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A New Model for a Frequency-Dependent Non-Linear Reactor

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***Summary*: Increase of power electronic devices has resulted in current/voltage harmonic pollution in power systems. Harmonics have adverse effects on equipment with magnetic materials. This paper investigates effects of harmonics on inductors and proposes a new model for a non-linear frequency-dependent inductor. The study is based on experiments in which inductors are subjected to sinusoidal inputs at different frequencies. The accuracy of the proposed formula is also investigated.**

Keywords:
frequency-dependent inductors,
harmon ic,
power quality

1. INTRODUCTION

Power system harmonics have been known to exist for a long time. However, with the increase of non-linear loads, their effects on power system equipment have become of special importance. Harmonics cause higher losses in electric machines, transmission lines, capacitor banks and may result in over voltage/ over current due to resonance conditions. The focus of this paper is to study effects of harmonics on inductors and develop a new model for a nonlinear frequency-dependent reactor. The study is carried out by experiments. An ac source is used to generate a sinusoidal voltage at various frequencies. The inductance of the sample inductor is measured. A new model is then developed for inductance estimation as a function of frequency. The sensitivity of the proposed model is also studied.

2. REVIEW ON PREVIOUS WORKS

There are few reports on behavior of inductors due to change in frequency of operation. In [1], a linear relationship is considered between the inductance and frequency of operation of an inductor. This model can be useful in harmonic studying of power systems even if it suffers from lack of accuracy. The reason is that higher frequencies are attended in power systems when they pass through power transformers. However, the model proposed in [1] cannot be used for harmonic filters which are exposed to higher order of harmonics.

In [2] a compilation of different models for a frequency-dependent load proposed in the literature has been reported. Electrical parameters are derived from the estimated active and reactive power, P and Q, and information about the composition and characteristics of the load. Also in this work a model taking into account electronic loads that produce harmonics is proposed. One of these proposed models composed of a parallel combination of an inductor and a resistor is shown in Figure 1. In this model, i.e. Hatzi model, the relations for the inductance and resistance are given by:

$$L(h)=\frac{U^2}{m(h).Q}$$

$$R(h)=\frac{U^2}{m(h).P} \tag{1}$$

$$m(h)=0.1h+0.9$$

However this model yields satisfactory results only if lower order of harmonics, e.g. h<10, are considered. Using finite element method is also a common approach for finding the inductance of a non linear reactor [3, 4]. In [3], the change in inductances and resistors of transformers is investigated in order to determine transformers de-rating factors in harmonic environments. Transformers behave like inductors at no load operations. The study in [3] is based on experiments and finite element methods. Table 1 lists the transformer data, and Figure 2 shows the transformer inductance as a function of frequency.

It can be seen from Figure 2 that up to 3 kHz, the transformer leakage inductance is almost constant. For frequency above 3 KHz, transformer inductance falls rapidly by frequency such that for frequencies over 90 kHz, the inductance changes to an air-inductance. The core of the inductance under study in [3] is made of ferrite, and therefore, the results cannot be extended to normal iron-core inductors. In another attempt [4], variation of resistance and inductance of a railway with frequency is computed by three different methods that are the FEM, coupled-inductance

