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Mathematical-information modeling of the operation of energy storages used in electric vehicles

Karol Bednarek, Leszek Kasprzyk, Emil Hłasko
Poznań University of Technology
60–965 Poznań, ul. Piotrowo 3a, e-mail: Karol.Bednarek@put.poznan.pl

The paper deals with the issues related to correctness of electric energy management – mainly in terms of the proper use of energy storages applied in mobile objects such as electric vehicles. Particular attention has been paid to mathematical models of lead-acid batteries, lithium-ion batteries and supercapacitors. While using the mathematical model of the lead-acid battery, the simulation of the process of its discharge was conducted using the MATLAB software. Characteristics of useful parameters of a model lead-acid battery obtained as a result of the conducted simulations were presented. Verification of the correctness of functioning of the created calculation model was carried by comparing the DC load characteristics of two specific batteries which were obtained as a result of conducted computer simulations, with characteristics determined on the basis of manufacturer's data sheets received as a result of performance of physical tests. As an additional element verifying the correctness of the functioning of the simulations, the characteristics of battery discharge was determined on the basis of the result of calculations which were performed in accordance with the normative definition of electric capacity of the battery.

KEYWORDS: energy storages, mathematical models, battery discharge simulations, mobile object power supply

1. Introduction

Man, in each sphere of its activity – both the economic one and the private one – commonly takes advantages of civilisational-technical achievements. Technical objects used by people very often require supply of energy to fulfill their functions. Owing to this, man is physically relieved. Initially, these energies included: heat energy, light energy and mechanical energy. At present, the most commonly used form of energy is electric energy in view of the most beneficial methods of its generation, transmission, processing and accumulation. Electric devices also require energy supply in order to ensure their operation. One of the ways to supply electric energy to receivers is the use of the mains. A substantial number of energy receivers, however, may not be powered from the grid, this is particularly the case with mobile devices. In such situations, various types of energy storages are applied.

Creation of complex technical objects requires precise knowledge regarding their changing parameters, multiple relationships between subassemblies and impact of external factors (environment). Such information can only be provided by measurements carried out using numerous prototypes (which is usually costly and labour-consuming), but also computer simulations based on mathematical models. For this reason, extensive computer software packages are developed to facilitate the processes of designing the produced objects.

The paper presents the mathematical models of a lead-acid battery, a lithium-ion battery and a supercapacitor. Based on the model of the lead-acid battery, the process of its discharge was subject to simulation using the MATLAB software. The DC characteristics created on the basis of the data obtained from the manufacturer of selected batteries were compared with the characteristics obtained as a result of the simulation. The created mathematical-information models of energy storages may be used for extended analyses of cooperation of power supply sources with mobile energy receivers. This allows economic and resource savings to be obtained in the processes of designing mobile objects (e.g. electric vehicles), taking into account the optimization of energy flows between the storages (sometimes operating as a hybrid) and energy receivers. Owing to this, the possibility of rational management of the resources and energy is obtained.

2. Energy storage

Energy may be stored in energy storages in the form of electric, chemical or mechanical energy. The most frequently used energy storages include: batteries (secondary electrochemical cells), supercapacitors (energy accumulation in the electric field), fuel cells (the electrochemical reaction of oxygenation of the supplied fuel takes place in them), kinetic energy storages (system with flywheels), pneumatic systems (with compressed air), superconductive energy storages (accumulation of energy in the magnetic field) and pumped storage power plants (storage of large amounts energy in the form of water potential energy). More information about the respective energy storages is provided in works [1-5].

In view of their useful parameters, energy storages (being of different nature) provide various options for their use in power supply systems of specific objects. The options for their use depend, to a great extent, on the value of the accumulated energies (functioning of power systems and e.g. portable RTV equipment), the way they are used (stationary and mobile systems), the expectations with regards to charge exchange dynamics (levels of charge and discharge currents), the incurred losses and achieved efficiencies, but also the ecological factors (effect on the environment).

The subject of consideration in this paper covers energy storages used in mobile systems. Systems in which the dynamics of energy consumption and accumulation changes intensively are subjected to particular analysis. A typical example of such conditions of functioning of the source and the receiver includes power supply systems for driving mechanisms of electric or hybrid vehicles, in which energy recovery during the vehicle braking process takes place.

