

A PROPOSAL FOR DESCRIPTIVE GEOMETRY TERM PAPERS

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Abstract: Designing roofs and embankments proves to be a good topic for a term paper for students of Descriptive Geometry. It is an excellent opportunity for actual application of the Monge method (2D structures) in 3D modelling using CAD software and also physical creation of a model of a roof (using paper) and embankment (using powders). The present paper includes an interesting proposal of conducting a project which could verify the theorem in practice as early as at the beginning of one's technical course at university.

Keywords: skeleton of roof, straight skeleton, circular cone, parabola, hyperbola, Dandelin's theorem, Voronoi diagram for polygon, medial axis, offset curve, internal friction angle, embankment

1 Introduction

Let us start our discussion with a paper describing application of roof geometry [1, 2, 3] for creating embankments with a natural slope [4]. Geometric solution of the shape of hipped roof ends is an excellent example of using basic geometric constructions in the Monge method. These include: planar drawing of the orthogonal projection of the hipped roof end obtained using the medial axis construction of the polygon of base, a frontal projection obtained through the incidence rule of elements to objects in projections, finding the actual shapes of its hipped roof ends through the construction of a cross section and finding the angles between hipped roof ends (the so called: bevel angle of hip rafter and valley rafter by constructing a perpendicular line to the plane as well as finding the actual measure of an angle by constructing the planar projection [2]). The solution of a roof lets us not only illustrate the application of all basic constructions in the Monge projections, such as: the incidence of elements, common elements of objects, a revolved section (alternatively auxiliary views), perpendicularity of elements (straight line to a plane), but above all, it justifies the choice of the necessary constructions used and discussed in the course of Descriptive Geometry. Introducing a geometric construction right at the moment when there is a practical need to discuss it is a crucial motivating act which encourages to study this construction. It is essential while developing course curriculum at times when massive amount of information is accumulated to be then passed on to students and when computers are commonly used to solve engineering problems. After students obtain a roof solution in a classic approach they may verify the method using 3D modelling tools or by designing and conducting the appropriate experiment and then providing documentation. A model of embankment obtained as a result of an experiment makes it possible to physically verify Dandelin's (well-known for his lecture on descriptive geometry) theory. 3D modelling is a virtual equivalent of a classic realization of a geometric model using paper, which used to be the ultimate verification tool of solution's correctness.

2 Create real and virtual models of the natural embankment

157 1st year Construction students (in their second semester) were divided into seven groups of 20 up to 24 students and, as part of their term project, were given individual assignments where variant diversification involved selecting an outline of a building of a zero-, one- or two-parameter set of building projections, induced by an appropriate drawing (note Fig. 1 - 6). Selections were made by the students themselves by following the rules described in B.

The term project consisted of 5 stages:

A: Students read paper [4], which is the basic source of knowledge required to complete the term project. The paper was sent by e-mail to 1st year students' inbox. It can also be found by following the link below: http://ogigi.polsl.pl/biuletyny/zeszyt_25/z_25_spis.aspx

B: Students make a virtual 3D model of a roof (using AutoCAD software). The assumptions for roof solution were selected out of the outlines shown below by building the appropriate matrix, applying the following procedure:

Group P1 (20 students): Students select in turn the roofs 1.01→1.06 (six), 2.01 (four), 2.02 (five), 2.03 (four), 2.04 (roofs with $a=0, b=0$; $a=0, b=-5$; ... parameters, i.e. pairs (0,0), (0, -5), (0,5)) (three) from the matrix

$$M_{4 \times 5}^{2.04} = \begin{bmatrix} 0,0 & 0,-5 & 0,5 & 0,-10 & 0,10 \\ -5,0 & -5,-5 & -5,5 & -5,-10 & -5,10 \\ 5,0 & 5,-5 & 5,5 & 5,-10 & 5,10 \\ 10,0 & 10,-5 & 10,5 & 10,-10 & 10,10 \end{bmatrix}$$

created from the pairs of shape parameters 2.04 of the second set (note Fig. 1). Parameters are selected in accordance with the numbers on the list in the Pn project group.