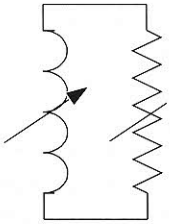

Fig. 1. Hatzi model for inductor

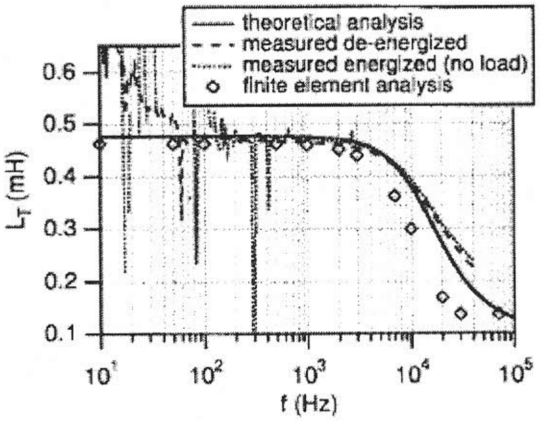

Fig. 2. Inductance variation curve

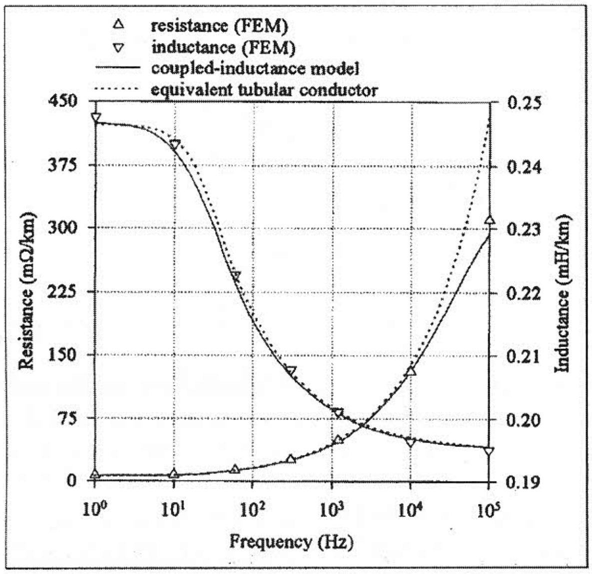

Fig. 3. Variation of resistance and inductance with frequency computed by 3 different methods

model and the equivalent cylindrical conductor model. By considering skin effect, the inductance has been divided into two different parts; the internal inductance which is frequency dependent and the internal inductance which is independent of frequency. The results of these approaches are shown in Figure 3. These results also demonstrate rapid decrease of the inductance by increasing the order of harmonics.

In some studies by supposing a laminated core and solving the related magnetic equations, the inductance has been derived. For example in [5] by using such a method the inductance is expressed as Equation:

$$L = \frac{2}{s\sqrt{j\omega\sigma\mu}} tg\left(\frac{s}{2}\sqrt{j\omega\sigma\mu}\right) L_0 \qquad (2)$$

Where s is the thickness of laminations, μ is the permeability of the core, σ is the electrical conductivity of laminations and L_0 is the reactor inductance without flux skin effect. This equation demonstrates that the reactor reactance is a function of frequency, laminated thickness, core conductivity and permeability. However this approach is straightforward but the applications of it is limited by non linearity characteristic of the reactors and also by the shape of them.

Table 1. Transformer characteristic

	primery winding	secondary winding
number of turns	165	104
number of layers	5	4
Turns per layer	33	26
wire gauge (AWG)	#16	#14
effective conductor thickness, b	1.14 mm	1.44 mm
window height, b_{win}	4.83 cm	4.83 cm
length of winding turn n, l_{Tn}	12 to 15 cm	16 to 20 cm
dc winding resistance (Ω)	0.28	0.16

3. INDUCTOR BEHAVIOR U NDER FREQUENCY CHANGE

This section presents the experimental results carried out to determine the change of inductance of an inductor under frequency variations.

3.1 Experimental Set-up

An ac source is used to generate voltage signals. A power quality analyzer is also used to capture voltage and current harmonics. Specifications of the ac source and power quality analyzer are given in the appendix. The inductor under study is an iron-core inductor made of 0.5 mm laminated core. The core size is $5\times10\times20$ cm^3 with a winding of 120 turns. At nominal frequency of 50Hz, its inductance is 700mH and its resistance is 1.2Ω.

The inductor is subjected to voltage signals with different frequencies, and the inductor active power, voltage, current, power factor, reactive power and apparent power are measured. The frequency range for the test is 500Hz to 1500Hz.

3.2. Method of Inductance Calculation

If an inductor is considered as a series combination of a resistance and an inductor, inductor parameters can be calculated using Equations (3) and (4). In order to reduce the effect of noise, a low-pass filter is also used. Figures 5 and 6 summarize the results.

