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About a new definition of kelvin

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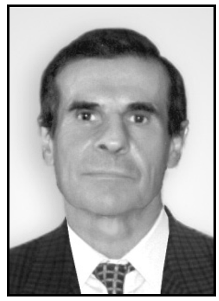

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Abstract

The kelvin, a unit of temperature T, is presently defined by the temperature of the triple point of water. Thus, the kelvin is linked with a material property. But in fundamental laws of physics temperature always appears as thermal energy kT. The 24th CGPM made the historical decision to redefine the kelvin through the Boltzmann constant k. The latest results on Boltzmann constant determination and practical realization of a new definition are reviewed. The new definition of thermodynamic temperature is fully independent of the material properties, technical implementation of the measurement, the temperature level and its span. The new definition of kelvin is now the motivation for the development of new measurement methods based on basic physical laws – primary thermometry, and finally it will lead to new temperature standards.

Keywords: SI base units, fundamental constant, Boltzmann constant, kelvin, temperature scale, temperature measurement.

O nowej definicji kelwina

Streszczenie

Kelwin to jednostka temperatury termodynamicznej, która obecnie określona jest przez temperaturę punktu potrójnego wody. Dotychczas, definicja jednostki kelwin była powiązana z właściwościami materiału. Zgodnie z podstawowymi prawami fizyki, pojęcie temperatury wynika z energii termicznej *kT*. W Międzynarodowym Biurze Miar (BIPM) podjęta została historyczna decyzja o ponownym zdefiniowaniu jednostki kelwin przy użyciu stałej Boltzmanna k. W artykule przestawiono najnowsze osiągnięcia metrologiczne dotyczące wyznaczenia stałej Boltzmanna i zmierzające do praktycznej realizacji nowej definicji kelwina. Omówiono podstawy wyznaczenia stałej Boltzmanna. Przewiduje się nowa definicja kelwina umożliwi pomiary temperatury ze standardową niepewnością względną dochodzącą do 10^{-6}. W pracy przedstawiono dokumenty metrologiczne Międzynarodowego Biura Miar związane z nową definicją kelwina i prace badawcze europejskich instytucji metrologicznych. Podano skutki jakie niesie nowa definicja dla rozwoju metrologii i pomiarów temperatury. Nowa definicja jednostki temperatury termodynamicznej będzie niezależna od właściwości substancji, realizacji technicznej i zakresu temperatury. Ułatwi to rozwój termodynamicznych metod pierwotnego pomiaru temperatury (*ang. primary thermometry*). Wpłynie to dokładność skali temperaturowej i pomiarów temperatury, w tym bezstykowych pomiarów radiacyjnych. Nowa definicja kelwina nie wymaga zmian w Międzynarodowej Skali Temperatury (ITS-90).

Słowa kluczowe: jednostki SI, stałe podstawowe, stała Boltzmanna, kelwin, skala temperatury, pomiary temperatury.

1. Introduction

It is commonly known that the question on redefinition of the four base units of the International System of Units (SI) towards their binding to natural physical constants or invariants have been actively disputed for the last ten years. The aim of the article is to consider the proposed redefinition of kelvin through the Boltzmann constant and the results of redefinition for metrology.

2. Background and matter of redefinition

Nowadays the temperature unit is the kelvin, symbol K, defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water (T_{tpw}) that is connected with properties of the matter.

The definition of the unit of thermodynamic temperature (T) was given in substance by the 10th General Conference of Weights and Measures (1954, Resolution 3) which selected the triple point of water as the fundamental fixed point and assigned to it the temperature 273.16 K exactly, so defining the unit. The 13th CGPM (1967/68, Resolution 3) taking into account that the unit of thermodynamic temperature and the unit of temperature interval are one and the same unit, which ought to be denoted by a single name and a single symbol adopted the name kelvin, symbol K, instead of "degree kelvin", symbol K, and defined the unit of thermodynamic temperature as follows (1967/68, Resolution 4)

"The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water."

It follows that the thermodynamic temperature of the triple point of water is exactly 273.16 kelvins, T_{tpw} = 273.16 K.