Group P2 (23 students): Students select parameters from a matrix $M_{4 \times 5}^{2.04}$ created for 2.04 variant (arranged as rows) starting from 0, -10 pair (the first row, fourth column). In this way 17 students will select their assumptions from the matrix $M_{4 \times 5}^{2.04}$. The remaining five students assume parameters from the matrix $M_{4 \times 5}^{2.05}$ which should be previously constructed.

Group P3 (23 students): Students select their assumptions by reading the subsequent rows in the matrix $M_{4 \times 6}^{2.06}$. This matrix should be previously constructed on the basis of 2.06 drawing in the second set.

Group P4 (24 students): Students select assumptions by reading the subsequent rows in the matrix $M_{7 \times 5}^{3.01}$. This matrix should be previously constructed on the basis of 3.01 drawing in the third set.

Group P5 (22 students): Students select their assumptions by reading the one-row matrixes $M_{1 \times 7}^{3.02}$, $M_{1 \times 7}^{3.03}$, $M_{1 \times 7}^{3.04}$. These matrixes should be previously constructed on the basis of 3.02, 3.03, 3.04 drawings from the third set.

Group P6 (22 students): Students select assumptions by reading one-row matrixes $M_{1 \times 5}^{4.01}$, $M_{1 \times 5}^{4.02}$, $M_{1 \times 5}^{4.03}$, $M_{1 \times 8}^{4.04}$. These matrixes should be previously created on the basis of drawings 4.01, 4.02, 4.03, 4.04 from the fourth set.

Group P7 (23 students): Students select their assumptions by reading the subsequent rows in the matrix $M_{7 \times 7}^{5.01}$. This matrix should be previously created on the basis of drawing 5.01 in the fifth set.

When determining the base of the roof we assume that right angles (convex and concave) remain as right angles and that the vertices in the circles do not change their position.

C: Making the physical model of the embankment with the base of the selected roof. Ideally, the model should be made in scale 2:1. The natural laboratory could be one's kitchen and

the powder to be used could be coarse-grained wheat flour. The powder is not wasted as it is used for baking (if we take good care and clean conditions are provided, it can be reused for cooking). First, we ought to cut out the shape of the embankment (roof) base from a hard piece cardboard, then we place the base on a support (e.g. glasses on a sheet of paper) and then we sift flour onto the surface using a sieve which we shake horizontally. When we finish sifting the flour, we should take a photograph with a camera or mobile phone.

D: Making a virtual model of a truncated embankment using any method in AutoCAD program.

E: Results: a 3D image of a roof obtained, for instance, by using a screenshot (PrtSc), a photograph of a physical model of an embankment, a 3D image of a truncated embankment inserted into a Word file, are solutions to the task and the content of the term project. The obtained file in *.doc (*.docx) format and the photograph (e.g. *.jpg) is sent to the instructor's e-mail inbox.