3. Demand for simulation of functioning of technical objects

The processes of designing complex mobile technical objects often require multi-variant analyses of correctness of cooperation between energy storages and receiving systems. The physical performance of such analyses is very frequently labour- and cost-consuming. Therefore, mathematical-information simulations are commonly used, as they enable the consideration of the effect of many external and internal factors on the functional properties of the created object [2, 3, 5-11]. It is only after theoretical development of the most beneficial technical solution that physical tests are carried out on prototypes created according to the mathematical-information calculations in order to confirm the correctness of the theoretical considerations.

In some of the mobile system solutions, e.g. in electric vehicles, in view of the dynamics of changes in energy flows, there is a need to determine the amount of energy which can still be used, and in the recuperation systems (with recovery), the level of energy which can be stored in order to ensure the rational management of the accumulated energy. Performance of multi-variant energy flow tests between the storages and receivers in physical systems is very costly, hence there is a need to develop mathematical models of the systems under consideration and to create the opportunities for conducting precise computer simulations which allow extensive functional analyses of the considered objects to be carried out, taking into account the pre-imposed criteria of performance of both energy storages and receivers. For this reason, the authors have presented the mathematical models of the selected energy storages and their information implementation in the following sections of this paper.

4. Mathematical models of energy storages

4.1. Mathematical model of the lead-acid battery

Figure 1 presents the equivalent circuit of the lead-acid battery cell, which comprises the main circuit and the parasitic circuit. The main circuit includes voltage source E_m, capacitor C_1 and resistors R_1 and R_2, while the parasitic circuit constitutes a cross branch with resistance R_{PN}. While the cell is fully charged, current I_{PN} with hardly traceable value flows through the parasitic branch. Therefore, in the process of the lead-acid battery discharge modeling, this branch can be omitted, just like resistance R_1, whose value is also low [4, 6].

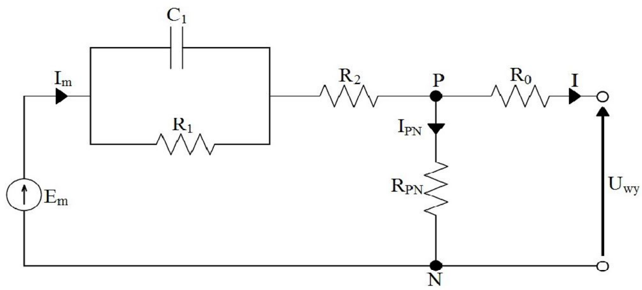

Fig. 1. Equivalent circuit of the lead-acid cell

Battery capacity $C(I,T_e)$ is the function of current I and electrolyte temperature T_e and is described by the following formula [4, 7]:

$$C(I,T_e)=\frac{K_C C_0\left(1+\frac{T_e}{-T_f}\right)^{\varepsilon}}{1+(K_C-1)\left(\frac{I}{I_n}\right)^{\delta}} \tag{1}$$

where: I_n – nominal current value [A], T_f – electrolyte freezing temperature [°C]. Coefficients K_C, C_0, ε, δ necessary to determine the cell capacity depend on the ambient temperature and the current consumption and are established on the basis of characteristics presented in Fig. 2.

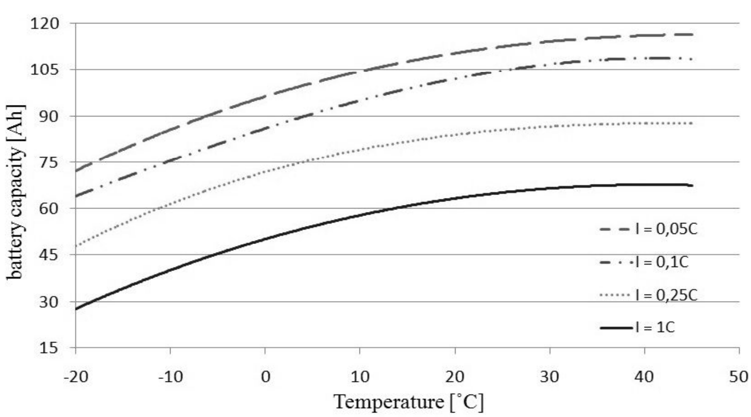

Fig. 2. Characteristics of changes in the battery capacity depending on the ambient temperature and discharge currents [6]

Coefficient ε may also be calculated on the basis of temperature coefficient α, using formula [4, 7]:

$$\varepsilon = \alpha\left(T_n - T_f\right) \tag{2}$$

where T_n – rated temperature [°C].

