Optica Applicata, Vol. XLV, No. 2, 2015
DOI: 10.5277/oa150202

Focusing properties of partially coherent dark hollow Gaussian beams through a thin lens system

YUAN DONG[1], CHENCHEN ZHAO[1], KUILONG WANG[2], CHENGLIANG ZHAO[1*], YANGJIAN CAI[1]

[1] College of Physics, Optoelectronics and Energy and Collaborative Innovation Center of Suzhou Nano Science and Technology, Soochow University, Suzhou 215006, China

[2] Department of Physics, Hangzhou Normal University, Hangzhou 310036, China

*Corresponding author: zhaochengliang@suda.edu.cn

This paper investigated the focal shift of partially coherent dark hollow Gaussian beams through a thin lens system. An analytic expression of the irradiance distribution of the focusing partially coherent dark hollow Gaussian beams in the back focal plane has been given by using the Collins formula. The focus shift of focused partially coherent dark hollow Gaussian beams in different parameters is studied in detail by numerical calculations. It is found that the absolute value of the focal shift of partially coherent dark hollow Gaussian beams decreases as the transverse coherence width or the order of the dark hollow Gaussian beams or a parameter of the dark hollow Gaussian beams increases.

Keywords: focusing properties, partially coherent, dark hollow beam, focal shift.

1. Introduction

When a light beam is focused by a converging lens, the axial coordinate where the intensity takes its maximum value does not coincide with the position of the focus as predicted by geometrical optics. The real focus – the point with maximum axial intensity of the focusing beams – is shifted away from the geometric focus, and rather somewhat closer to the diffraction plane, which is called the focal shift (see Fig. 1) [1–4]. In some cases, such as some imaging applications, the magnitude of the focal shift is sufficiently small, therefore, the focal shift can be neglected. However, in some cases, such as in some laser cavities, the influence of the focal shift cannot be neglected, so the focal shift becomes very important. Therefore, it is interesting and necessary to study the focal shift. This phenomenon, especially the focal shift of the beams with

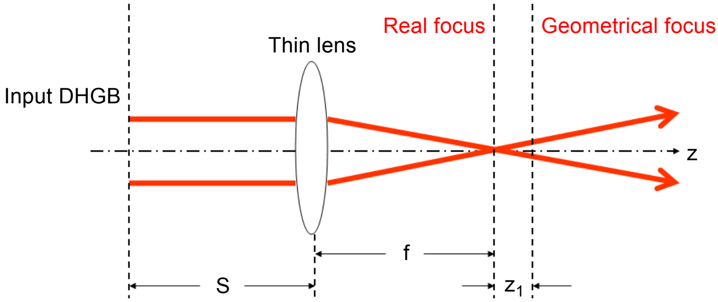

Fig. 1. The schematic of a lens optical system.

zero-central intensity, has been attracting more and more attention because of the applications in guiding and manipulating the neutral atoms and micro-sized particles [5–7]. The trapping characters by laser beams are affected by the changes in the partial coherence of the beams [8].

At present, there are two different definitions of focal shifts reported. One which is proposed by LI and WOLF and is called the LW method [3, 4] is to find the position of the maximum intensity along the optical axis. After the beams are passed through a thin lens system, it is then viewed as the real focus [1, 3, 4]. This is determined by the root of the equation $dI(z)/dz = 0$, where $I(z)$ is the axial intensity distribution of the beams. Another one based on the encircled-power method which is proposed by GREENE and HALL and is called the GH method [9] is usually used for some different cases, such as high-order Bessel–Gaussian and Hermite–Gaussian beams [10, 11]. Until now, there have appeared many papers concerning the focal shift in different types of focused beams through a variety of systems. But most of them are for coherent beams, and rarely for partially coherent beams.

The purpose of this paper is to investigate the dependence of the focal shift of the partially coherent dark hollow Gaussian beams (DHGBs) on the parameters of the beams. In Section 2, we obtain the expression for the cross-spectral density of partially coherent DHGBs through a thin lens system from the Collins formula. It can help us in describing the intensity distribution of focused partially coherent DHGBs in the focal plane. In Section 3, based on the expression derived above, the focal shift with different parameters is given. It is found that the relative focal shift is strongly dependent on these parameters: the transverse coherence width σ_g, the order N and the parameter p of the DHGBs. In Section 4, we finally conclude the main results.