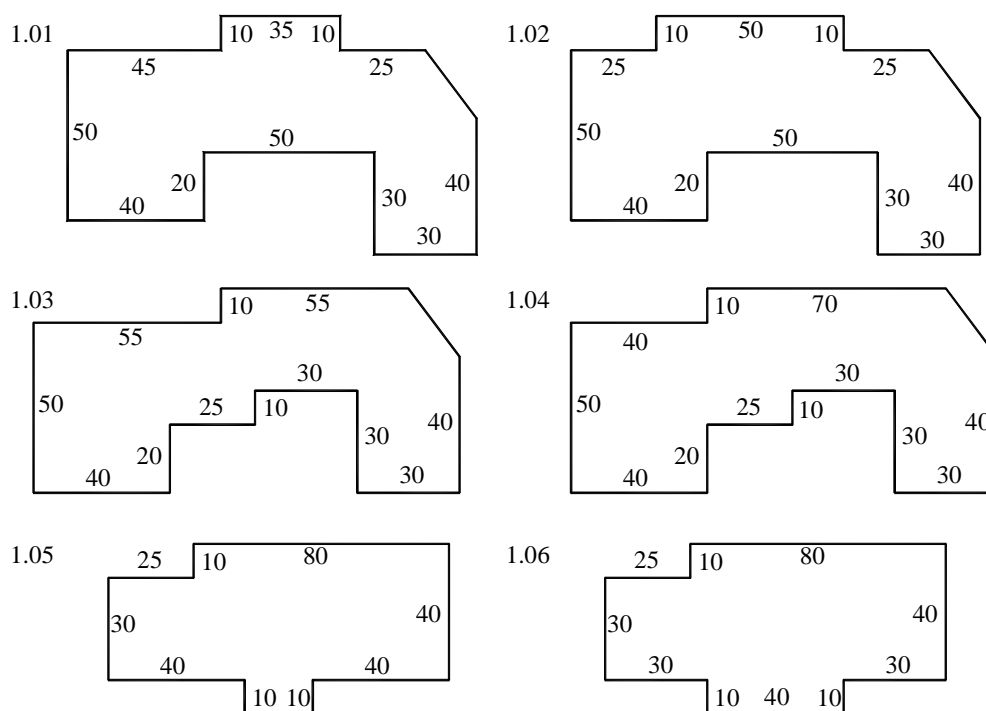


Figure 1: Set_01 of variants of the roof base (embankment base)

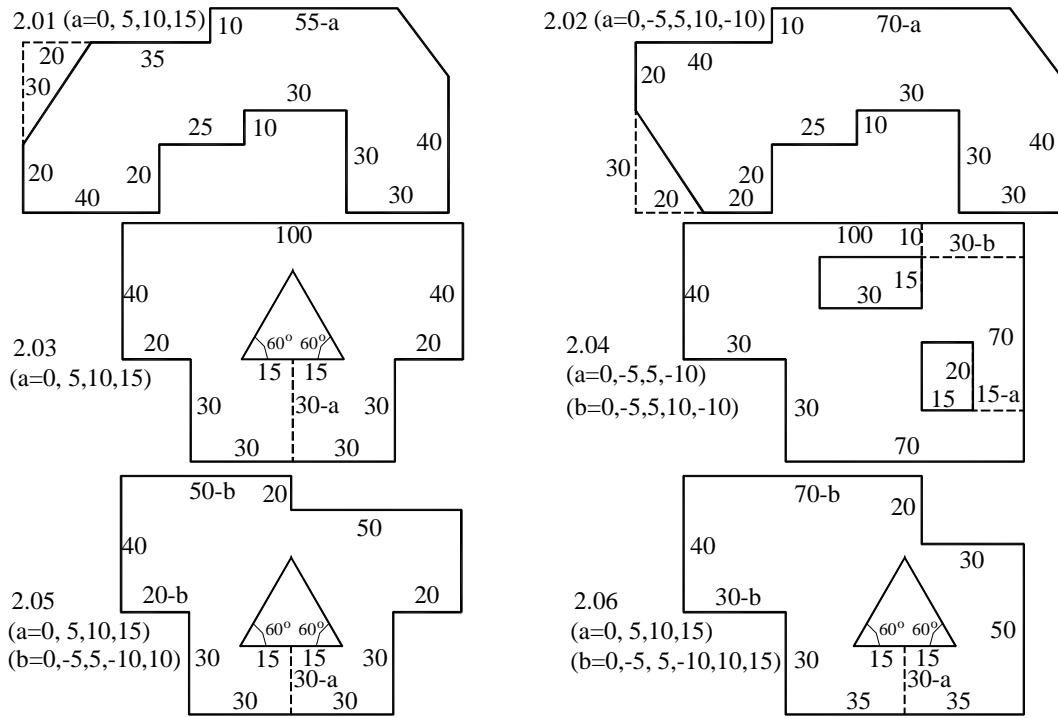


Figure2: Set_02 of variants of the roof base (embankment base)