Electrolyte temperature T_e changes over time and is calculated on the basis of the following relations [4, 6]:

$$T_e(t) = \int_0^{t_w} \frac{1}{C_t}\left(P_s - \frac{T_e - T_a}{R_t}\right) dt \tag{3}$$

where: T_a – ambient temperature [°C], P_s – dissipated power on resistances R_0 and R_2 [W], C_t – temperature capacity [Wh/°C], R_t – thermal resistance [°C/W], t_w – charge exchange time (charge or discharge) [s].

Battery state of charge *SOC* and depth of charge *DOC* are determined on the basis of the following formulas [4, 8]:

$$SOC = 1 - \frac{Q_e}{C(0, T_e)} \tag{4}$$

$$DOC = 1 - \frac{Q_e}{C(I_{avg}, T_e)} \tag{5}$$

in which: I_{avg} – mean battery discharge current [A], Q_e – charge accumulated or drawn from the cell, specified on the basis of relation [4, 6]:

$$Q_e = \int_0^{t_w} I_m(t)\, dt \tag{6}$$

where: I_m – current flowing through the main circuit of the cell [A].

Current I_{PN} flowing through the parasitic branch depends on the electrolyte temperature, its freezing temperature, voltage U_{PN} on the branch and constants U_{P0}, G_{P0}, A_p determined on the basis of tests. It is expressed by the following formula [4, 6]:

$$I_{PN} = U_{PN} G_{P0} \exp\left(\frac{U_{PN}}{U_{P0}} + A_p\left(1 + \frac{T_e}{-T_f}\right)\right) \tag{7}$$

The values of elements of which the model consists, depending on the electrolyte temperature and battery state of charge, can be calculated by using the following expressions [4, 8]:

$$E_m = E_{m0} - K_E\left(273 + T_e\right)\left(1 - SOC\right) \tag{8}$$

$$R_0 = R_{00}\left(1 + A_0\left(1 - SOC\right)\right) \tag{9}$$

$$R_1 = -R_{10}\ln\left(SOC\right) \tag{10}$$

$$C_1 = \frac{\tau}{R_1} \tag{11}$$

$$R_2 = R_{20} \frac{\exp\left(A_{21}(1 - SOC)\right)}{1 + \exp\left(\frac{A_{22} I_m}{I_n}\right)} \tag{12}$$

In the above formulas: τ – time constant of the circuit [s], E_{m0}, K_E, R_{00}, A_0, R_{10}, R_{20}, A_{21}, A_{22} are constants specified on the basis of tests (exemplary values are given in works [6, 9]). Upon determination of the value of elements of the model, it is possible to calculate the outlet voltage based on the following formula [4, 6]:

$$U_{wy} = E_m - I \cdot Z \tag{13}$$

where: I – current drawn from or supplied to the cell [A], Z – equivalent cell impedance.

4.2. Mathematical model of the lithium-ion battery

The model of the lithium-ion battery cell comprises voltage source V_{OC}, resistance R_S, resistances R_{TS} and capacities C_{TS} connected in parallel, and resistances R_{TL} and capacities C_{TL} connected in parallel [4, 6]. The equivalent circuit of a cell is presented in Fig. 3.

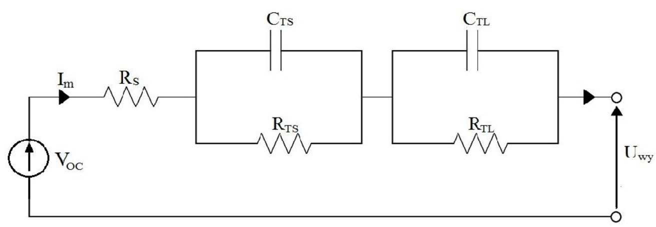

Fig. 3. Equivalent circuit of the lithium-ion cell

Values of all the elements of the model depend on the battery state of charge SOC as well as parameters specified on the basis of manufacturer's data sheets. They are determined by means of the following formulas [4, 6]:

$$V_{OC} = V_{OC0} + V_{OC1} \cdot e^{A_1 \cdot SOC} + V_{OC2} \cdot SOC + V_{OC3} \cdot SOC^2 + V_{OC4} \cdot SOC^3 \tag{14}$$