At its 2005 meeting the International Committee of Weights and Measures (CIPM) affirmed that: this definition refers to water having the isotopic composition defined exactly by the following amount of substance ratios: 0.000 155 76 mole of ^{2}H per mole of ^{1}H, 0.000 379 9 mole of ^{17}O per mole of ^{16}O, and 0.002 005 2 mole of ^{18}O per mole of ^{16}O.

Because of the manner in which temperature scales used to be defined, it remains a common practice to express a thermodynamic temperature, symbol T, in terms of its difference from the reference temperature T_0 = 273.15 K, the ice point. This difference is called the Celsius temperature, symbol t, which is defined by the quantity equation:

$$t = T - T_0. \qquad (1)$$

The unit of Celsius temperature is the degree Celsius, symbol °C, which is by definition equal in magnitude to kelvin.

A difference or interval of temperature may be expressed in kelvins or in degrees Celsius (13th CGPM, 1967/68, Resolution 3, mentioned above), the numerical value of the temperature difference being the same. However, the numerical value of a Celsius temperature expressed in degrees Celsius is related to the numerical value of the thermodynamic temperature expressed in kelvins by the relation

$$t/°C = T/K - 273.15. \qquad (2)$$

The kelvin and the degree Celsius are also units of the International Temperature Scale of 1990 (ITS-90) adopted by the CIPM in 1989 in its Recommendation 5.

Although the triple point of water is a natural invariant, but its thermodynamic temperature to some extent depends on the impurity content and isotopic composition of water volume. Inconsistence T_{tpw} for different realizations of the triple point is within 50 mK, through uncertainties of the other reference points determining International Temperature Scale of 1990 (ITS-90) will be quite larger.

This circumstance has become the main prerequisite for the redefinition of kelvin in term of fundamental physical constant (FPC) – the Boltzmann constant [1].

The 24th CGPM (2011, Resolution 1 "On the possible future revision of the International System of Units the SI") made the historical decision that the International System of Units would be the system of units in which:

- the ground state hyperfine splitting frequency of the cesium 133 atom $\Delta\nu$ (133Cs)$_{hfs}$ is exactly 9 192 631 770 hertz,
- the speed of light in vacuum c is exactly 299 792 458 meter per second,
- the Planck constant h is exactly 6.626 06X×10^{-34} joule second,
- the elementary charge e is exactly 1.602 17X×10^{-19} coulomb,
- the Boltzmann constant k is exactly 1.380 6X×10^{-23} joule per kelvin,
- the Avogadro constant N_A is exactly 6.022 14X×10^{23} reciprocal mole,
- the luminous efficacy K_{cd} of monochromatic radiation of frequency 540×10^{12} Hz is exactly 683 lumen per watt,

where:

(i) the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, meter, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to Hz = s^{-1}, J = m^2 kg s^{-2}, C = s A, lm = cd m^2 m^{-2} = cd sr, and W = m^2 kg s^{-3},

(ii) the symbol X in this Resolution represents one or more additional digits to be added to the numerical values of h, e, k, and N_A, using values based on the most recent Committee on Data for Science and Technology (CODATA) adjustment, from which it follows that the SI will continue to have the present set of seven base units, in particular.

"The kelvin will continue to be the unit of thermodynamic temperature, but its magnitude will be set by fixing the numerical value of the Boltzmann constant to be equal to exactly 1.380 6X×10^{-23} when it is expressed in the SI unit m^2 kg s^{-2} K^{-1}, which is equal to J K^{-1}."

But there are other reasons for the redefinition also. A temperature measurement, based on the international temperature scale, ITS-90 in particular, in comparison with direct measurements of thermodynamic temperature is implemented more easily and well reproducible.

However, ITS-90 has some inherent drawbacks. In particular, noticeable differences may occur between measurements made for the same temperature but under different definitions while measurements of higher precision. Even using one definition differences may occur when using different interpolation tools between the reference points. These effects are known in temperature metrology as a "lack of unity" and "inconsistency ranges" [2].