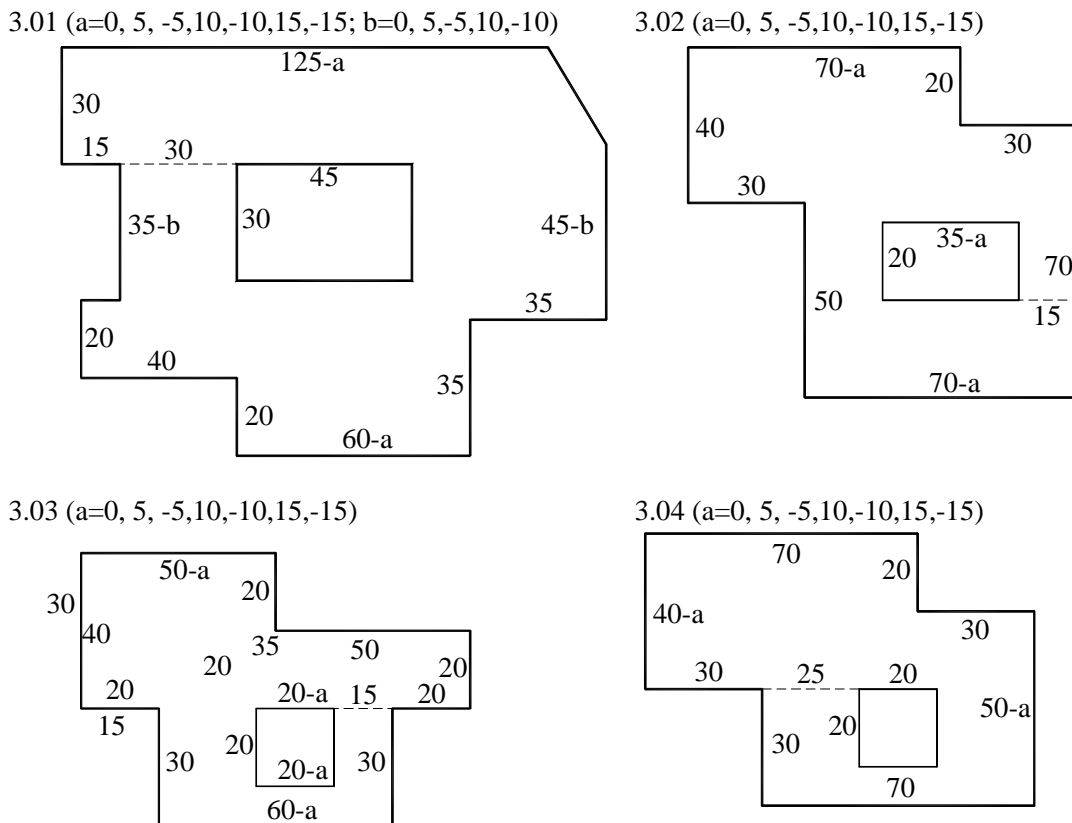
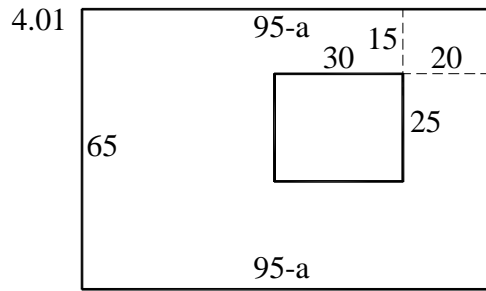
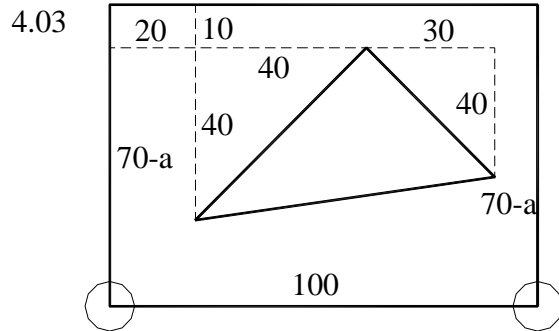
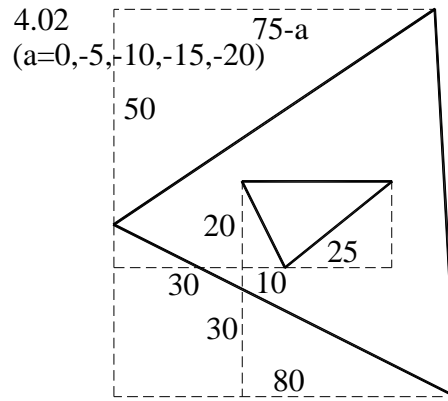