2. The cross-spectral density of the focused partially coherent DHGBs

The cross-spectral density of the focused partially coherent DHGBs at the input plane ($z = 0$) is defined as a superposition of a finite sum of partially coherent Gaussian Schell-model beams [12]:

$$W_I(x_1, y_1, x_2, y_2, 0) = \sum_{m=1}^{N} \sum_{n=1}^{N} \frac{(-1)^{m+n}}{N^2} \binom{N}{m} \binom{N}{n} (X_1 - X_2 - X_3 + X_4) Y \quad (1)$$

where:

$$X_1 = \exp\left(-\frac{nx_1^2 + mx_2^2}{w_0^2} - \frac{ny_1^2 + my_2^2}{w_0^2}\right)$$

$$X_2 = \exp\left(-\frac{nx_1^2}{w_0^2} - \frac{mx_2^2}{pw_0^2} - \frac{ny_1^2}{w_0^2} - \frac{my_2^2}{pw_0^2}\right)$$

$$X_3 = \exp\left(-\frac{nx_1^2}{pw_0^2} - \frac{mx_2^2}{w_0^2} - \frac{ny_1^2}{pw_0^2} - \frac{my_2^2}{w_0^2}\right)$$

$$X_4 = \exp\left(-\frac{nx_1^2 + mx_2^2}{pw_0^2} - \frac{ny_1^2 + my_2^2}{pw_0^2}\right)$$

$$Y = \exp\left[-\frac{(x_1 - x_2)^2}{2\sigma_g^2} - \frac{(y_1 - y_2)^2}{2\sigma_g^2}\right]$$

while both $\binom{N}{m}$ and $\binom{N}{n}$ denote binomial coefficients, w_0 determines the beam waist width, p is a real positive parameter and satisfies $p < 1$ (we can adjust the central dark size of the DHGBs by varying p), σ_g is the transverse coherence width, N is the order of the DHGBs. The irradiance distribution of a partially coherent beam is given by $I(x, y, z) = W(x, y, x, y, z)$. We calculate in Fig. 2 the normalized irradiance distribu-

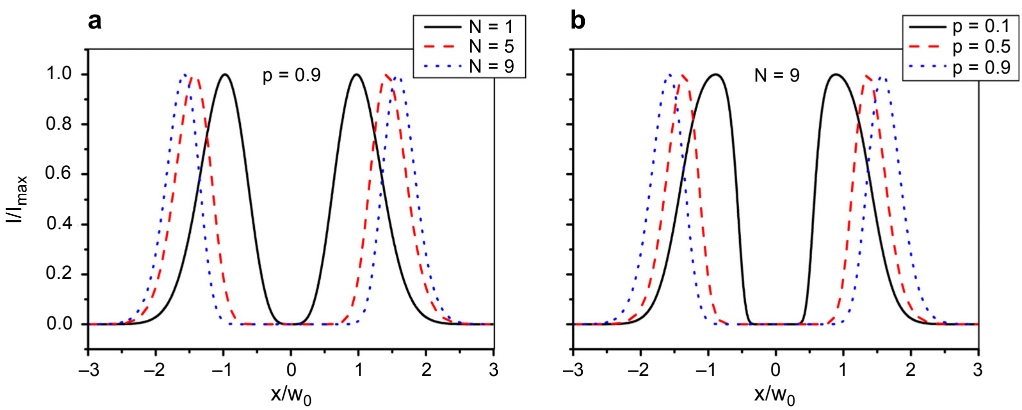

Fig. 2. Normalized intensity distributions ($y = 0$) of the input DHGBs, for different N with $p = 0.9$ (**a**), and for different p with $N = 9$ (**b**).

tions ($y = 0$) of the DHGBs for different values of N and p. It is shown that the range of the zero-central increases as N or p increases.