For many years, the CIPM had a long-term objective definition of all basic SI units through the FPC to eliminate the dependence of properties of any artifacts or materials and provide long-term stability units.

In 2005 in the Consultative Committee for Thermometry (CCT) the Task Group on the SI units (TG-SI) was formed, the purpose of which was the evaluation of introduction results of a new kelvin definition.

The TG-SI summarized the results of all studies relating to the possible new definition and recommended the kelvin redefinition of the unit through a fixed Boltzmann constant. Choice of the Boltzmann constant is quite natural, as thermal energy kT will always appear in the fundamental laws of physics.

Thus, the Boltzmann constant is the coefficient of proportionality between the real (true) characteristic of thermal systems - thermal energy, and the unit of thermodynamic temperature T, which was historically formed as the kelvin. Gibbs was the first who had established this [3]

$$E = kT \qquad (3)$$

and the numerical value of k was determined by Plank [4]. The value of k precisely describes the unit of thermodynamic temperature in energy terms.

This conclusion is in good agreement with the statistical definition of temperature. The development of the kinetic theory of gases led to the molecular interpretation of thermodynamic temperature, according to which the temperature is a parameter that characterizes the energy distribution of the molecules. This interpretation was improved thanks to the advanced by Gibbs (1902) statistical mechanics and the output equation (3). Thus, there was established the link between statistical and thermodynamic temperature.

3. Increasing accuracy of measuring the Boltzmann constant is a necessary condition for redefinition of the kelvin

As already mentioned, the present uncertainty of the triple point of water is approximately 50 mK, or in relative terms, about $2 \cdot 10^{-7}$. Obviously, the Boltzmann constant and uncertainty should be close to this value in order the redefinition of the kelvin did not lead to a significant loss of precision of temperature measurements.

By decision of the CIPM, the standard uncertainty value for the Boltzmann constant redefinition of the kelvin should not exceed $1 \cdot 10^{-6}$. The method that can be used to measure the Boltzmann constant is the same as those of primary thermometry, where T is related to other quantities that can be measured, as well as known constants. It uses the fundamental physical laws and the laws of thermodynamics.

The basis of the primary thermometry is formed with gas temperature-indicating measurements. Gas thermometric temperature is determined on the basis of the equation of state for a gas thermometer.

The latest in a simplified form, represents the vessel in which the gas (usually nitrogen, argon or helium, thermal properties of which are well-known) is, and which has known constant mass (or, what is equivalently, a constant number of moles). The state of the gas is changeable.

According to the equation of state, the change in temperature causes a change in pressure and / or volume through which the temperature is determined. Gas temperatures are divided into different kinds. To the initial methods of thermometry there also belong: noise thermometry; thermometry on broadening the spectral lines by the Doppler effect; spectral radiation thermometry. Here are the most promising primary thermometers for measuring the Boltzmann constant and, the minimum uncertainties of these measurements achieved with their help (Tab. 1) [5].

Tab. 1. The relative standard uncertainty of kelvin
Tab. 1. Wielkości niepewności określenia kelwina

Primary thermometer	The relative standard uncertainty
Acoustic gas thermometer	$1\cdot10^{-6}$
Dielectric constant gas thermometer	$2\cdot10^{-6}$
Refractive index gas thermometer	$10\cdot10^{-6}$
Johnson noise thermometer	$5\cdot10^{-6}$
Doppler broadening thermometer	$10\cdot10^{-6}$

4. The latest results on Boltzmann constant determination

Determination of *k* were especially activated from 2005, when the decision on preparation of redefinition of kilogram, ampere, kelvin and mole was made. These efforts were carried out most actively at such National Metrology Institutes (NMIs) as: *NIST* (USA), *PTB* (Germany), *NPL* (United Kingdom), *LNE-CNAM* (France).