Figure 3: Set_03 of variants of the roof base (embankment base)



($a=0,-5,-10,-15,-20$)



($a=0,-5,-10,-15,-20$)

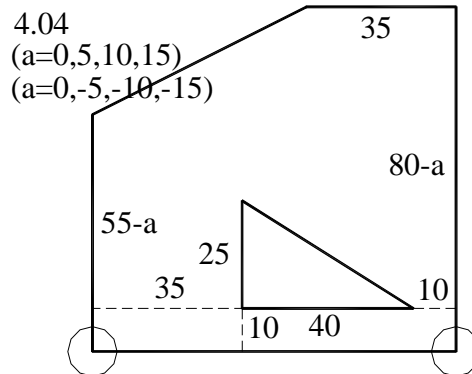
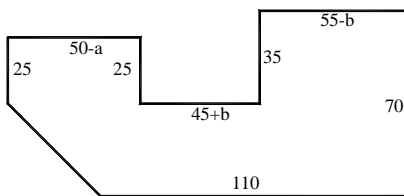


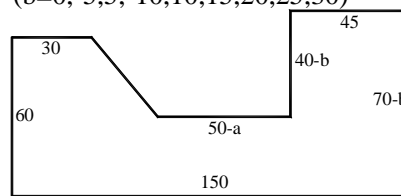
Figure 4: Set_04 of variants of the roof base (embankment base)

5.01($a=0,-5,5,-10,10,-15,15$; $b=0,-5,5,-10,10,-15,15$)

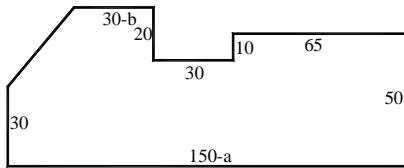


5.02($a=0,-5,5,-10,10,15,20,25,30$)

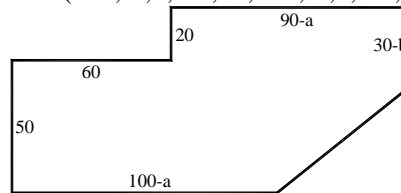
($b=0,-5,5,-10,10,15,20,25,30$)



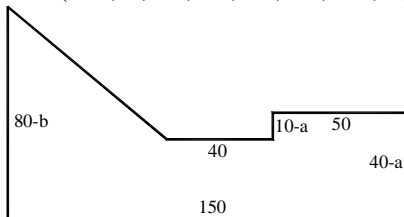
5.03($a=0,-5,5,-10,10,15$; $b=0,-5,5,-10,10,-15,15$)



5.04($a=0,-5,5,-10,10$; $b=0,-5,5,-10,10,15$)



5.05($a=0,-5,-10,-15,-20,-25$; $b=0,-5,5,-10,10,15,-15$)



5.06($a=0,-5,5,-10,10$; $b=0,-5,5,-10,10,15$)

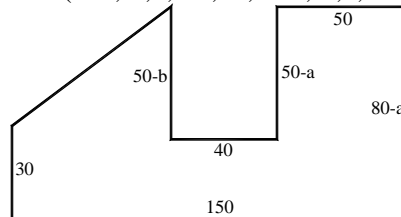


Figure 5: Set_05 of variants of the roof base (embankment base)