$$R_S = R_{S0} + R_{S1} \cdot e^{A_2 \cdot SOC} \tag{15}$$

$$R_{TS} = R_{TS0} + R_{TS1} \cdot e^{A_3 \cdot SOC} \tag{16}$$

$$R_{TL} = R_{TL0} + R_{TL1} \cdot e^{A_4 \cdot SOC} \tag{17}$$

$$C_{TS} = C_{TS0} + C_{TS1} \cdot e^{A_5 \cdot SOC} \tag{18}$$

$$C_{TL} = C_{TL0} + C_{TL1} \cdot e^{A_6 \cdot SOC} \tag{19}$$

Details regarding the presented mathematical model are included in work [6].

4.3. Mathematical model of the supercapacitor

The equivalent circuit of the supercapacitor, which consists of serial resistance R_s, parallel resistance R_r and capacity C, is presented in Fig. 4.

The supercapacitor capacity $C(u(t))$ is the function of voltage which changes over time and is calculated by means of the following formula [4, 10]:

$$C(u(t)) = C_0 + k_k \cdot U(t) \tag{20}$$

where: C_0 – capacity at voltage 0 V [F], U – voltage [V], k_k – coefficient of capacitance as the function of voltage.

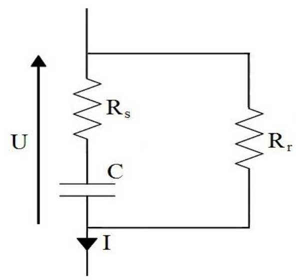

Fig. 4. Equivalent circuit of the supercapacitor

Coefficient of capacitance k_k is determined on the basis of the following relation [4, 10]:

$$k_k = \frac{\left(C_0 + \frac{4}{3} k_k U\right) U^2}{\left(C_0 + \frac{4}{3} k_k U_{\max}\right) U_{\max}^2} \tag{21}$$

Detailed information related to the presented model can be found in works [10, 11].

5. Computer simulation of lead-acid battery discharge in the MATLAB environment

5.1. Description of the simulation

The authors performed the simulation of discharge of the EH 65–12 battery manufactured by EUROPOWER, using the MATLAB software. For this purpose, the model presented in section 4.1, which was simplified by leaving out the cross branch, was used. The impact of this branch on the results obtained during the battery discharge process is insignificant due to the low value of the current which flows through it (~ nA). The expressions, which described the respective elements, being part of the circuit presented in Fig. 5, were implemented in the software.

The first step was to determine the value of all elements of the model in the state of full charge of the battery. Then, the analysed values were periodically calculated, taking into consideration the fact of a change in the battery state of charge when a load was applied to it. The conducted simulation allowed the observation of a change in the capacitance, voltage or state of charge of the battery during the process of its discharge.

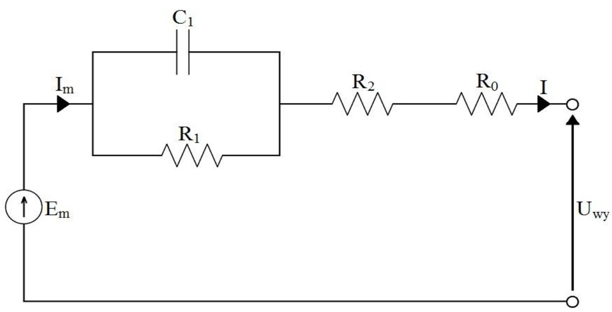

Fig. 5. Diagram of the lead-acid battery used for the simulation

5.2. Simulation results

Figure 6 presents the characteristics of discharge of a single cell of the EH 65–12 lead-acid battery with the capacitance of 65 Ah, obtained as a result of simulation for the DC load amounting to I = 3.25 A. If the load is applied for 20 hours, the voltage value on a single cell of the lead-acid battery drops in a non-linear manner from about 2.12 V to about 1.75 V. The non-linear nature of the discharge process results from the fact that the values of elements being part of the equivalent circuit of the model also change in a non-linear manner.

The characteristics of changes in electromotive force E_m of the battery during a discharge is presented in Fig. 7.

The value of the electromotive force depends on the battery state of charge (SOC) and electrolyte temperature, and as the time of effect of the DC load increases, the said value decreases in a linear manner. The linear nature is also demonstrated by battery state of charge (SOC) and depth of charge (DOC) processes over time (Fig. 8) which depend on the amount of charge drawn from the cell and its capacity.

