



Shear Capacity and Residual Strengths of Steel Fibre Reinforced Waste Sand Concrete (SFRWSC)

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1. Introduction

Concrete is the most commonly used structural material today and concrete construction has been developing dynamically throughout the world in recent years (Neville 2000, Czarnecki & Kurdowski 2005, Piekarski et al. 2008, Jasiczak et al. 2008) Generally available and relatively inexpensive to produce, concrete is widely used in both industrial and residential construction. However, modern constructions pose a serious challenge to engineers who must meet requirements regarding limit states under extremely varied conditions of static and dynamic interactions. Unfortunately, concrete is a brittle and hardly deformable material, which means it is vulnerable to cracking. Its low tensile strength and vulnerability to crack propagation are a major reason why new types of concrete, whose physico-mechanical parameters would meet the current requirements, are constantly developed. The addition of steel fibres is one way to improve the properties of brittle concrete. Concretes reinforced with steel fibres, generally referred to as fibre-reinforced composites, are becoming increasingly popular. In some cases, they are used as an alternative to ordinary concrete. The addition of e.g. steel (most commonly used) fibres increases the plasticity and cracking resistance of these materials (Leung 2004). Fibre-reinforced concrete originated in the 19th century when A. Bernard first patented concrete reinforced with steel fibres in 1874. Since that year, attempts have been made to assess both the influence of fibres on concrete's properties and fibre distribution within fibre-reinforced concrete. Consequently, fibre-reinforced concrete has become an alternative to ordinary concrete. The most important advantages of fibre-reinforced concrete are: higher compressive strength at early stages of concrete hardening in comparison to ordinary concrete (Ding & Kusterle 2000), higher tensile strength (Yazici et al. 2007, Głodkowska & Kobaka 2010) and flexural tensile strength (Leung

2004), higher splitting tensile strength (Głodkowska & Kobaka 2010), high dynamic resistance (Tso-Liang 2008, Zhi-Liang 2008), limited crack propagation in construction elements (Uygunoğlu 2008), better behaviour at failure (Wang et al. 2010) (failure does not occur abruptly), possible reduction of conventional reinforcement, resistance to high temperatures (Sukontasukkul 2010). Fibre-reinforced composites are also characterized by higher abrasion resistance and better durability compared with ordinary concrete. On the negative side, steel fibres worsen the workability of the mix and the cost of steel fibres is relatively high – a composite with the volume content of steel fibres equal to 1% constitutes 90% of the price for C25/30 ordinary concrete. However, this cost can be reduced by roughly 30% if expensive gravel is replaced with waste sand.

The field of application for fibre-reinforced concretes in construction is varied. Some examples (Bank 2009, Shakya 2012, Zollo 1997, Salehian & Barros 2015, Arnau & Molins 2011, Schimmelpfennig & Borgerhoff 1995) include: beams, slabs, elements of thin shells, industrial floors, elements of bridges and tunnels, elements reinforcing excavation sites and constructions exposed to seismic hazard (Sevil et al. 2011), elements of nuclear power plants when a high level of safety must be ensured e.g. during a terrorist attack or an earthquake. They are used particularly in elements subjected to significant dynamic loads (Borgerhoff 1995).

Flexural/shear design play an important role in the production of structural elements made from fibre-reinforced composites. The issue of shearing concrete elements reinforced with steel fibres has been an object of research and theoretical analyses for years (Kaushik et al. 1987, Roberts et al. 1983, Swamy et al. 1985, Narayanan & Darwish 1987). For more than a quarter-century, experimental studies have been conducted to investigate the shear behaviour of fibre-reinforced concrete beams with changes of parameters having a major influence on the transmission of transverse forces. It has been confirmed that steel fibres not only increase shear capacity by transmitting tensile stresses but also prevent crack propagation in the same way stirrups and bent up bars do. In spite of extensive research confirming steel fibres' beneficial influence on the properties of concrete, no comprehensive and reliable method for calculating shear capacity has been found.

In Europe, the methods for calculating shear capacity of fibre-reinforced concrete elements are presented in RILEM TC 162-TDF and fib ModelCode 2010 provisions. They are based on the knowledge of residual strengths, which serve as basic values for classifying SFRWSC. Knowing the values of residual strengths, one can determine ultimate moment capacity as well as shear capacity of an element made from a fibre-reinforced composite. It should be pointed out that calculation methods using residual strengths are based on the currently valid standard Eurocode 2 for designing concrete structures.

2. Aim and significance of research

In the Pomerania region of Poland, aggregate deposits occur as a mix of fine and coarse aggregates. Huge demand for coarse aggregates has contributed to the development of hydroclassification (an extraction technology involving washing out aggregates from deposits). A by-product of hydroclassification are piles of sand without coarse fractions (Fig. 1).



Fig. 1. View of heaps of waste sands in Pomerania (Poland)

Excavation sites thus created should become an object of costly reclamation. However, the use of waste sand as a structural material can offer an alternative to reclamation. Partially replacing ordinary concrete with SFRWSC may considerably limit environmental degradation. It will also contribute to the gradual reduction of sand piles and make exploitation of regional aggregates possible.

Partially replacing ordinary concrete with materials which exhibit the same or even better properties is an excellent solution for the regions where natural deposits of coarse aggregate are scarce e.g. the region of Pomerania in Poland (where only 4% of total Polish coarse aggregates occur), the Middle East and North Africa.

This article presents physico-mechanical properties of SFRWSC as an alternative to ordinary concrete. It also contains research results concerning residual strengths (a basic feature in designing structural elements) of fine-grained composites with varying steel fibre content. The next task was to calculate shear capacities of beams made from SFRWSC in accordance with RILEM TC 162-TDF and fibModelCode 2010. It has been demonstrated that SFRWSC, whose properties are similar or better than those of ordinary concrete, can be used to produce structural elements as regards shear capacity.