Till 2010 the most accurate value of the Boltzmann constant was considered the value which was received at *NIST* with the uncertainty equal to $1.7\cdot10^{-6}$ at 1988 [6] using an acoustic gas thermometer with a spherical cavity. Improvement of devices and the measurement procedure was continued and in 2011 the news appeared that *LNE-CNAM* received the value of the Boltzmann constant with the standard uncertainty $1.2\cdot10^{-6}$. In 2012, based on the latest experimental results, CODATA published a new recommended values of the Boltzmann constant k=1.3806488(13)$\cdot10^{-23}$J/K with the standard uncertainty $9.1\cdot10^{-7}$ [7]. This value shows the progress in determination of the Boltzmann constant and the execution of conditions for achieving the uncertainty level of $1\cdot10^{-6}$.

5. Kelvin redefinition results. Status ITS-90

It is expected that a new definition of kelvin will have a small influence on ITS-90 status. Though, ITS-90 will no longer be the only practical basis for temperature measurements. Thus, the most direct and beneficial effects of the changes are expected for temperatures below ~ 20 K and above ~ 1300 K, where the primary thermometers can provide users with less uncertainty than it is possible using ITS-90.

However, ITS-90 will continue to be used in the foreseeable future as a scale that reproduces accurately and conveniently and has a good approximation to the thermodynamic temperature scale. Long-term consequence of the changes is that the primary methods will be developed to achieve a higher accuracy and will gradually replace ITS-90 as the basis of temperature measurements. With the application of the new definition of kelvin it will be important to indicate which uncertainty of the thermodynamic temperature will be used (Tab.2).

In this table in column 5 $u(T_{k\,fixed})$ there is the future uncertainty in the thermodynamic temperature of the listed phase transitions (which presently serve as fixed points on ITS-90) when assuming that the new definition for the kelvin is adopted. All the values are quoted as standard uncertainties.

The values in columns 3 and 4 have been taken from Table 1.2 “Supplementary Information for the International Temperature Scale of 1990”[8]. In the foreseeable future, the majority of temperature measurements in the range from minus 200 °C to plus 960 °C will be made using the standard platinum thermometers, calibrated by ITS-90.

As ITS-90 will remain unchangeable, the uncertainties in T_{90} will not change implementations of reference points (column 3 of the table). It should be taken into account that a relative standard uncertainty of $1\cdot10^{-6}$ in k corresponds to a standard uncertainty of about 0.25 mK in the temperature of the triple point of water (T_{tpw}) after the redefinition, instead of existing 0.49 mK.

Tab. 2. Defining fixed points of the ITS-90 with uncertainties $u(T_{90})$ of the best practical realization (T_{90} in terms of ITS-90) and uncertainties $u(T)$ of the thermodynamic temperature
Tab. 2. Niepewności temperatury dla punktów ITS-90

Fixed points	T_{90} K	$u(T_{90})$ mK	$u(T)$ mK	$u(T_{k\,fixed})$ mK
1	2	3	4	5
Cu	1357.77	15	60	60.1
Au	1337.33	10	50	50.1
Ag	1234.93	1	40	40.1
AL	933.473	0.3	25	25.1
Zn	692.677	0.1	13	13.1
Sn	505.078	0.1	5	5.10
In	429.7485	0.1	3	3.11
Ga	302.9146	0.05	1	1.15
H_2O	273.16	0.02	0	0.49
Hg	234.3156	0.05	1.5	1.55
Ar	83.8058	0.1	1.5	1.50
O_2	54.3584	0.1	1	1.00
N_e	24.5561	0.2	0.5	0.50
e-H_2	≈ 20.3	0.2	0.5	0.50
e-H_2	≈ 17.0	0.2	0.5	0.50
e-H_2	13.8033	0.1	0.5	0.50
^{4}He	4.2221	0.1	0.3	0.30

As all thermodynamic measurements are defined relatively to the triple point of water, the uncertainty of 0.25 mK covers all thermodynamic temperature measurements. In practice, the change in definition will only effect the measurements near 273 K, because far from this range the uncertainty of the thermodynamic temperature will be significantly greater than 0.25 mK .