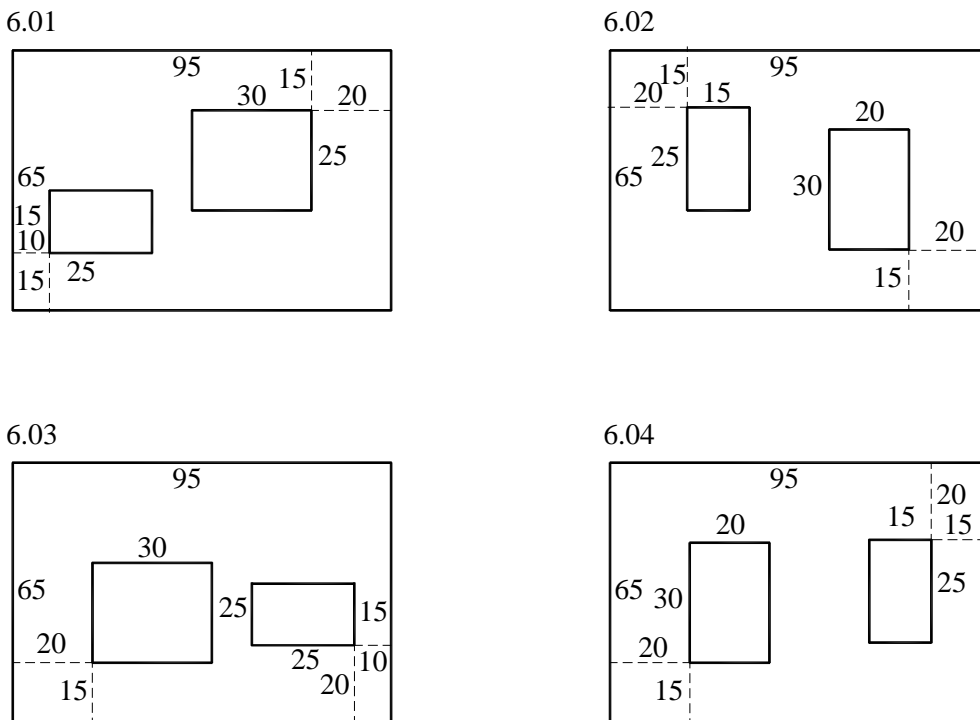


Figure 6: Set_06 of variants of the roof base (embankment base)



Figure 7a: The natural embankment with input data 2.01: $a=0$ (made by P. Białojan, civil engineering student, IIst semester)

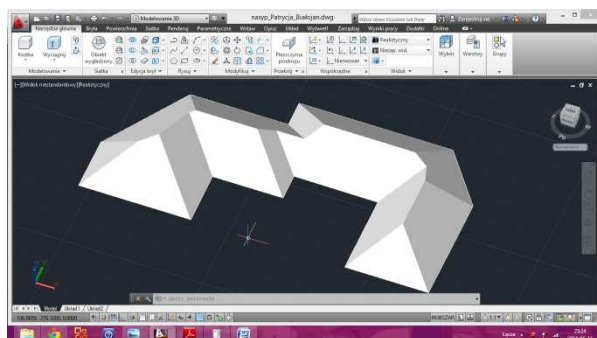


Figure 7b: 3D model of appropriate roof with input data 2.01: $a=0$ (made by P. Białojan, civil engineering student, IIst semester)

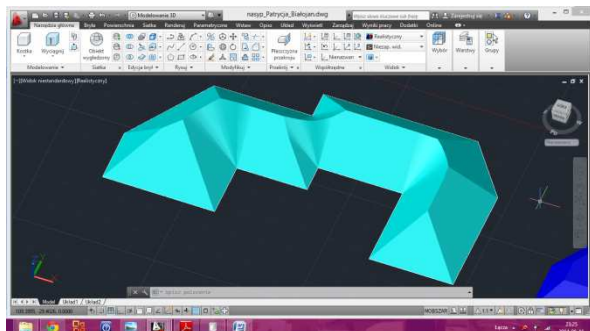


Figure 7c: 3D model of the appropriate embankment with input data 2.01: $a=0$ (made by P. Białojan, Civil Engineering student, 2nd semester)