3. Materials and test specimens

In order to produce test specimens with determined residual strengths, portland cement CEM II/A-V 42,5R (420 kg/m³), silica fume (21 kg/m³), superplasticizer (16.8 kg/m³) and municipal water (160 kg/m³) were used. Considerable homogeneity of grading as well as smooth grading curve characterised the aggregate (Fig. 2). The waste sand used, as demonstrated by the analysis of the test results [3], meets the requirements for mineral aggregates for regular concrete. Reinforcement is steel hooked-end fibres with the aspect ratio $\lambda = l/d = 62.5$ ($l = 50$ mm, $d = 0.8$ mm). The matrix of the composite was designed using an analytical-experimental method. The composition of the matrix was modified by adding superplasticizer and silica fume, which enabled a $w/c = 0.38$ ratio to be obtained. Steel fibre content was adopted as the member variable – steel fibres were dosed in 0.5% increments, in relation to the volume of the composite, up to the content of 2.5%. The composite mix was characterised by random distribution of steel fibres.

The research showed that the fine-grained composite reinforced with steel fibres (Table 1) can provide an alternative to ordinary concrete as regards structural design. Partially replacing ordinary concrete with a fibre-reinforced composite which exhibits the same or even better properties is an excellent solution for the regions where natural deposits of coarse aggregate are scarce e.g. the region of Pomerania in Poland (where only 4% of total Polish coarse aggregates occur), the Middle East and North Africa. The detailed characteristics of the fibre-reinforced composite used in the research are presented in the paper (Głodkowska & Kobaka 2010, Głodkowska & Laskowska-Bury 2015).

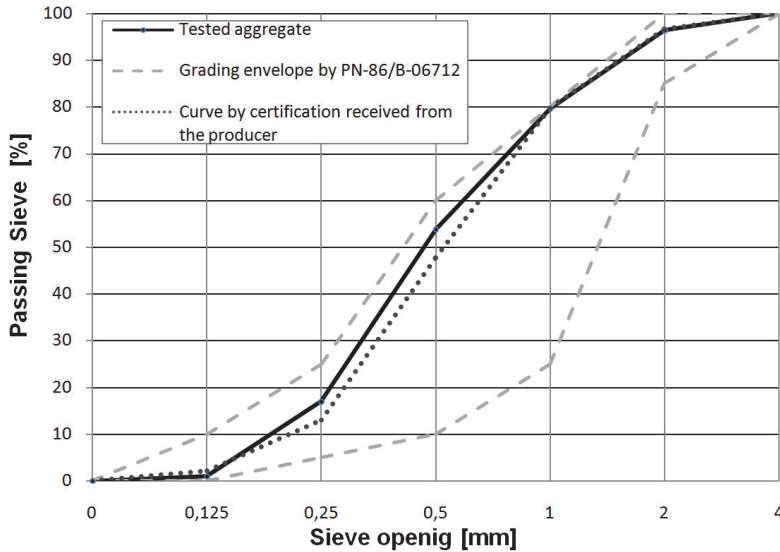


Fig. 2. Smooth grading curve of the aggregate used in the research

Table 1. Basic properties of SFRWSC and ordinary concrete (Głodkowska & Kobaka 2010, Głodkowska & Laskowska-Bury 2015)

Properties of fiber composite	Fibre volume fraction, V_f [%]							regression function / correlation coefficient	properties of ordinary concrete
	0.0	0.5	1.0	1.2	1.5	2.0	2.5		
Apparent density $[\text{g}/\text{cm}^3]$	2.1	2.2	2.3	2.3	2.3	2.3	2.3	$\rho = 2.352 - 0.216e^{-0.938V_f}$ $r = 0.97$	2.0-2.6 (PN-EN 12390-7)
compressive strength $f_{c,cyl}$ $[\text{N}/\text{mm}^2]$	44	51.8	61.4	64.4	61.6	61.3	61.9	$f_c = 62.77 - 19.22e^{-1.61V_f}$ $r = 0.90$	1-50 (PN-EN 1992-1-1)
compressive strength $f_{c,cube}$ $[\text{N}/\text{mm}^2]$	44.5	51.8	61.4	67.6	61.6	61.3	61.9	$f_{c,cube} = 62.77 - 19.22e^{-1.61V_f}$ $r = 0.90$	15-60 (PN-EN 1992-1-1)
split tensile strength $f_{t,split}$ $[\text{N}/\text{mm}^2]$	3.3	5.5	7.7	7.3	8.3	8.8	9.2	$f_{t,split} = 9.75 - 6.52e^{-0.99V_f}$ $r = 0.97$	3.0-3.7 (PN-EN 12390-6)
static modulus of elasticity E_{cm} [GPa]	32.9	33.3	34.5	36.7	34.7	34.0	33.9	$E_c = 32.7 + 2.35V_f - 0.76V_f^2$ $r = 0.88$	29-37 (PN-EN 12390-13)
dynamic modulus of elasticity E_d [GPa]	41.5	43.7	45.8	45.9	46.3	46	45.5	$E_d = 41.44 + 5.7V_f - 1.65V_f^2$ $r = 0.9$	$E_{cm} = 0.83 E_d$ (wg Neville A.M. 2000)
shrinkage ϵ_{cs} [%]	0.91	0.89	0.85	0.88	0.84	0.78	0.75	$\epsilon_s = 0.898(1 - e^{-0.2t^{0.6}}) - 0.048V_f$ $r = 0.93$	0.2-0.6 (ITB 194/98)
Workability Ve-Be test [s]	4.2	6.4	9.5	12	14.3	21.5	32.2	$K = 4.24e^{0.8V_f}$ $r = 0.87$	3-31 (PN-EN 12350-3:2009)
Abrasion resistance A $[\text{cm}^3/50\text{cm}^2]$	5.32	6.90	8.17	9.0	8.20	8.16	8.24	$A = 0.04f_{cyl}^{1.28}$ $r = 0.99$	1.,5-22 (PN-EN13892-3)
Dynamic modulus of rigidity G_d [GPa]	17.8	18.54	18.33	-	18.81	18.60	18.23	$G_d = 17.83 + 1.146V_f - 0.391V_f^2$ $r = 0.79$	-

The composite's residual strengths serving to calculate shear capacity were determined on forty-seven $150 \times 150 \times 700$ mm prisms. Six prisms were not reinforced with steel fibres whereas other prisms contained steel fibres in the following amounts $V_f = 0.5\%$ (8 prisms) and $V_f = 0.9\%$ (8 prisms). The research presented in the paper (Głodkowska & Kobaka 2010, Głodkowska & Laskowska-Bury 2015) showed that the optimal (exceeding the properties of ordinary concrete), maximum fibre content in SFRWSC is equal to 1.2%. For this fibre content, residual strengths were determined on 31 prisms. Before being tested, the prisms were stored for 28 days at a temperature of $20 \pm 2^\circ\text{C}$ and at a relative air humidity of about 100%. Next, they were stored for 24 hours under laboratory conditions (temp. $20 \pm 2^\circ\text{C}$, relative humidity $50 \pm 5\%$). The specimens were tested after 29 days of curing.