The TG-SI considers that a small increase in the thermodynamic temperature uncertainty is not a problem for metrology or general community and researchers. It is expected that any changes in a future temperature scale will be much smaller than the tolerances associated with current documentary standards for thermoelectric converters and industrial platinum resistance thermocouples.

The triple point of water will continue to play an important role in practical thermometry. In fact, the difference in T_{tpw}, which is implemented in various reference ampoules of the triple point of water, can be as low as 50 mK (which corresponds to the relative standard uncertainty $2\cdot10^{-7}$) and even less.

Therefore, the long-term experiments requiring the maximum accuracy at or near T_{tpw} will still rely on the realization of the triple point of water.

6. Practical realization of the new definition

To help users to take accurate and reliable measurements of CCT through the CIPM, the compilation of guidelines on the temperature measurement similar to the current supporting information on the international temperature scale was published in 1990. The guidance relating to the practical realization of the definition of kelvin (*The Mise en Pratique - MeP-K*) [8], in the future will be updated and expanded to describe the recognized primary methods of temperature measurement or scale and sources of uncertainties associated with measurements.

Although there will be immediate changes of ITS-90, the future majority of views will include clarification on the basis of the best thermodynamic measurements. When determining the current day through the kelvin temperature of the triple point of water, the procedure to ensure traceability and traceability to T_{tpw} can be presented in the scheme in Fig. 1.

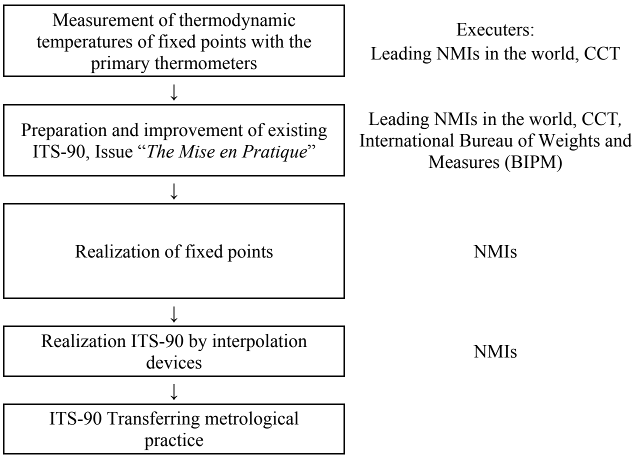

Fig. 1. ITS-90 realization scheme
Rys. 1. Schemat realizacji ITS-90

The essential features of the system should be emphasized:
- primary thermometers may have only a few most developed countries participating in the development and determination of existing ITS-90;
- CIPM and CCT developing the *MeP-K* and publish it BIPM web-site.

Hereby most countries may have no primary thermometers within their national standards. For the realization ITS-90 it is enough to have an equipment for realizing the fixed points and interpolation devices for scaling.

This is very significant because the primary thermometers are very complex and expensive.

In case of redefining kelvin through the Boltzmann constant the procedure becomes different (Fig. 2). Based on the most accurate measurements of the Boltzmann constant, CODATA and CIPM establish the exact value of the Boltzmann constant and publish it for use by NMIs in all countries.

Further differentiation of the new definition implementation is possible. The most developed NMIs which are able to have their own primary thermometers can actually realize the thermodynamic temperatures (scale) and provide traceability to the Boltzmann constant, that has primary standards of kelvin.

But the majority of NMIs obviously have no possibility to have a primary thermometer. Even if they have equipments for the ITS-90 fixed points realization they will not realize thermodynamic temperature.