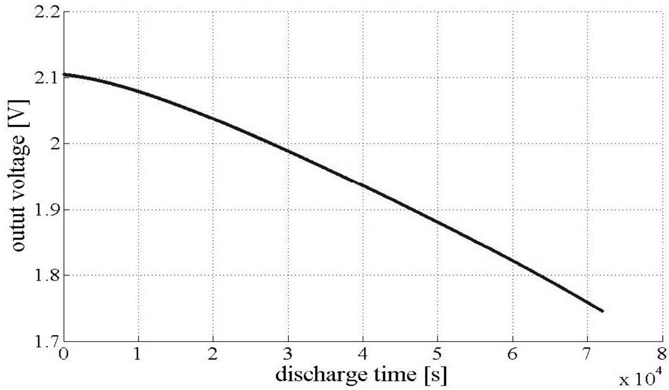

Fig. 6. Characteristics of the EH 65–12 lead-acid battery discharge

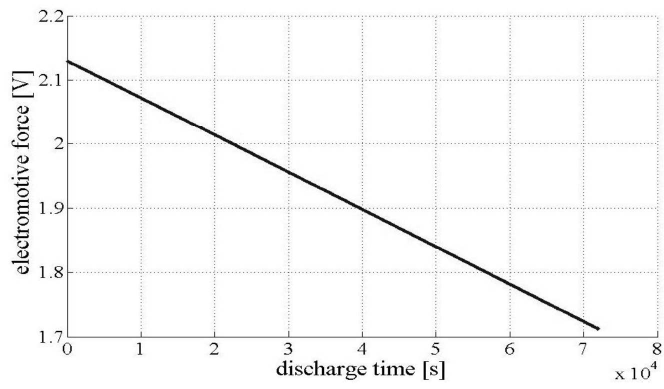

Fig. 7. Change in the electromotive force of the EH 65–12 lead-acid battery vs time

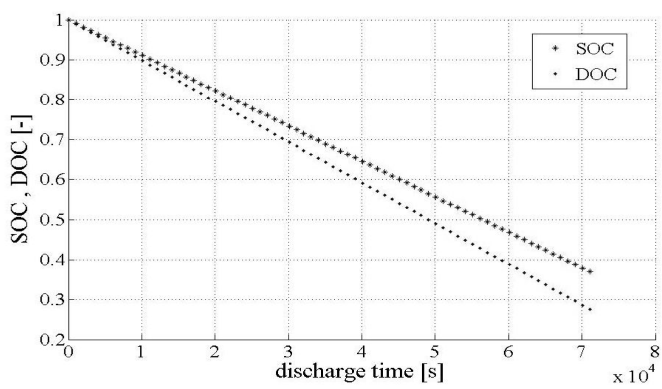

Fig. 8. Characteristics of change in the battery state of charge (SOC) and depth of charge (DOC) over time

5.3. Comparison of simulated characteristics with the characteristics given by the battery manufacturer

Based on the data sheets for the EUROPOWER EH 65–12 lead-acid battery, whose basic parameters are given in Table 1, the DC characteristics of discharge was specified [12]. The authors conducted the simulation of the DC discharge process with regards to the battery under consideration, using the MATLAB software. The characteristics drawn up on the basis of the simulation was compared with the chart obtained from the manufacturer by way of performance of physical tests. Both processes were presented in Fig. 9.

Table 1. EH 65–12 battery rated data [12]

Parameter		**Value**
Rated voltage		12 V
Electric capacitance		65 Ah
Number of cells		6
Technology		AGM
Inner resistance		< 8 mΩ
Charging current		6,5 A (max 19,5 A)
Operating temperature range	charge	0°C ÷ 40°C
	discharge	–20°C ÷ 50°C
weight		20 kg