4. Test methodology

A research on SFRWSC residual strengths was conducted in accordance with PN-EN 14651. Fig. 3 shows a test set-up where cracking resistance in bending was determined in the form of the so-called residual strengths.

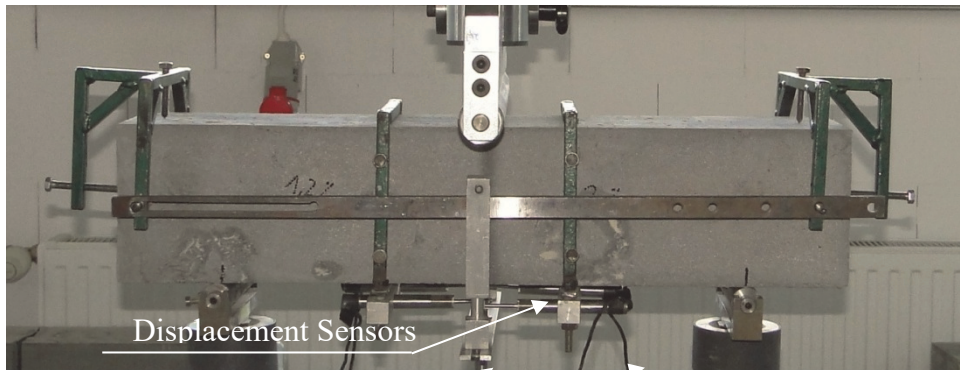


Fig. 3. View on the prisms' set-up, load application and displacement sensors location in the residual strength test

In order to test residual strengths, constant static load was applied to the prisms. Load increase values were set depending on crack mouth opening displacement (*CMOD*). In the case of prisms without steel fibres, the test was terminated when the specimen underwent flexural failure. In the case of fibre-reinforced prisms, the test ended when the specimen's deflection was equal to 5 mm. Ultimate deflection of the prisms was set in accordance with PN-EN 14651 standard in order to achieve all *CMOD* values and to determine residual strengths ($f_{R,i}$)

for the corresponding value $CMOD_j$, in which $j = 1, 2, 3, 4$. The quantities $f_{R,1}, f_{R,2}, f_{R,3}, f_{R,4}$ correspond to the tensile stresses associated to the force at a given $CMOD$, which were equal to 0.5, 1.5, 2.5, 3.5 mm respectively.

Residual strength can be determined by following expression:

$$f_{R,j} = \frac{3 \cdot F_j \cdot l}{2 \cdot b \cdot h_{sp}^2} \quad (1)$$

where:

$f_{R,j}$ – residual strength [N/mm²],

F_j – load value [N],

l – length of test specimen [mm],

b – width of test specimen [mm],

h_{sp} – distance between tip of the notch and top of cross section [mm].

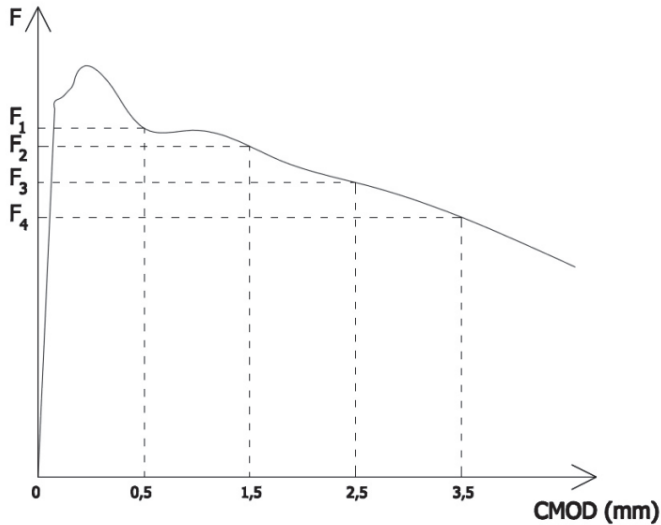


Fig. 4. Graphical interpretation of $F_1 - F_4$ determination

An important parameter enabling a fibre-reinforced composite to be classified is the chart shape “load- $CMOD$ ” from the point of attaining limit of proportionality until ultimate deflection. Two shapes of the chart can be defined: the first shape is characterised by a decrease in load and an increase in $CMOD$ value following the appearance of the first crack (*post-crack softening – pcs*), the second shape is defined by an increase in both load and $CMOD$ value (*post-crack hardening – pch*).

5. Residual strengths – research results and their analysis

Fig. 5-8 present crack width – load relation. In order to facilitate interpretation of the research results in Fig. 6-8, envelopes of the charts (solid lines) as well as average load-*CMOD* relation (broken line) were shown. The prisms without fibres failed when concrete attained tensile strength. It must be emphasized that a three-point bending test according to PN-EN 14651 does not include specimens without fibres and applies only to fibre-reinforced materials. Specimens without fibres were tested to highlight the considerable influence of steel fibres on tensile stresses. An analysis of the test results reveals that the material, even at 0.5% steel fibre content in volume, significantly changes its properties under the action of load. The major observation is that a test specimen made from fibre-reinforced composite did not fail as abruptly as a specimen made from concrete without fibres did. In the case of fibre-reinforced prisms, a crack appeared and increased its width until the ultimate deflection, which ended the testing. This illustrates the considerable influence of steel fibre content on residual strengths. Comparing the chart shapes in Fig. 6-8, one can observe that the prisms with 0.9% and 1.2% fibre content are characterised by *pcs* which stands for a slow decrease in load and an increase in *CMOD* value following the appearance of the first crack. However, the prisms reinforced with fibres in the amount of $V_f = 0.5\%$ are characterised by an intermediate behaviour between *pcs* and *pch* – the load remained constant while *CMOD* value increased. Furthermore, one observed that fibre content influences crack width (an evident relation: the higher the fibre content V_f , the smaller the crack width). For prisms with varying fibre content (V_f), attaining the same *CMOD* values means applying different loads. In other words, higher fibre content in the prisms requires higher load to reach a given *CMOD* value.