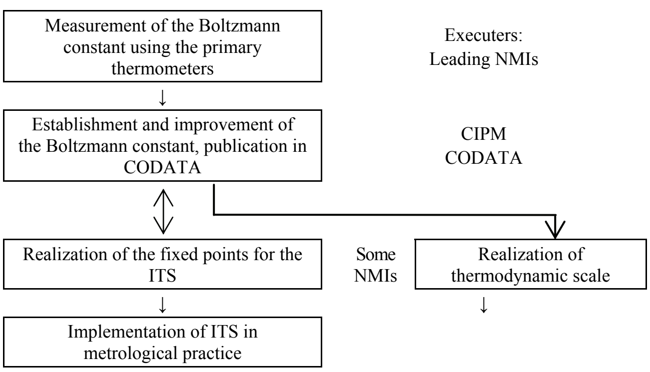

Fig. 2. New definition of kelvin realization scheme
Fig. 2. Nowy schemat realizacji definicji jednostki temperatury Kelwin

As it is shown above, this is not essential for practical metrology and can be some restriction only in the scientific use of equipment. In the future, it may be appropriate to develop some transfer mechanism for the "corrections" from thermodynamics to practical scale, if needed.

7. Conclusions

The Boltzmann constant is not associated with other FPC, unlike, for example, the molar gas constant. Thus, there are no alternative connections between kelvin and some other constants except the Boltzmann constant. Resolution of the 24th CGPM for the new definition of the kelvin by fixing the Boltzmann constant continues the same way as with other units to ensure long-term sustainability. To simplify the usage of the new definition, it will be accompanied by text *MeP-K* explaining how this affects the primary and practical thermometry.

The new definition should be harmonized with a modern science, which is characterized by the statistical nature of temperature and thermodynamics and which includes the equivalence of energy and temperature expressed by the equation (1). In principle the temperature can be deduced from the measurement of energy. But in practice there is no simple and universal tool for measuring energy, and it becomes in various forms, for example, in the form of temperature. Due to the Boltzmann constant, the temperature can be recalculated into energy values. It is not necessary that the new definition of the SI base unit directly allows realizing a unit with the lower uncertainty. Redefinition of kelvin through the exact value of the Boltzmann constant, according to the Resolution 1 of the 24th CGPM, is much more important for metrology and science in general than any increase in the accuracy when determining the thermodynamic temperature. At very low and very high temperatures it will not be necessary in the future to return to the triple point of water. TG-SI considers that this result will give the main practical advantage of the new definition. It is expected that the kelvin redefinition will not lead to a significant change in status of ITS-90. However, later ITS-90 will stop to be the singular basis for measuring temperature, but will be used as a precision and reliable scale connected with the thermodynamic one. It is predicted that a new definition will stimulate further improvement of primary thermometers in terms of both design and accuracy, and they will gradually replace ITS-90. In the near future the majority of temperature measurements in the most important range from minus 200 °C to plus 960 °C will be made with standard platinum resistance thermometers, calibrated in accordance with ITS-90.

8. References

[1] Mills I.M., Mohr P.J., Quinn T.J., Taylor B.N., Williams E.R.: Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI - 2005), Metrologia 43, pp. 227-246 (2006).
[2] Konogonov S.A.: Metrology and fundamental physical constants. Moskow: Standartinform. 2008, p.269.
[3] Kuni F.M.: Statistical Physics and Thermodynamics. Moskow: Nauka. 1981, p. 352.
[4] Planck M.: Ständiger Bejbachter der Preuβischen Akademie der Wissenschaften.-1899. - P.440; Annalen der Physik. – 1990.- V.1.- P.69.
[5] Fischer J., Zandt T.: New Definition of the Kelvin and Determination of the Boltzmann Constant, 56th International Scientific Colloquium, Ilmenau University of Technology, 12 – 16 September 2011, URN (Paper): urn:nbn:de:gbv:ilm1-2011iwk-013:1
[6] Moldover M.R., Trusler J.P.M., Edwards T.J., Mehland T.J., Davis R.S.: Measurement of the universal gas constant R using a spherical acoustic resonator, J. Res. Natl. Bur. Stand. 93 pp. 85-114 (1988).
[7] Reviews of Modern Physics, volume 84, October-December 2012, doi: 10.1103/RevModPhys.84.1527.
[8] Preston-Thomas H., Bloembergen P., Quinn T. J.: Supplementary Information for the International Temperature Scale of 1990 (BIPM, Sèvres, 1990).
[9] Mise en pratique for the definition of the kelvin http://www.bipm.org/utils/en/pdf/MeP_K.pdf.

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