Under the paraxial approximation, the propagation of any linear-polarized field through an optical system can be treated by the Collins formula of the cross-spectral density, which is given by

$$W_O(\rho_{x1}, \rho_{y1}, \rho_{x2}, \rho_{y2}, z) = \left(-\frac{ik}{2\pi B}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_I(x_1, y_1, x_2, y_2, 0) Z \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}y_1 \mathrm{d}y_2 \tag{2}$$

where

$$Z = \exp\left\{-\frac{ik}{2B}\left[A(x_1^2 + y_1^2) - 2(x_1\rho_{x1} + y_1\rho_{y1}) + D(\rho_{x1}^2 + \rho_{y1}^2)\right] + \right.$$
$$\left. + \frac{ik}{2B^*}\left[A(x_2^2 + y_2^2) - 2(x_2\rho_{x2} + y_2\rho_{y2}) + D(\rho_{x2}^2 + \rho_{y2}^2)\right]\right\}$$

and $W_I(x_1, y_1, x_2, y_2, 0)$ is the cross-spectral density of the partially coherent DHGBs in the input plane as Eq. (1), and z is the distance from the input plane to the output plane, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, A, B, C and D are the elements of this matrix for the paraxial optical system between the input plane and the output planes [13]. Here, we consider the partially coherent DHGBs propagating through an apertureless lens (as shown in Fig. 1). The elements of the transfer matrix of this optical system between the input and the output planes can be given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & z_1 + f \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -z_1/f & f + (1-\alpha)z_1 \\ -1/f & 1-\alpha \end{bmatrix} \tag{3}$$

where f is the focal length of the lens, s is the distance from the input plane to the lens plane, the ratio $\alpha = s/f$; z_1 is the distance between the geometric focus and the real focus, which is called focal shift, and $z_1 = z - f - s$.

Substituting Eqs. (1) and (3) into Eq. (2), and after integration, the cross-spectral density of the partially coherent DHGBs at the output plane can be described as

$$W_O(\rho_{x1}, \rho_{y1}, \rho_{x2}, \rho_{y2}, z) = \sum_{m=1}^{N}\sum_{n=1}^{N}\frac{(-1)^{m+n}}{N^2}\binom{N}{m}\binom{N}{n} P \times$$
$$\times\left(\frac{\pi^2}{M_{11}M_{12}}Q_1 - \frac{\pi^2}{M_{11}M_{22}}Q_2 - \frac{\pi^2}{M_{31}M_{32}}Q_3 + \frac{\pi^2}{M_{31}M_{42}}Q_4\right) \tag{4}$$

where

$$P = -\frac{k^2}{4\pi^2 B^2}\exp\left[-\frac{ikD}{2B}(\rho_{x1}^2 + \rho_{y1}^2 - \rho_{x2}^2 - \rho_{y2}^2)\right]$$

$$Q_1 = \exp\left[-\frac{k^2(\rho_{x1}^2+\rho_{y1}^2)}{4M_{11}B^2} - \frac{k^2}{4M_{12}B^2}\left(\rho_{x2} - \frac{\rho_{x1}}{2\sigma_g^2 M_{11}}\right)^2 - \frac{k^2}{4M_{12}B^2}\left(\rho_{y2} - \frac{\rho_{y1}}{2\sigma_g^2 M_{11}}\right)^2\right]$$

$$Q_2 = \exp\left[-\frac{k^2(\rho_{x1}^2+\rho_{y1}^2)}{4M_{11}B^2} - \frac{k^2}{4M_{22}B^2}\left(\rho_{x2} - \frac{\rho_{x1}}{2\sigma_g^2 M_{11}}\right)^2 - \frac{k^2}{4M_{22}B^2}\left(\rho_{y2} - \frac{\rho_{y1}}{2\sigma_g^2 M_{11}}\right)^2\right]$$

$$Q_3 = \exp\left[-\frac{k^2(\rho_{x1}^2+\rho_{y1}^2)}{4M_{31}B^2} - \frac{k^2}{4M_{32}B^2}\left(\rho_{x2} - \frac{\rho_{x1}}{2\sigma_g^2 M_{31}}\right)^2 - \frac{k^2}{4M_{32}B^2}\left(\rho_{y2} - \frac{\rho_{y1}}{2\sigma_g^2 M_{31}}\right)^2\right]$$

$$Q_4 = \exp\left[-\frac{k^2(\rho_{x1}^2+\rho_{y1}^2)}{4M_{31}B^2} - \frac{k^2}{4M_{42}B^2}\left(\rho_{x2} - \frac{\rho_{x1}}{2\sigma_g^2 M_{31}}\right)^2 - \frac{k^2}{4M_{42}B^2}\left(\rho_{y2} - \frac{\rho_{y1}}{2\sigma_g^2 M_{31}}\right)^2\right]$$