The limit of proportionality and residual strength values for the test composite with varying fibre content V_f are presented in Fig. 9.

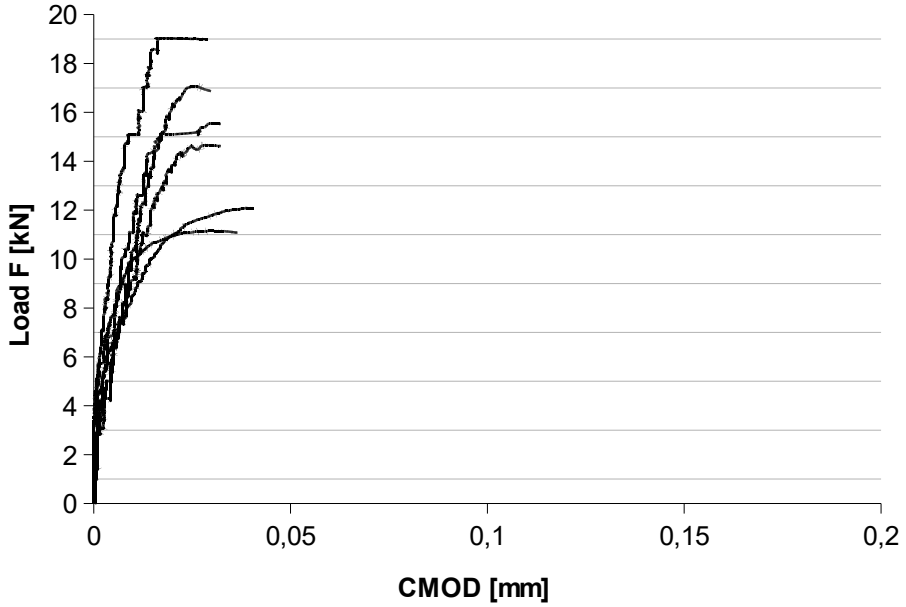


Fig. 5. Load-CMOD relation for $V_f = 0\%$

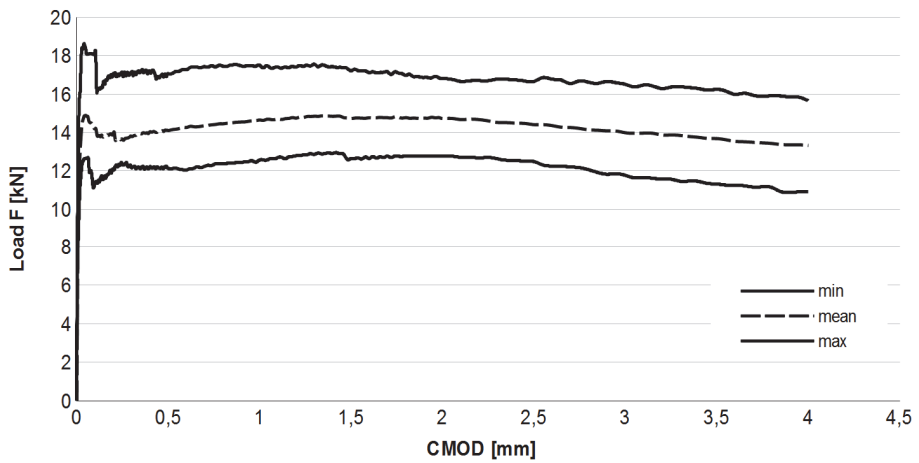


Fig. 6. Load-CMOD relation for $V_f = 0.5\%$

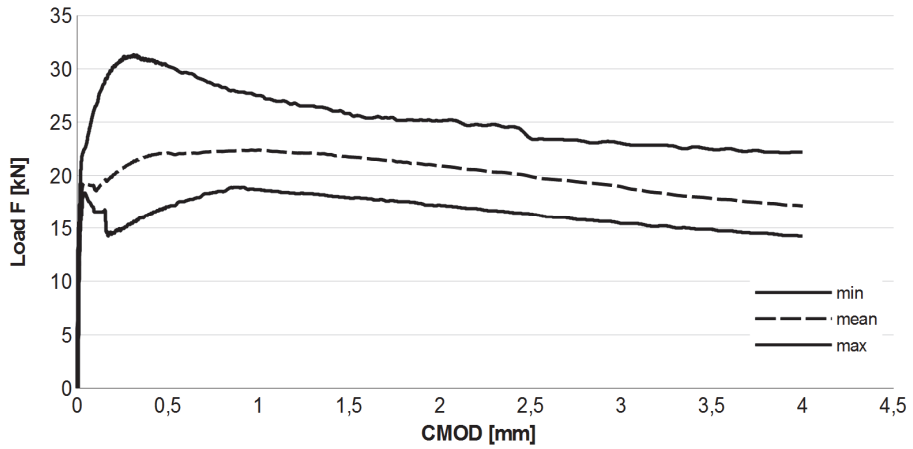


Fig. 7. Load-CMOD relation for $V_f = 0.9\%$

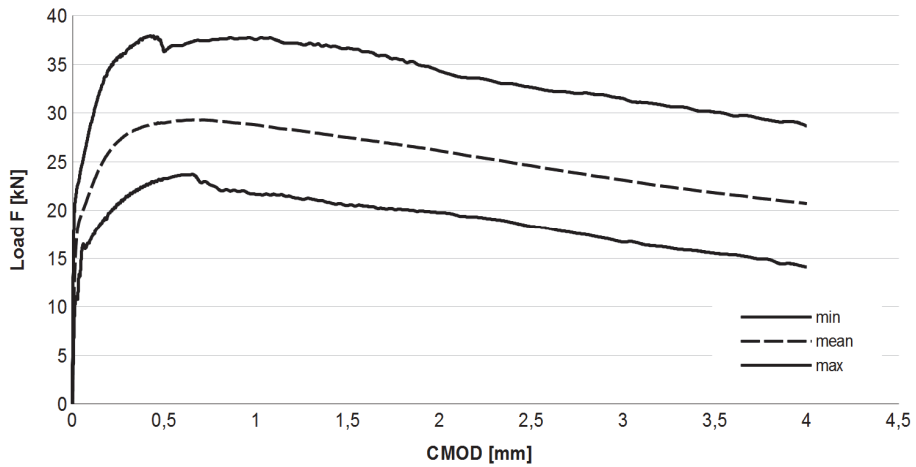


Fig. 8. Load-CMOD relation for $V_f = 1.2\%$

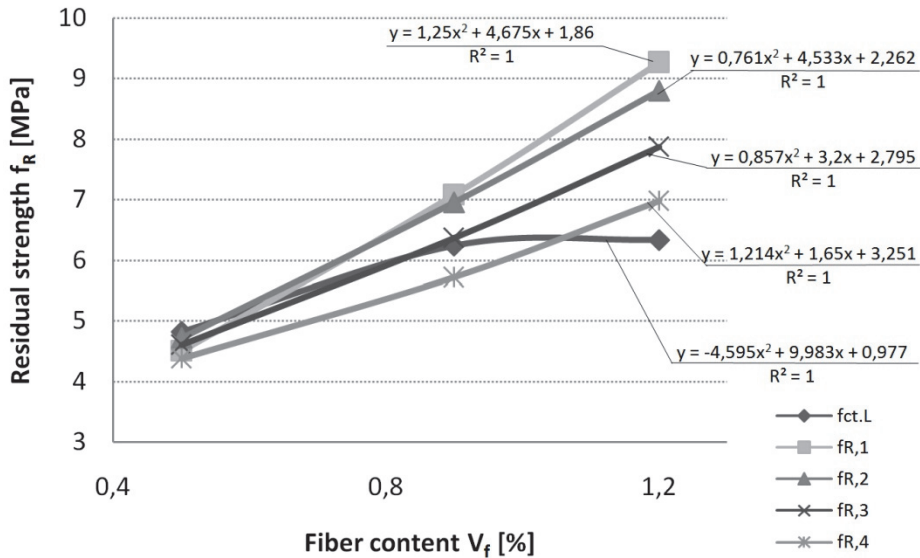


Fig. 9. Residual strength-fibre content V_f relation

Having analysed the presented data, one may conclude that residual strengths increase along with fibre content in the composite mixture. However, it must be pointed out that limit of proportionality was more or less the same in the case of fibre content (V_f) being equal to 0.9% and 1.2%. In view of the fact that the fibre content of 1.2% was optimal (Głodkowska & Laskowska-Bury 2015) for the test composite, characteristic values (Table 3) were determined to calculate shear capacities.

Table 3. Parameters for statistical analysis of residual strengths for the composite with $V_f = 1,2\%$

Property symbol	Characteristic value [MPa]	standard deviation s [MPa]	coefficient of variation v [%]	homogeneity index k [-]	confidence interval [MPa]	material classification by fib Model Code
$f_{ct,L,k}^f$	5.24	0.67	11	0.78	6.09-6.60	7b
$f_{R,1,k}$	7.30	1.20	13	0.80	8.82-9.74	