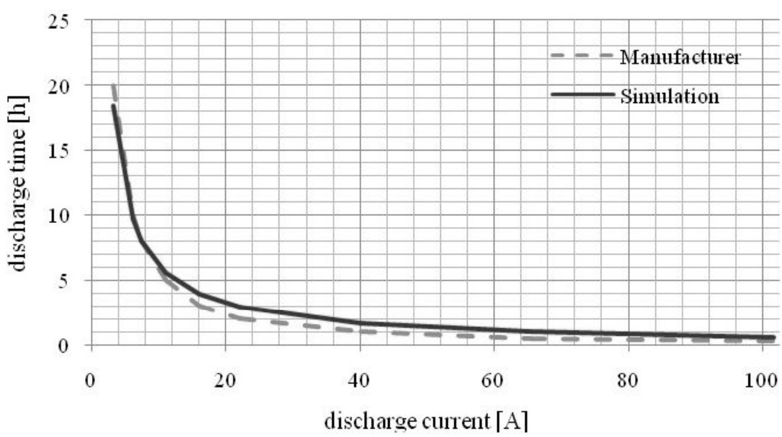

Fig. 9. Comparison of the DC characteristics obtained on the basis of the data from the EUROPOWER EH 65–12 battery manufacturer and the characteristics obtained as a result of simulation

The DC characteristics of discharge was also determined for the EUROPOWER EH 100–12 battery, whose technical data were given in

Table 2 [12]. Just as is the case with the first battery, the authors conducted the simulation on the basis of which the discharge time was specified for the discharge currents provided by the manufacturer. The DC characteristics of discharge obtained in this way was compared with the characteristics obtained from the manufacturer's data, which is presented in Fig. 10.

The results of the conducted simulations approximate the provided technical data, which proves that the model created by the authors is correct.

Table 2. EH 100–12 battery rated data [12]

Parameter		Value
Rated voltage		12 V
Electric capacitance		100Ah
Number of cells		6
Technology		AGM
Inner resistance		< 6 mΩ
Charging current		10 A (max 25 A)
Operating temperature range	charge	0°C ÷ 40°C
	discharge	–20°C ÷ 50°C
weight		28,5 kg

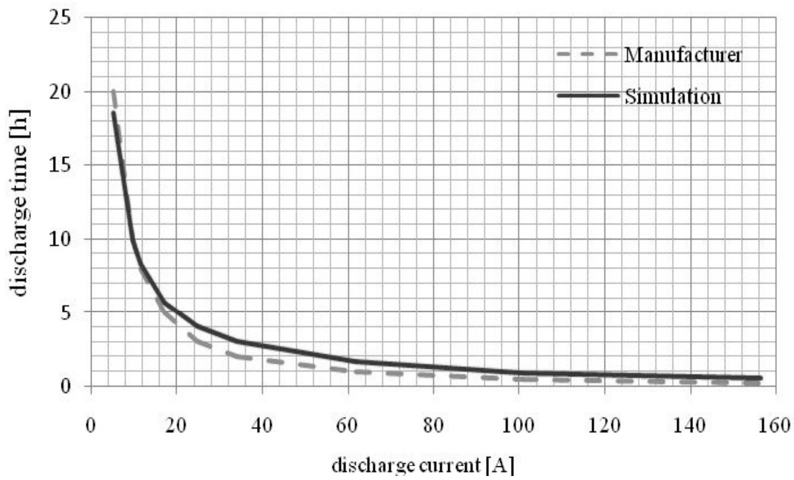

Fig. 10. Comparison of the DC characteristics obtained on the basis of the data from the EUROPOWER EH 65–12 battery manufacturer and the characteristics obtained as a result of simulation

6. Final remarks and conclusions

The paper deals with a very important and valid subject of correctness of the use of energy storages as well as rationalization of energy management in mobile systems. Mathematical models of selected energy storages, namely, the lead-acid battery, the lithium-ion battery and the supercapacitor were presented. This constituted the basis for conducting the computer simulation (in the MATLAB environment) of the lead-acid battery discharge. The mathematical-information model was verified in two ways: by the definitional discharge of the battery in accordance with the normative requirements (according to the definition of the electric capacitance) and by comparison of DC discharge characteristics of specific batteries, drawn up on the basis of technical parameters provided by the manufacturer (obtained in the process of testing of physical objects) and the characteristics obtained as a result of conducted computer simulations. The convergence of the compared measurement results and the computer simulations is satisfactory.

The properly developed mathematical-information models of the functioning of energy storages may be used to organize the rational management of energy flows between energy storages (including two different ones operating as a hybrid) and the vehicle's driving system, in which energy may be recovered during the vehicle's driving speed reduction and braking processes.

The accuracy of obtained calculations of functional parameters of storages, the correctness of simulation of the energy storage charge and discharge processes, and as a consequence of this, the quality of options of management of energy flows between supply sources and receivers all depend on the precision of the developed models.

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