and

$$M_{11} = \frac{n}{w_0^2} + \frac{1}{2\sigma_g^2} + \frac{ikA}{2B}$$

$$M_{12} = \frac{m}{w_0^2} + \frac{1}{2\sigma_g^2} - \frac{ikA}{2B} - \frac{1}{4M_{11}\sigma_g^4}$$

$$M_{22} = \frac{m}{pw_0^2} + \frac{1}{2\sigma_g^2} - \frac{ikA}{2B} - \frac{1}{4M_{11}\sigma_g^4}$$

$$M_{31} = \frac{n}{pw_0^2} + \frac{1}{2\sigma_g^2} + \frac{ikA}{2B}$$

$$M_{32} = \frac{m}{w_0^2} + \frac{1}{2\sigma_g^2} - \frac{ikA}{2B} - \frac{1}{4M_{31}\sigma_g^4}$$

$$M_{42} = \frac{m}{pw_0^2} + \frac{1}{2\sigma_g^2} - \frac{ikA}{2B} - \frac{1}{4M_{31}\sigma_g^4}$$

3. The focal shift of partially coherent DHGBs

In this section, we will study the phenomenon of the focal shift for partially coherent DHGBs passing through an apertureless lens system. The irradiance distribution of partially coherent DHGBs at the output plane can be given by

$$I(x, y, z_1) = W_O(x, y, x, y, z_1) \tag{5}$$

The position of the maximum irradiance point along the axis can be a root of the following equation [1, 4, 14]:

$$dI(0, 0, z_1)/dz_1 = 0 \quad (6)$$

According to the LW method, in our case, a root (z_1) of Eq. (6) is just the focal shift. On substituting Eq. (5) into Eq. (6), we can easily find the root of the equation, which means that we have obtained the focal shift z_1, the point of the maximum irradiance along the axis.

Based on the equations obtained above, some numerical calculations were performed to illustrate the focal shift of partially coherent DHGBs passing through a thin lens. In our following calculations, we choose and fix the parameters as: $\lambda = 1.06$ µm, $f = 0.1$ m, $w_0 = 1$ mm, and $s = 0$.

In order to study the effect of transverse coherence width on the irradiance distributions of the focused partially coherent DHGBs, we calculate in Fig. 3 the 3D-nor-