$f_{R,2,k}$	6.68	1.29	15	0.74	8.30-9.28	
$f_{R,3,k}$	5.82	1.25	15	0.74	7.39-8.34	
$f_{R,4,k}$	5.07	1.16	17	0.71	6.53-7.42	

The classification 7b in accordance with [24], defines the test material as a composite with a very high f_{RI} value (ranging from 1 to 8). The letter 'b' indicates that the test composite is characterised by pcs , which was determined by means of the ratio f_{R3}/f_{RI} (according to (Model Code 2010) 'a' and 'b' – pcs , 'd' and 'e' – psb). The strength values presented in the table may be used to design flexural/shear structural elements made from SFRWSC.

Measuring crack widths while testing residual strengths may be problematic. For that reason, the standard (PN-EN 14651, 2005) allows the possibility of calculating $CMOD$ value using the prism's deflection value δ according to the formula:

$$\delta = 0,85 \cdot CMOD + 0,04 \quad (2)$$

Since both crack width $CMOD$ and deflection δ were measured in the test, Figures 10-12 present the relation between these two values against the backdrop of a theoretical relation in accordance with PN-EN (PN-EN 14651, 2005).

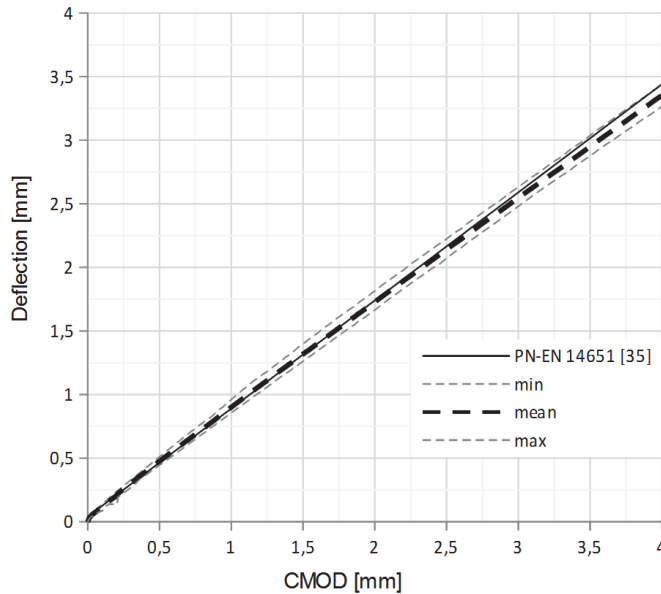


Fig. 10. Deflection-CMOD relation for the composite with the fibre content $V_f = 0.5\%$

According to the charts above, the experimental average values of $CMOD$ -deflection relation correspond closely to the values proposed in the standard. Therefore, it is fully justified to use the standard for calculating residual strengths.

