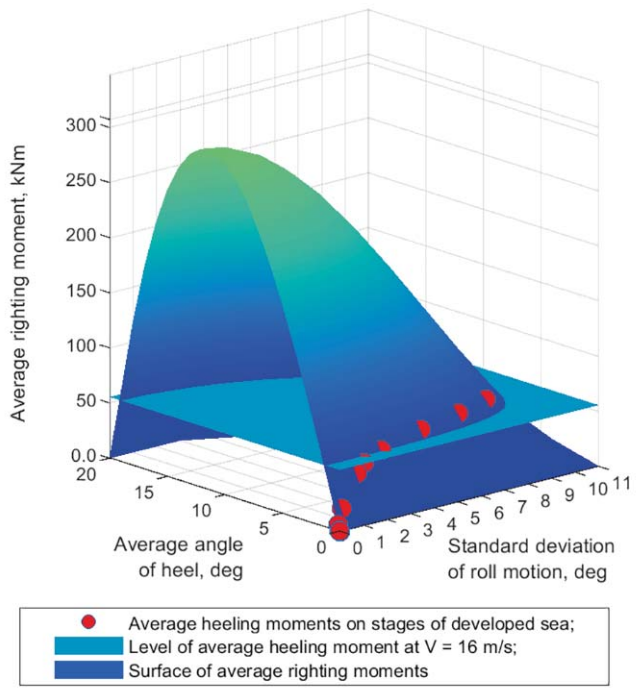

*Fig. 5. Diagram of average righting moments with average values of heeling moments $\overline{M}_h$ of the **wind action** on the stages of storm growth presented in Tables 1, 2*

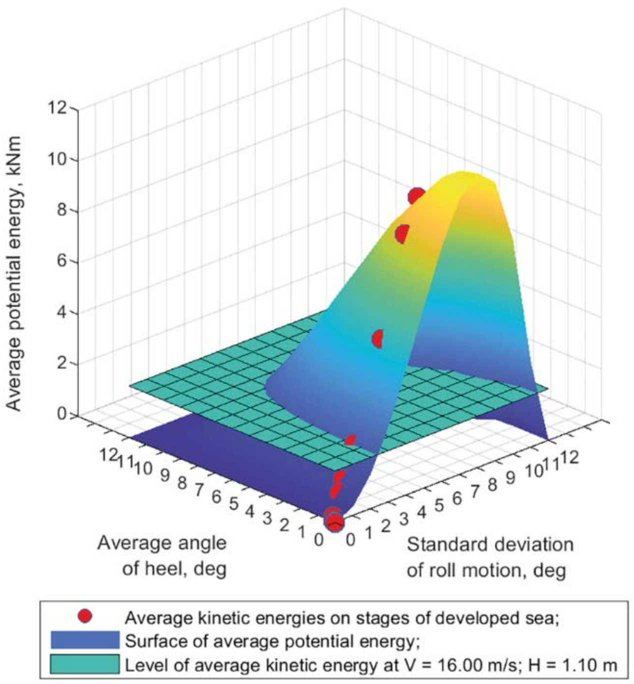

*Fig. 6. Diagram of average potential energies $2\overline{E}_p$ with values of average kinetic energies $2\overline{E}_k$ of the **wind and waves** action on the stages of storm growth presented in Tables 1, 2*

The algorithm for reflecting the region of nonlocal stability of a ship, from the space of kinematic characteristics of motion to the space of the parameters of the acting wind–waves excitations described in [5], was implemented in the considered example when solving the sequence of tasks for the equations of statistical moments (20) by the method of nonlinear programming with the objective function (15). As a result of using the one-parameter spectra of wind velocity $S_v(\omega)$ and sea $S_\zeta(\omega)$, the boundary $\boldsymbol{\Gamma}_2$ of the region $\boldsymbol{\Omega}_2$ of the ship nonlocal stability in the mean square in the space parameters of wind–waves modes (regimes) such as the average wind velocity V, m/s and the characteristic wave height $H_{1/3}$, m was received. This boundary is shown in Fig. 7.

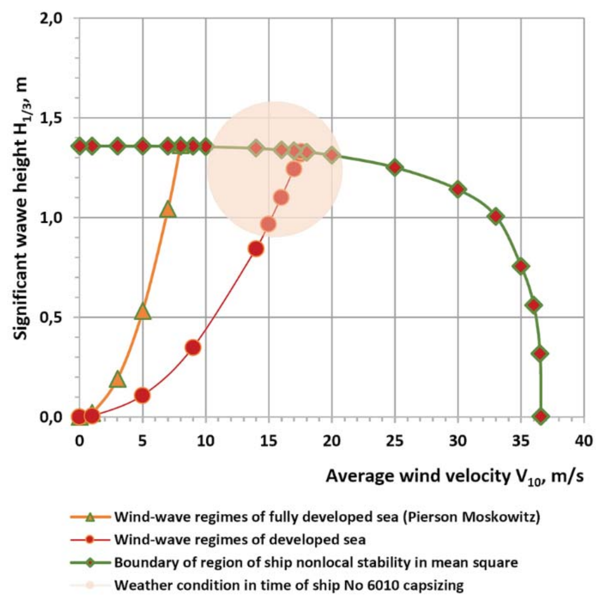

Fig. 7. Boundary $\boldsymbol{\Gamma}_2[V, H_{1/3}]$ of the region $\boldsymbol{\Omega}_2\{V, H_{1/3}\}$ of ship nonlocal stability in the mean square and wind-waves modes withstood by the ship at the stages of storm intensity increasing specified in Tables 1, 2