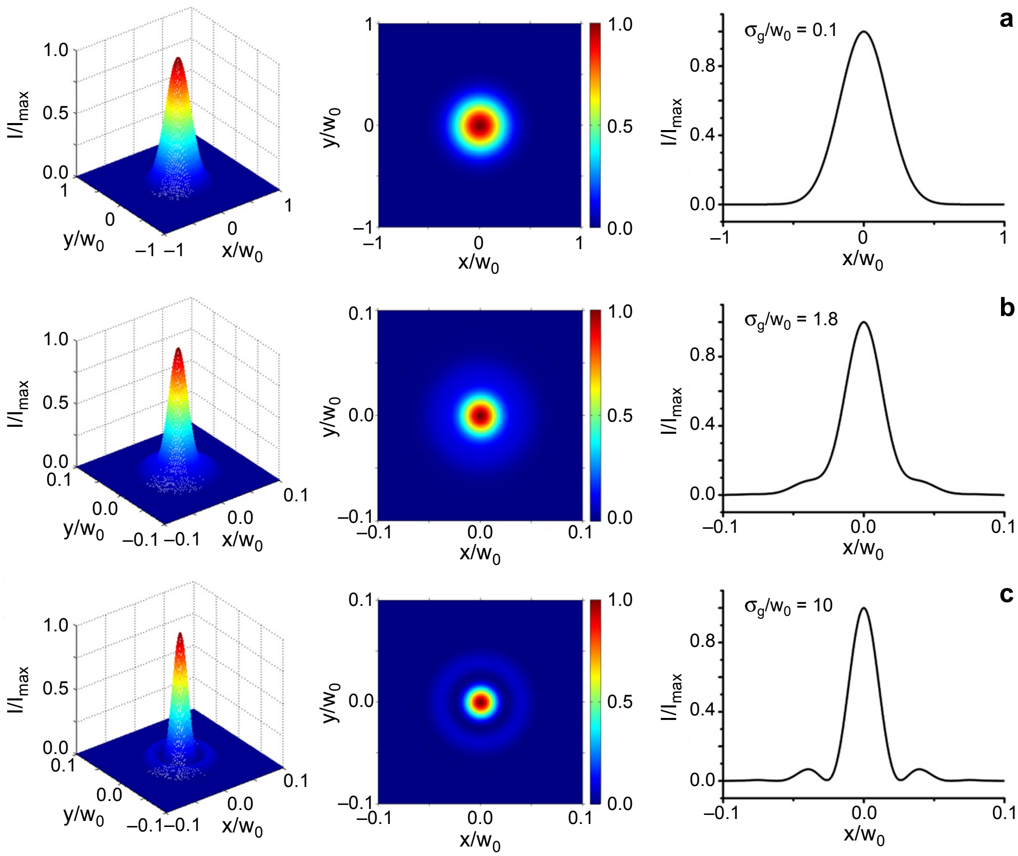

Fig. 3. 3D-normalized intensity distributions and corresponding contour graphs and corresponding cross-lines ($y = 0$) of the focused partially coherent DHGBs at the focal plane for different coherence width σ_g with $p = 0.8$ and $N = 9$; $\sigma_g/w_0 = 0.1$ (**a**), $\sigma_g/w_0 = 1.8$ (**b**), and $\sigma_g/w_0 = 10$ (**c**).

malized irradiance distributions and corresponding contour graphs and corresponding cross-lines ($y = 0$) of the focused partially coherent DHGBs at a geometrical focal plane for different coherence width σ_g with $p = 0.8$, and from Fig. 3, we can find that the focused irradiance distribution of the partially coherent DHGBs is related to its initial coherence width. When the coherence width is large (see Fig. 3**c**), the focused beam profile is Gaussian distribution and there is a small bright ring around the brightest circular solid beam spot. But as the coherence width decreases, the bright ring gradually disappears and the focused beam profile gradually becomes only Gaussian distribution, and the focused beam spot size increases.

Figure 4 shows the focal shift z_1/f of partially coherent DHGBs with coherence width $\sigma_g = 0.1w_0$ *versus* the parameter p and the order of the DHGBs N at $N = 5$ in Fig. 4**a** and at $p = 0.5$ in Fig. 4**b**. From Fig. 4, we can see that the focus shift of partially coherent DHGBs increases with the rise in the parameter p or the parameter N.

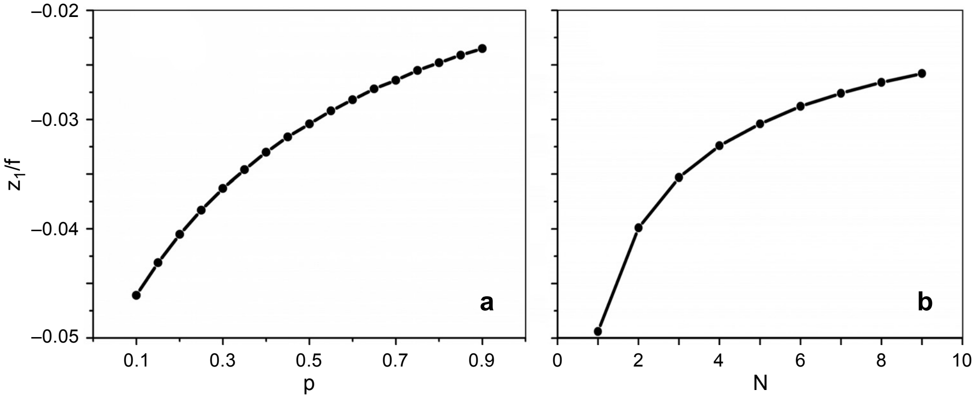

Fig. 4. Focal shift of partially coherent DHGBs with coherence width $\sigma_g = 0.1w_0$ *versus* the parameter p and the order N of the DHGBs: $N = 5$ (**a**), and $p = 0.5$ (**b**).

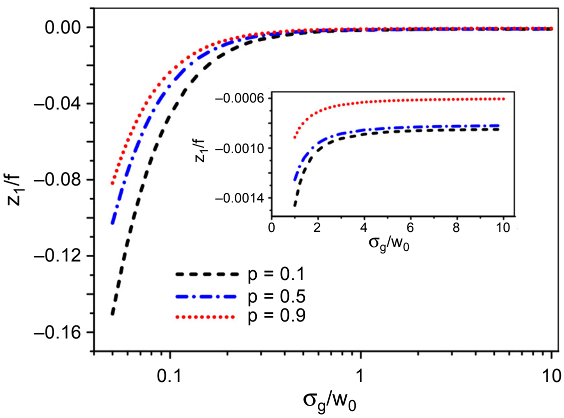

Fig. 5. Focal shift of partially coherent DHGBs with $N = 5$ *versus* the transverse coherence width σ_g for different parameter p.