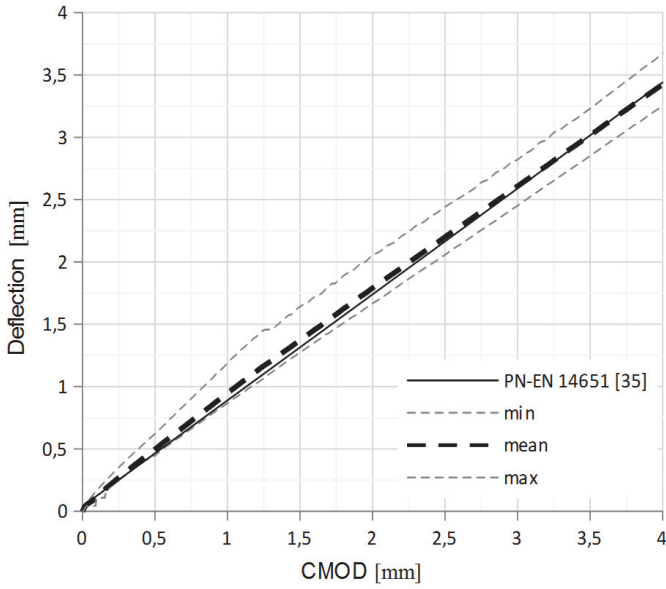


Fig. 11. Deflection-CMOD relation for the composite with the fibre content $V_f = 0.9\%$

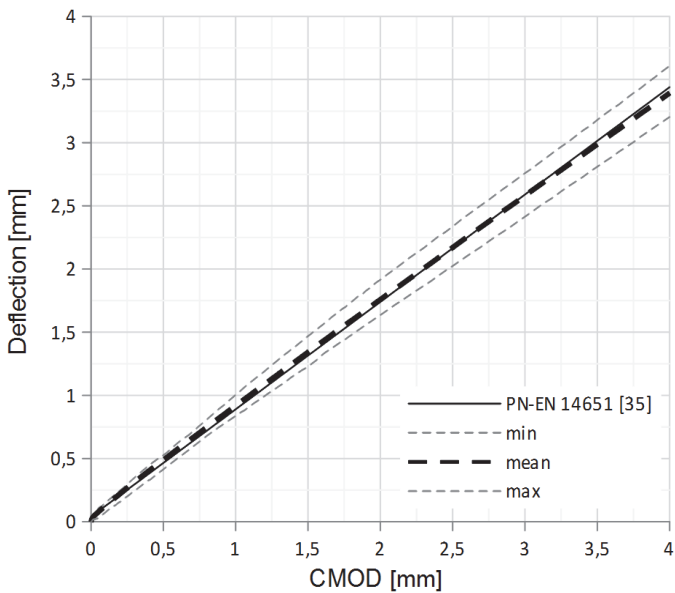


Fig. 12. Deflection-CMOD relation for the composite with the fibre content $V_f = 1.2\%$

6. Selected shear design methods – general characteristics

RILEM TC-162-TDF

Determining shear capacities of specimens made from fibre-reinforced composites in accordance with RILEM TC-162-TDF is based on Eurocode 2 (EN 1992 -1-1. 2004), which means it is based on truss system model S-T. The influence of fibres on shear capacity is defined as shear stress increment τ_{fd} , which is determined by means of residual strength $f_{R,4,k}$.

Shear capacity V_{Rd} is a sum of three factors:

$$V_{Rd} = V_{cd} + V_{fd} + V_{wd} \quad [\text{N}] \quad (3)$$

V_{cd} – specimen's shear capacity without shear reinforcement,

$$V_{cd} = \left[0,12 \cdot k \cdot \left(100 \cdot \rho_l \cdot f_{ck} \right)^{\frac{1}{3}} \right] \cdot b_w \cdot d \quad [\text{N}] \quad (4)$$

where:

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2 \quad (5)$$

$$\rho_l = \frac{A_s}{b_w \cdot d} \leq 0,002 \quad [-] \quad (6)$$

b_w , d width and effective depth of cross section (EN 1992 -1-1. 2004), [mm],

A_s – cross sectional area of reinforcement which is bonded beyond the considered section (EN 1992 -1-1. 2004), [mm²],

f_{ck} – characteristics cylinder compressive strength for concrete with fibers, [MPa],

V_{fd} – shear capacity increase due to steel fibre reinforcement:

$$V_{fd} = 0,7 \cdot k_f \cdot k \cdot \tau_{fd} \cdot b_w \cdot d \quad [\text{N}] \quad (7)$$

where:

k_f – factor taking into account the shape of the cross-section; for rectangular cross section beams $k_f = 1,0$

τ_{fd} - design value of increase in shear strength due to fibres, given by:

$$\tau_{fd} = 0,12 \cdot f_{R,4,k} \quad [\text{MPa}] \quad (8)$$

$f_{R,4,k}$ – residual strength corresponding to CMOD of 3,5 mm,
 V_{wd} – shear capacity increase due to conventional shear reinforcement in accordance with (EN 1992-1-1.2004).

Fib Model Code

The fib Model Code method does not take residual strengths into direct consideration for determining shear capacities of fibre-reinforced composites. In tension zone, according to this method, the contribution of steel fibre reinforcement is defined by two strengths: serviceable residual strength f_{Fts} (post-crack strength for serviceability limit state), and ultimate residual strength f_{Ftu} . In order to determine shear capacities, fib Model Code recommends to use linear model describing cracked tension zone where tensile stresses are directly proportional to crack width w_u (Fig. 13). It is noteworthy that the stresses may increase or decrease along with crack width increments.

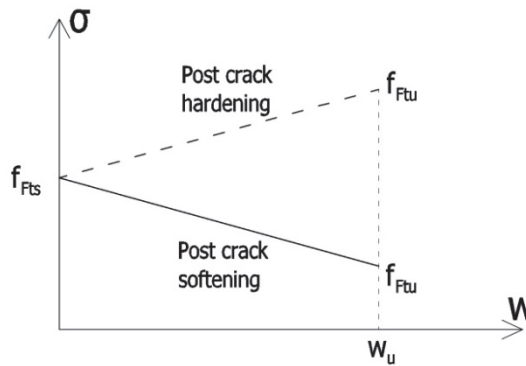


Fig. 13. Linear model describing cracked tension zone in bending cross-section of fibre-reinforced concrete in accordance with fib Model Code 2010: f_{Fts} – post-crack residual strength for serviceability limit state); f_{Ftu} – ultimate residual strength