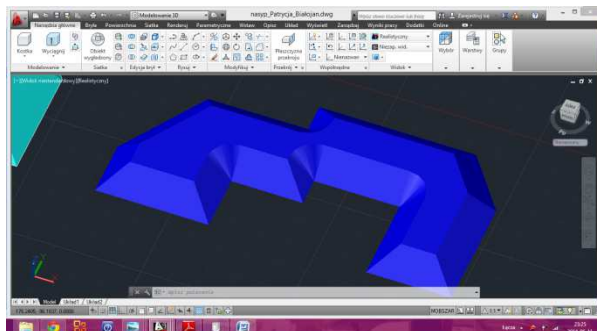


Figure 7d: 3D model of the appropriate truncated embankment with input data 2.01: $a=0$ (made by P. Białojan, Civil Engineering student, 2nd semester)



Figure 8a: The natural embankment with input data 2.04: $a=0$, $b=10$ (made by D. Ślepowroński, Civil Engineering student, 2nd semester)

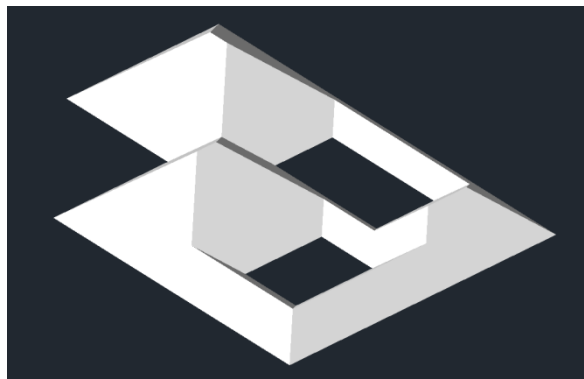


Figure 8b: 3D model of appropriate roof with input data 2.04: $a=0$, $b=10$ (made by D. Ślepowroński, Civil Engineering student, 2nd semester)



Figure 8c: 3D model of the appropriate embankment with input data 2.04: $a=0$, $b=10$ (made by D. Ślepowroński, Civil Engineering student, 2nd semester)

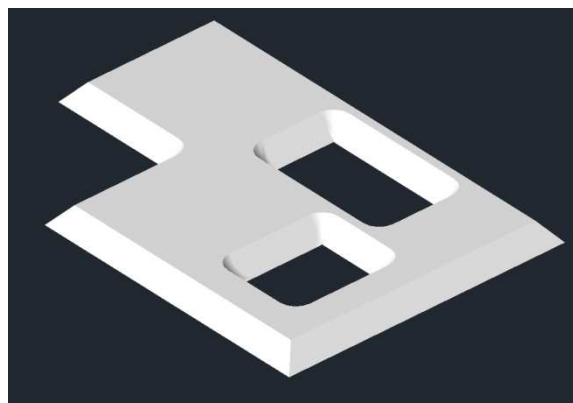


Figure 8d: 3D model of the appropriate truncated embankment with input data 2.04: $a=0$, $b=10$ (made by D. Ślepowroński, Civil Engineering student, 2nd semester)

3 Conclusion

The students solved the set task (note Fig. 7 and 8) successfully and with interest. Using their skills and knowledge of descriptive geometry, as early as in their 2nd semester of their course, they completed a fairly complex project: a flat solution of a roof – it is a standard problem in the curriculum of Descriptive Geometry in the faculty of Construction and Civil Engineering; building a 3D model by applying the Dandelin model. It seems the proposal can well be used with a large group of students. Note that students select the input data that cannot be used by another group. At the same time they apply the notion of matrix, which they studied in the course of Mathematics, they also learn how to use papers published in scientific journals and use computer software. The way the geometric assumptions are generated indicates unlimited possibilities of individual approach to solving the problem in a large group of students.

References:

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PROPOZYCJA PRACY SEMESTRALNEJ Z GEOMETRII WYKREŚLNEJ

Kształtowanie dachów oraz nasypów okazuje się być dobrym tematem na prace semestralne dla studentów w ramach przedmiotu geometria wykreślna. Stwarza znakomitą okazję do realnego zastosowania w praktyce metody Monge'a (konstrukcje 2D) i modelowania 3D za pomocą programu CAD, a także wykreowania fizycznego modelu dachu (konstrukcja z papieru) i nasypu (model utworzony z materiału sypkiego). Praca zawiera propozycję zrealizowania ambitnego projektu z zakresu weryfikacji teorii w praktyce już na początku studiów technicznych.