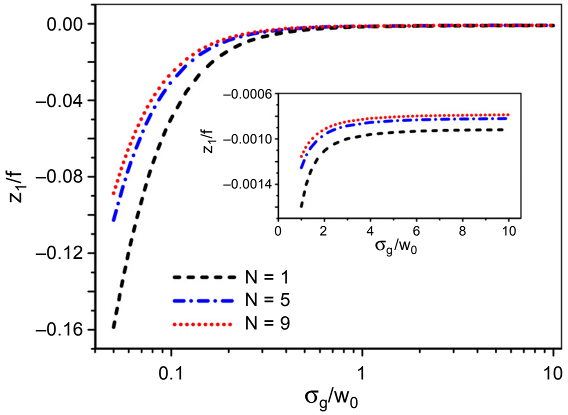

Fig. 6. Focal shift of partially coherent DHGBs with $p = 0.5$ *versus* the transverse coherence width σ_g for different order N of the DHGBs.

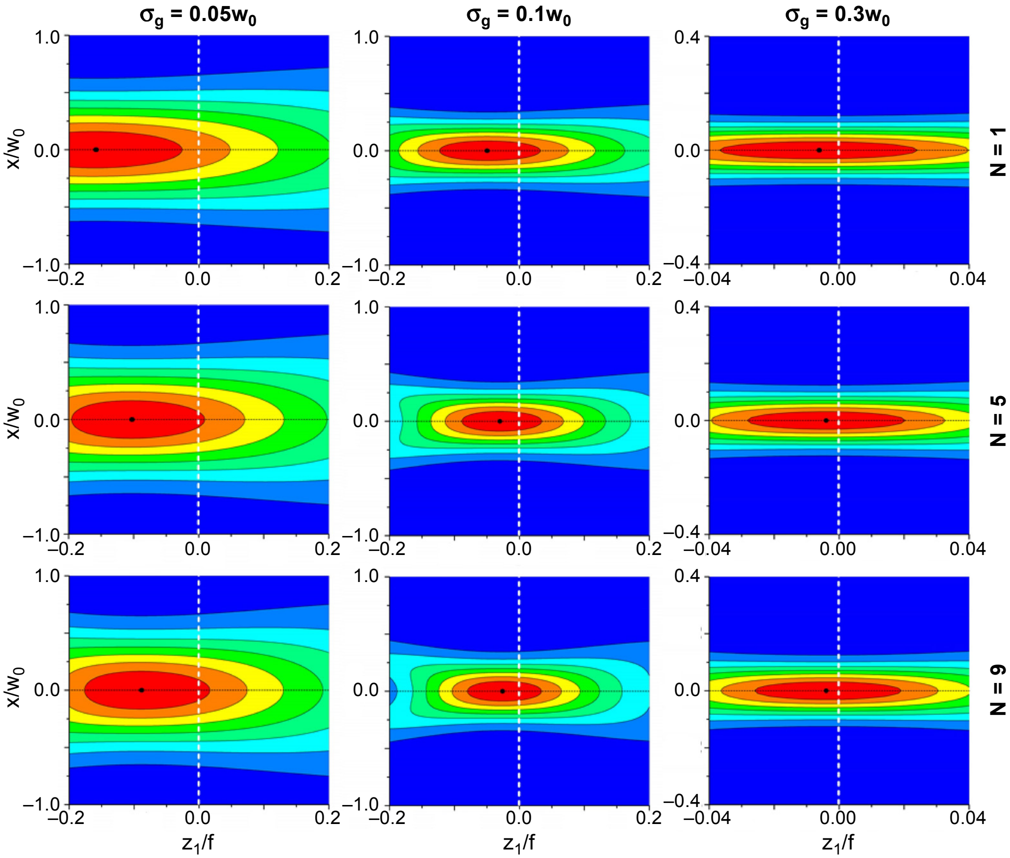

Fig. 7. Contour lines of the focused irradiance distributions of the partially coherent DHGBs with $p = 0.5$ near the geometrical focus point for different N and σ_g. The black point is the position of the real maximum intensity.