Values f_{Fts} and f_{Ftu} are defined as:

$$f_{Fts} = 0,45 \cdot f_{R,1,k} \text{ [MPa]} \tag{9}$$

$$f_{Ftu} = f_{Fts} - \frac{w_u}{CMOD_3} \cdot (f_{Fts} - 0,5 \cdot f_{R,3,k} + 0,2 \cdot f_{R,1,k}) \geq 0; \quad w_u = 1,5mm \text{ [MPa]} \tag{10}$$

According to fib Model Code [24], the shear capacity of fibre-reinforced composites is presented as the sum of two capacities:

$$V_{Rd} = V_{Rd,f} + V_{Rd,s} \text{ [N]} \quad (11)$$

$V_{Rd,f}$ – design value of shear resistance in members with conventional longitudinal reinforcement and without conventional shear reinforcement is given by:

$$V_{Rd,f} = \left\{ \frac{0,18}{\gamma_c} \cdot k \cdot \left[100\rho_l \cdot \left(1 + 7,5 \cdot \frac{f_{Ftuk}}{f_{ctk}} \right) \cdot f_{fck} \right]^{1/3} \right\} \cdot b_w \cdot d \text{ [N]} \quad (12)$$

where:

γ_c – the partial safety factor for the concrete matrix without fibres,

f_{ctk} – characteristic value of tensile strength for the concrete matrix, [MPa],

V_{wd} – shear capacity increase due to conventional shear reinforcement in accordance with (EN 1992 -1-1. 2004).

An important aspect raised by fib Model Code is the minimum amount of traditional shear reinforcement in fibre-reinforced specimens:

$$f_{Ftuk} \geq 0,08 \cdot \sqrt{f_{ck}} \quad (13)$$

If the condition (13) is met, the minimum amount of transverse reinforcement is not required.

7. Calculating shear capacity using residual strengths

In order to prove the possibility of using SFRWSC with the fibre content equal to 1.2% as a construction material transmitting bending moment and transverse force, shear capacities of the prisms were calculated according to fib Model Code 2010 and RILEM- TC-162-TDF. In the analysis, two variants were considered. In the first variant, an assumption was made that the prisms have a constant cross-section ($b \times h = 200 \times 300$ mm) and their flexural reinforcement is variable. In the second variant, it was assumed that the flexural reinforcement ratio is constant ($\rho_l = 0.01$) and the prism's height is variable (h) with the proportion $h/b = 1.5$ maintained. A partial factor of safety equal to $\gamma_f = 1.5$ was used in the calculations.

Table 4 presents calculation results for shear capacities of the prisms with steel fibres and without steel fibres as well as the relation between these two values. This relation demonstrates the influence of steel fibres on shear capacity. The analysis revealed that adding steel fibres to the test composite significantly increases the specimen's shear capacity. According to Model Code 2010, the increase in shear capacity of the specimen with steel fibres is constant, regardless

of the amount of main reinforcement, and is equal to 124% when compared to the increase in shear capacity of the specimen without fibres. In the case of RILEM – TC-162-TDF however, the opposite was observed. In the extreme case (i.e. for $\rho_l = 0.02$), shear capacity increase is equal to 75%.

Table 4. Selected calculation results for shear capacities of SFRWSC prisms in variant I

ρ_l [-]	Design methods						
	RILEM TC-162-TDF			Model Code 2010			
	V_{cd} [kN]	$V_{cd}+V_{fd}$ [kN]	$(V_{cd}+V_{fd})/V_{cd}$ [-]	$V_{Rd,c}$ [kN]	$V_{Rd,f}$ [kN]	$V_{Rd,f}/V_{Rd,c}$ [-]	f_{Ftu} [MPa]
0.008	41.02	81.13	1.98	41.02	91.89	2.24	2.18
0.010	44.19	84.67	1.92	44.19	98.99		
0.012	46.96	87.77	1.87	46.96	105.19		
0.014	49.44	90.53	1.83	49.44	110.74		
0.016	51.69	93.05	1.80	51.69	115.78		
0.018	53.76	95.36	1.77	53.76	120.41		
0.020	55.68	97.51	1.75	55.68	124.72		
ρ_l – tensile reinforcement ratio							

Analysing the shear capacity values (Table 5) calculated using RILEM method (variant II), it is safe to say that cross-section dimensions have a considerable influence on shear capacity increase with regard to the addition of fibres. The larger the cross-section of the prism is, the more the composite's shear capacity increases. However, analysing the results according to Model Code, one can conclude that changing the prism's cross-section has no influence on shear capacity increase in the case of a specimen made from fibre-reinforced composite. Another conclusion is that the fiber-reinforced specimen's shear capacity always increases proportionally by the same value in relation to the shear capacity of a specimen without fibres. In both calculation methods, shear capacity increase for different cross-sections at $\rho_l = 0.01$ exceeded 100%.

An important aspect raised in Model Code is that minimum shear reinforcement is required. This is understandable due to the problem of brittle fracture in specimens reinforced with steel fibres. In the case of the test composite, according to (13) one obtained:

$$f_{Ftuk} = 2,18MPa > 0,08 \cdot \sqrt{f_{ck}} = 0,6MPa$$

Table 5. Selected calculation results for shear capacities of SFRWSC prisms in variant II

$b \times h$ [mm]	Design methods						
	RILEM TC-162-TDF			Model Code 2010			
	V_{cd} [kN]	$V_{cd} + V_{fd}$ [kN]	$(V_{cd} + V_{fd}) / V_{cd}$	$V_{Rd,c}$ [kN]	$V_{Rd,f}$ [kN]	$V_{Rd,f} / V_{Rd,c}$	f_{fu} [MPa]
160x240	29.19	60.85	2.08	29.19	65.49	2.24	2.18
187x280	38.88	82.67	2.13	38.88	87.24		
213x320	49.81	107.67	2.16	49.81	111.77		
240x360	61.97	135.84	2.19	61.97	139.04		
267x400	75.33	167.16	2.22	75.33	169.02		
293x440	89.88	201.60	2.24	89.88	201.67		
320x480	105.61	239.15	2.26	105.61	236.97		

8. Conclusions

Residual strengths are the major strength characteristics of a material. They may serve as basic values for designing structural elements reinforced with steel fibres. They also enable the composite's cracking resistance in bending to be determined. The defined four values of residual strength depend on $CMOD$ value. It is noteworthy that the residual strength values are also related to the prisms' behaviour following the appearance of the first crack. In other words, these values vary depending on whether a given specimen is characterised by a decrease (pcs) or an increase in load along with an increase in $CMOD$ value (pch). A decrease in load is related to a decrease in the other residual strength values just as an increase in load is related to an increase in the residual strength values.