Fig. 7 also shows a number of the wind–waves modes withstood by the ship when searching for the boundary of the region of the existence of real solutions of equations (20) by using the nonlinear programming algorithm with the objective function (9). At the limit point of the sequence of solutions with the objective function (9), the same results were obtained as in determining the boundary of the region of ship nonlocal stability in the mean square, the solution of which is realised using the nonlinear programming task with the objective function (15). This means that the boundary of the region of nonlocal stability in the mean square of the ship equilibrium position $\theta \equiv 0$ is the boundary of the real solutions of the equations for the statistical moments (20), and at this boundary the potential resources of the ship are exhausted with respect to increasing constantly acting wind–waves excitations.

AN EXAMPLE OF SOLVING THE PROBLEM OF SHIP RELIABILITY UNDER ACTION OF WIND AND WAVES

An event of maintaining the nonlocal stability of ship under the action of the wind–waves modes of its operating area is considered. The probability of such event $P_{kl}(\varphi)$ when using one-parameter characteristics of the wind and waves is calculated by the formula

$$P_{kl}(\varphi) = \iint\limits_{0}^{\Gamma_2(\bar{V}, H_{1/3})} f(\bar{V}, H_{1/3}) d\bar{V} dH_{1/3}. \quad (25)$$

The calculation scheme for the determination of such reliability indicator $P_{kl}(\varphi)$ is presented in Fig. 8. This indicator determines the relative number of wind–waves modes of the operation area withstood by the ship.

When calculating the reliability indicator by formula (25), we use in the equations for statistical moments (20) the one-parameter wave spectrum $S_\zeta(\omega)$, which depends only on $H_{1/3}$. The use of the two-parameter ITTC spectrum, which depends on the characteristic height $H_{1/3}$ and the average wave period T, allows us to clarify the position of the boundary of the region of ship nonlocal stability in the mean square in the space of such parameters of the acting excitations as $\bar{V}$, $H_{1/3}$, T [5,6]. The corresponding solution of the system of equations for statistical moments (20) with the definition of the boundary $\boldsymbol{\Gamma}_2$ [V, $H_{1/3}$, T] of region $\boldsymbol{\Omega}_2$ {V, $H_{1/3}$, T} of ship nonlocal stability in the mean square is shown in Fig. 9.

This solution allows us to clarify also the reliability indicator determined by the formula (15). The result of the refined definition of the boundary of the region of nonlocal stability in the mean square and the interpretation of the use of formula (15) to find the value of the reliability indicator P_{kl} in the space V, $H_{1/3}$, are presented in Fig. 10.

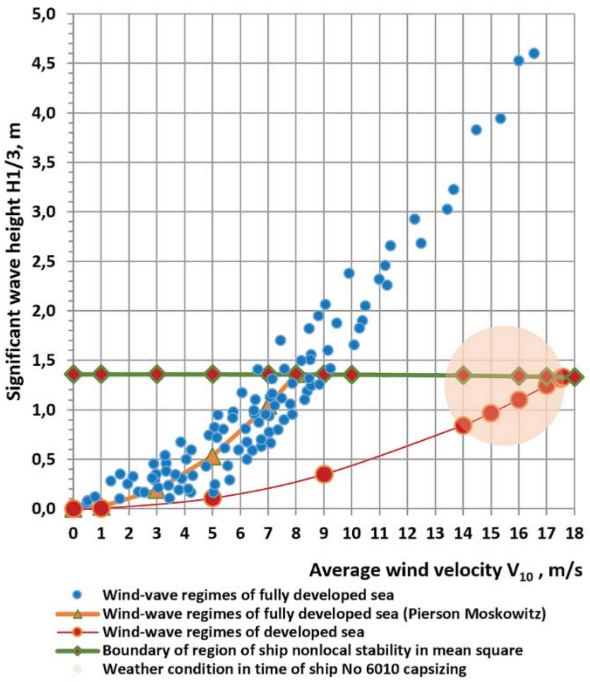

Fig. 8. The determination of the reliability indicator $P_{kl}(\varphi)$ of the ship in given area of operation, i.e. the probability of its operation without capsizing in accordance with formula (25)