Figures 5 and 6 show the focal shift z_1/f of partially coherent DHGBs *versus* the transverse coherent width σ_g/w_0 for different values of the parameter p and different order N of the DHGBs. From Figs. 5 and 6, we can see that the focus shift of partially coherent DHGBs increases with the rise in the transverse coherent width σ_g/w_0 for different parameter p (or different order N). For the same transverse coherent width σ_g/w_0, the bigger the value of p (or N), the larger the focal shift. But for the focus shifts of partially coherent DHGBs with different parameter p (or different order N), the differences between them become smaller and smaller as the transverse coherent width σ_g/w_0 increases. From Figs. 4–6, we can also find that the effects of the parameter p and order N on the focal shift of partially coherent DHGBs are similar.

Figure 7 shows the contour lines of the focused irradiant distributions of the partially coherent DHGBs with $p = 0.5$ near the geometrical focus point for different N and σ_g, where the black point is the position of the real maximum intensity. From Fig. 7, we can clearly see that the real focus is closer to the diffraction plane (here it

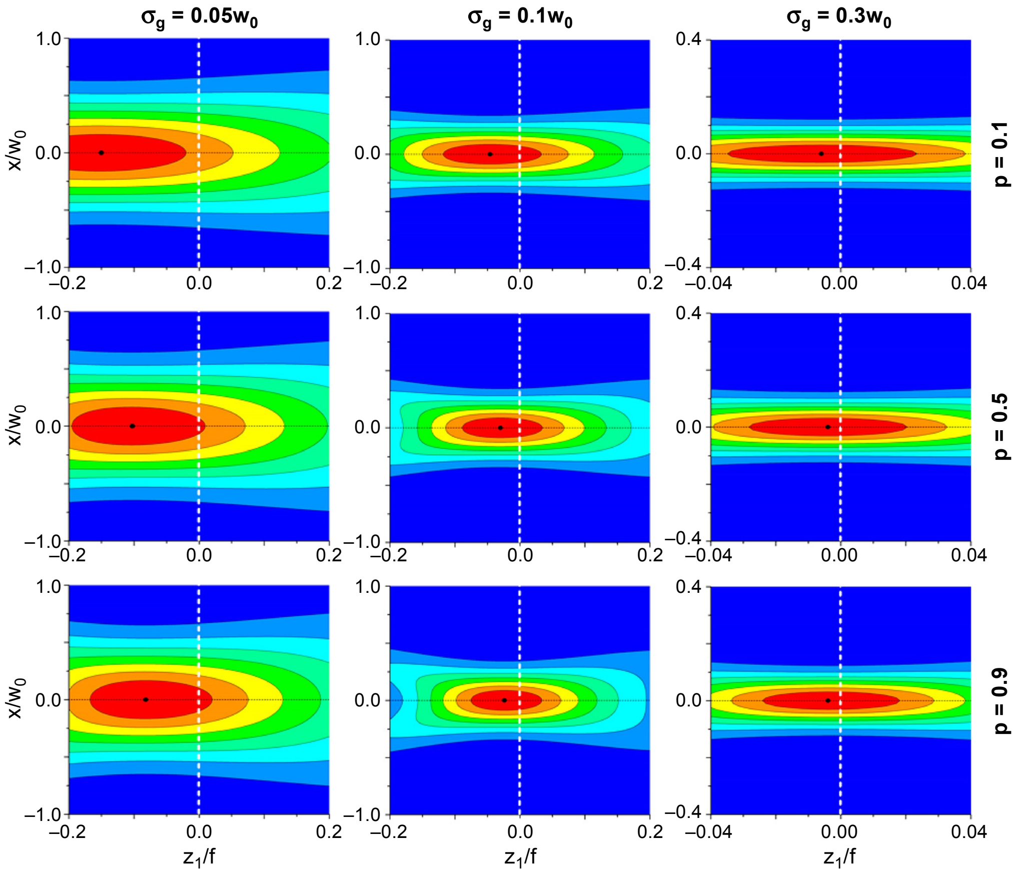

Fig. 8. Contour lines of the focused irradiance distributions of the partially coherent DHGBs with $N = 5$ near the geometrical focus point for different p and σ_g. The black point is the position of the real maximum intensity.

is a thin lens), and the focal shift decreases as the order N or the coherence width σ_g increases. One can also see that these focus shift phenomena are in agreement with the results given above.