Presented in article SFRWSC with the fibre content equal to 1.2% meets the requirements of construction materials and can be used, in some cases, as an alternative to ordinary concrete. The composite subject to analysis fulfils the standard requirements (Model Code 2010): $f_{R1}/f_{ik} > 0,4$, $f_{R3}/f_{R1} > 0,3$, which means that steel fibre reinforcement may partially replace conventional reinforcement at the ultimate limit state. Calculations done according to RILEM, fib Model Code show a considerable increase in the shear capacity of composites with the steel fibre content of 1.2%, which computationally leads to reduction of conventional shear reinforcement. Comparing the two methods, one should emphasize that RILEM regulations define shear capacity in a more simplified manner. Using

only one residual strength $f_{R,4,k}$, a major aspect of the composite's behaviour following the appearance of the first crack (*pcs* or *pch*) would be omitted. Model Code, however, describes the issue of transmitting post-cracking stresses in more detail. Firstly, it defines the value f_{F1sk} , which is determined directly from the value $f_{R1,k}$ – this occurs at the initial stage of cracking during the residual strength test. Next, the standard defines the value f_{F1uk} which takes residual strengths into account for crack width equal to 0.5 mm and 2.5 mm as well as for limit crack width used in structural design.

Therefore, it is possible to conclude that the Model Code recommendation defines more accurately SFRWSC strength characteristics, which are used in structural design. Furthermore, the important issue of minimum conventional shear reinforcement should be emphasized. The RILEM method sets no conditions for reducing or omitting the minimum ratio of stirrup reinforcement in fibre-reinforced prisms.

Another conclusion is that, according to RILEM, the designed SFRWSC elements have to meet the requirements regarding the minimum reinforcement in the form of stirrups just like traditional reinforced concrete elements do. However, the Model Code standard does not require minimum stirrup reinforcement according to the formula (13), which has been proved in this article. Therefore, the condition (13) may evidently help improve reinforcement fixing at construction sites.

It should be also underlined that the composite is made from waste sand, which has an important ecological meaning. The possibility of using sand as an aggregate to produce structural material on an industrial scale would help manage waste sand accumulating in the Polish region of Pomerania. It would also make good use of the plentiful sand occurring in the Middle East and North Africa. Vast resources of fine aggregates in these regions may become a source of wealth by providing a component for the production of structural elements.

Notation

$CMOD$ – crack mouth opening displacement

E_{cm} – static modulus of elasticity

E_d – dynamic modulus of elasticity

$f_{c,cyl}$ – cylinder compressive strength

$f_{c,cube}$ – cube compressive strength

$f_{ct,L}$ – limit of proportionality

f_{F1s} – serviceable residual strength

f_{F1u} – ultimate residual strength

$f_{R,i}$ – residual tensile strength

$f_{t,spl}$ – split tensile strength

G_d – dynamic modulus of rigidity

k – homogeneity index

s – standard deviation

SFRWSC – steel fibre reinforced waste sand concrete
 V_{cd} – specimen's shear capacity without shear reinforcement,
 V_f – fibre content by volume
 V_{fd} – shear capacity increase due to steel fibre reinforcement,
 V_{Rd} – shear capacity
 V_{wd} – shear capacity increase due to conventional shear reinforcement
 γ_f – partial factor of safety
 δ – prism's deflection value
 ε_{cs} – shrinkage
 λ – aspect ratio (l/d)
 v – coefficient of variation
 ρ_l – tensile reinforcement ratio
 τ_{fd} – shear stress increment

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Abstract

This article presents calculation results for shear capacity which were obtained using SFRWSC's residual strengths in accordance with fib Model Code and RILEM standards. Shear capacity was calculated in two variants: for constant cross-section of a specimen and for constant ratio of flexural reinforcement. The article also presents residual strength test results for SFRWSC with varying steel fibre content and shows the influence of fibre content on the composite's residual strengths. It has been proved that SFRWSC, which is an ecological composite and whose properties are similar or better than those of ordinary concrete, can be useful in the production of structural elements with regard to shear capacity. Additionally, one has observed that SFRWSC's strength characteristics, which are used in structural design, are determined more accurately in fib Model Code than in RILEM standard. Furthermore, the issue of minimum conventional shear reinforcement should be emphasized as RILEM recommendation sets no conditions for reducing or omitting this reinforcement.

Keywords:

Steel fiber-reinforced concrete, waste sand, residual strength, shear capacity

Nośność na ścinanie i wytrzymałości resztkowe fibrokompozytu na bazie piasków odpadowych**Streszczenie**

W artykule przedstawiono wyniki obliczeń nośności na ścinanie przy zastosowaniu wytrzymałości resztkowych analizowanego materiału wg fib Model Code i RILEM. Obliczenia nośności na ścinanie wykonano w dwóch wariantach: przy stałym przekroju elementu oraz przy stałym stopniu zbrojenia na zginanie. Zaprezentowano także wyniki badań wytrzymałości resztkowych dla fibrokompozytu o różnej zawartości włókien stalowych wytworzonego na bazie piasków odpadowych takiego fibrokompozytu. Przedstawiono wpływ zawartości zbrojenia rozproszonego na jego wytrzymałości resztkowe. Wykazano, że opracowany ekologiczny fibrokompozyt na bazie piasków odpadowych, którego właściwości są zbliżone lub lepsze niż betonu zwykłego, może być przydatny do wykonywania elementów konstrukcyjnych w aspekcie nośności na ścinanie. Ponadto stwierdzono, że norma fib Model Code bardziej wnikliwie niż norma RILEM określa cechy wytrzymałościowe fibrokompozytu, które następnie wykorzystuje się do wymiarowania elementów konstrukcyjnych. Dodatkowo należy podkreślić kwestię minimalnego zbrojenia konwencjonalnego na ścinanie. Rekomendacja RILEM nie wskazuje żadnych warunków zredukowania bądź pominięcia tego zbrojenia.

Słowa kluczowe:

fibrokompozyt, piasek odpadowy, wytrzymałość resztkowa, nośność na ścinanie