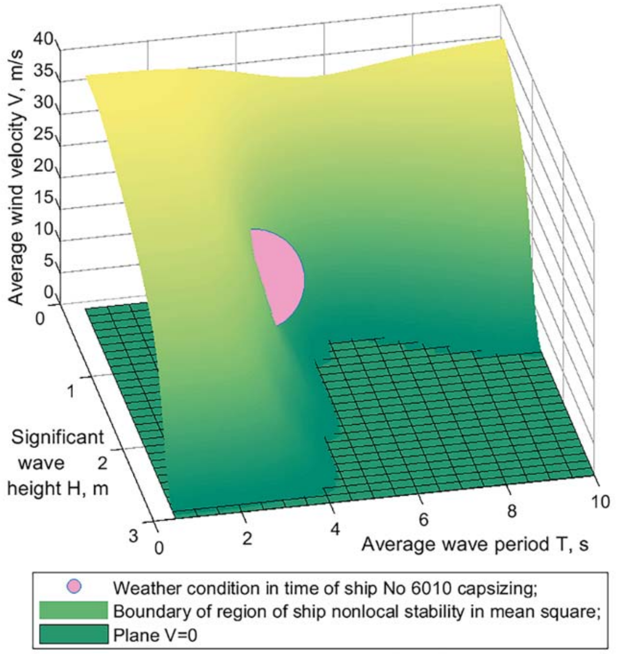

Fig. 9. Boundary $\boldsymbol{\Gamma}_2$[V, $H_{1/3}$, T] of the region $\boldsymbol{\Omega}_2${V, $H_{1/3}$, T} of nonlocal stability in the mean square and weather conditions for ship No 6010 accident

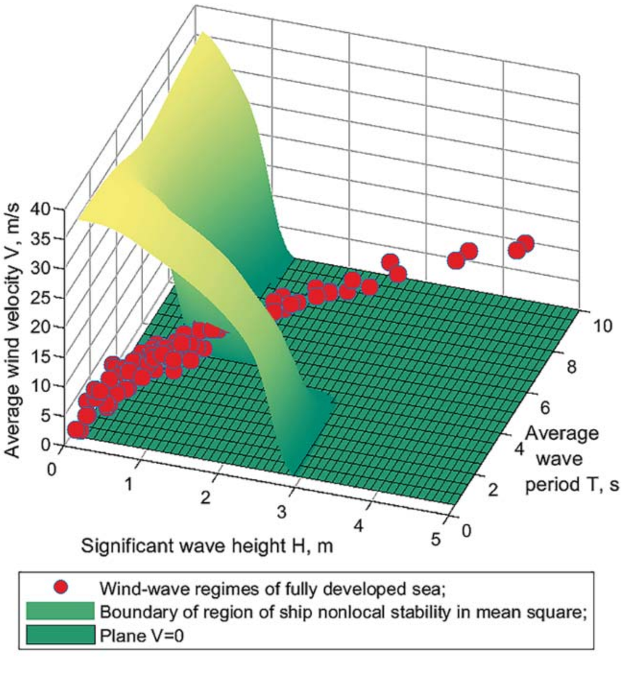

Fig. 10. On determining the reliability indicator of the ship in area of its operation $P_{kl}(\varphi)$, i.e. the probability of ship operation without capsizing in accordance with formula (15)

CONCLUSIONS

1. The proposed complex method of investigating the stochastic nonlocal stability and reliability of the ship in storm conditions of operation has two characteristic features that condition its novelty in ship seakeeping theory. Firstly, it

continues the traditions of the widely known deterministic stability theory of ships at large angles of inclination, both from the point of view of creating new and more realistic approaches for finding the boundaries of areas of nonlocal stability of the ship under actions of wind–waves modes, and from the point of view of improving the physical interpretation of the studied phenomena. Secondly, the method complements the procedure for determining the losses of ship nonlocal stability under the action of different wind–waves modes with an algorithm for calculating the corresponding indicator of ship reliability with the help of the long-term distribution of these modes characteristics for the given area of ship operation.

2. The results of the application of the proposed method of investigating mean-square nonlocal stability are in satisfactory agreement with the results of ship No. 6010 capsizing under the action of the wind–waves mode of the developing sea.

REFERENCES

1. Himmelblau D. M. (1972): *Applied nonlinear programming*. McGraw Hill, New York.

2. Khasminsky R. Z. (1969): *Stability of systems of differential equations under random perturbations of their parameters*. Fizmatgiz, Moscow.

3. Krasovsky N. N. (1959): *Some problems of the theory of stability of motion*. Fizmatgiz, Moscow.

4. Lyapunov A. M. (1950): *The general problem of motion stability*. GITTL, Moscow, Leningrad.

5. Nekrasov V. A. (1978): *Probabilistic problems of ships seakeeping*. Shipbuilding, Leningrad.

6. Nekrasov V. A. (1985): *Seakeeping and technical stability of the ship. Hydrodynamics of the ship*. Collection of NSI scientific papers. Nikolayev Shipbuilding Institute, Nikolayev.

7. Nekrasov V. A. (1985): *Criteria of seakeeping and reliability indicators for the use of the ship and its technical means in given area of operation. Hydrodynamics of the ship*. Collection of NSI scientific papers. Nikolayev Shipbuilding Institute, Nikolayev.

8. Nekrasov V. A. (2013): *Nonlocal stability of mobile marine objects*. MAST 2013, Gdansk, Poland.

9. Stratonovich R. L. (1961): *Selected issues of the theory of fluctuations in radio techniques*. Soviet Radio, Moscow.

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