Figure 8 shows the contour lines of the focused irradiance distributions of the partially coherent DHGBs with $N = 5$ near the geometrical focus point for different p and σ_g, where the black point is the position of the real maximum intensity. Through the transverse and longitudinal comparison in Fig. 8, it is obvious that the focal shift decreases as the coherence width σ_g or the parameter p increases.

4. Conclusion

In summary, we have studied the properties of the focal shift of partially coherent DHGBs. The cross-spectral density of partially coherent DHGBs through a thin lens system has been obtained by using the Collins formula. By calculating the point of the maximum irradiance in focused beams, the position of the real focus has been found. Through theoretical analysis and numerical simulations, it is easily found that the focal shift of partially coherent DHGBs can be controlled by changing the beam parameters (such as p, N and σ_g) properly. Further research on the manipulating and trapping of neutral atoms and micro-sized particles by focused DHGBs will be carried out in the future.

Acknowledgments – This research is supported by the National Natural Science Foundation of China under Grant Nos. 11274005, 11374222, the Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions, and the Universities Project Sponsored by the Scientific Research Foundation for the Returned Overseas Chinese Scholars, the State Education Ministry, the Qing Lan Project of Jiangsu Province, the 2013 High Educational Reform Project of Soochow University under Grant No. 5731501713, the Zhejiang Provincial Natural Science Foundation of China Grant No. LY12A04012.

References

[1] CHENGLIANG ZHAO, LIGANG WANG, XUANHUI LU, *Focal shift of hollow Gaussian beams through a thin lens*, Optics and Laser Technology **40**(1), 2008, pp. 58–63.

[2] ARIMOTO A., *Intensity distribution of aberration-free diffraction patterns due to circular apertures in large F-number optical systems*, Optica Acta **23**(3), 1976, pp. 245–250.

[3] YAJUN LI, WOLF E., *Focal shifts in diffracted converging spherical waves*, Optics Communications **39**(4), 1981, pp. 211–215.

[4] YAJUN LI, WOLF E., *Focal shift in focused truncated gaussian beams*, Optics Communications **42**(3), 1982, pp. 151–156.

[5] OVCHINNIKOV YU.B., MANEK I., GRIMMET R., *Surface trap for Cs atoms based on evanescent-wave cooling*, Physical Review Letters **79**(12), 1997, pp. 2225–2228.

[6] KUGA T., TORII Y., SHIOKAWA N., HIRANO T., SHIMIZU Y., SASADA H., *Novel optical trap of atoms with a Doughnut beam*, Physical Review Letters **78**(25), 1997, pp. 4713–4716.

[7] SONG Y., MILAM D., HILL W.T., *Long, narrow all-light atom guide*, Optics Letters **24**(24), 1999, pp. 1805–1807.

[8] Chengliang Zhao, Yangjian Cai, Xuanhui Lu, Eyyuboğlu H.T., *Radiation force of coherent and partially coherent flat-topped beams on a Rayleigh particle*, Optics Express **17**(3), 2009, pp. 1753–1765.
[9] Greene P.L., Hall D.G., *Focal shift in vector beams*, Optics Express **4**(10), 1999, pp. 411–419.
[10] Mahajan V.N., *Axial irradiance and optimum focusing of laser beams*, Applied Optics **22**(19), 1983, pp. 3042–3053.
[11] Baida Lü, Runwu Peng, *Focal shift in Hermite–Gaussian beams based on the encircled-power criterion*, Optics and Laser Technology **35**(6), 2003, pp. 435–440.
[12] Xiang Lü, Yangjian Cai, *Partially coherent circular and elliptical dark hollow beams and their paraxial propagations*, Physics Letters A **369**(1–2), 2007, pp. 157–166.
[13] Collins J.R., Stuart A., *Lens-system diffraction integral written in terms of matrix optics*, Journal of the Optical Society of America **60**(9), 1970, pp. 1168–1177.
[14] Ji Xiaoling, Baida Lu, *Focal shift and focal switch of flattened Gaussian beams in passage through an aperture bifocal lens*, IEEE Journal of Quantum Electronics **39**(1), 2003, pp. 172–178.

Received October 21, 2014
in revised form January 6, 2015