$$R = \frac{P}{I^2} \qquad (3)$$

$$Z = \frac{U}{I}$$

$$X = \sqrt{Z^2 - R^2} \qquad (4)$$

$$L = \frac{X}{h\omega_0}$$

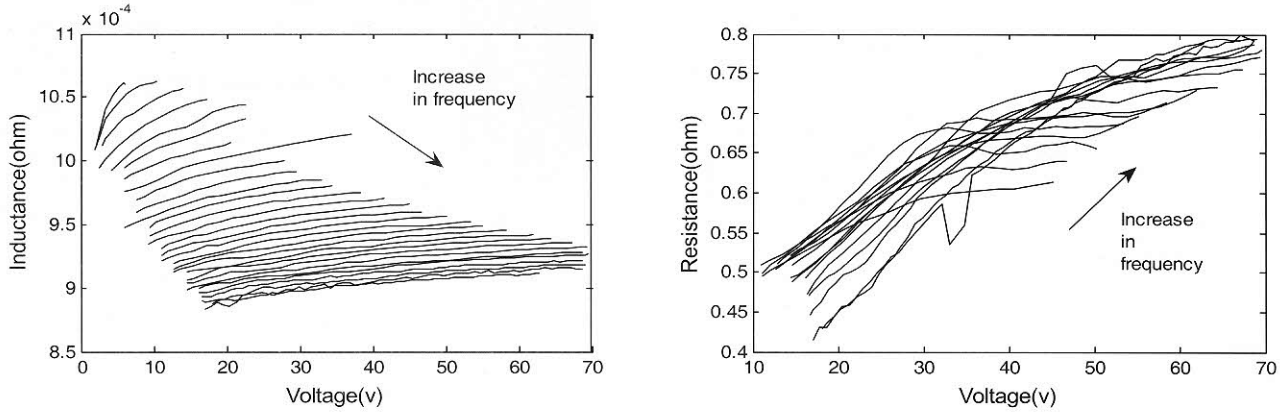

Fig. 4. Inductance and resistance vs. voltage and frequency

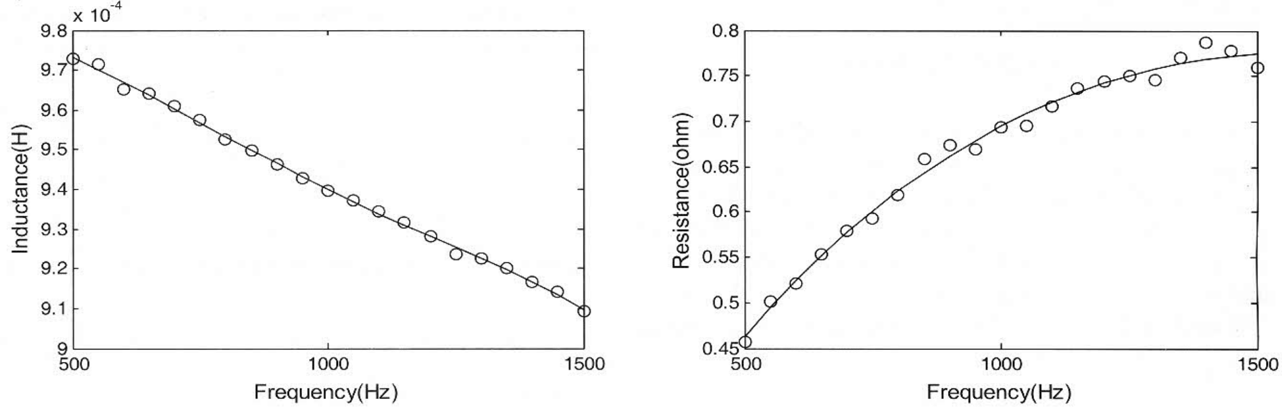

Fig. 5. Variation of resistance and inductance with frequency when voltage fixed at 100 volt

Figure 4 shows the change of inductance and resistance when the voltage and the frequency is varied. It can be seen from these figures that the inductance decreases as the frequency increase. Also at higher frequencies the change of inductance is lower. The resistance in contrast with the inductance increases by increasing the frequency. Figure 5 shows the change of inductance and resistance when the frequency is varied and the voltage level is fixed at 100 volt.

3.3 New Inductor Model

In order to find a relation for the inductance of an inductor at a specific frequency, e.g. 500Hz, the inductance value is first determined when the voltage level is varied. At 500Hz, a model is first assumed according to the following equation, and the parameters are found based on curve fitting method:

$$L(U) = aU^{b} + c \qquad (\mu H) \tag{5}$$

$$a = 889$$

$$b = 0.02582$$

$$c = 22.31$$

Now, a new compensating factor is defined in order to consider the effect of frequency change on the inductance.

To determine this penalty factor, the inductance values at different frequencies at any voltage level is found. The resulted inductance is then divided by the value obtained from Equation (5). At each frequency, an average value is obtained over the values determined at different voltage levels. Equation (6) is used as the penalty factor, and the relation between the average values is determined using a curve fitting method. In general, the inductance can be written by Equation (7) which is the multiplication of Equations (5) and (6). This equation shows simultaneous effects of voltage and frequency change:

$$Penalty\ Term = \alpha e^{\beta h} + \varphi \tag{6}$$

$$h = Harmonic \quad Order$$

$$\alpha = 0.2599$$

$$\beta = -0.06631$$

$$\gamma = 0.8681$$

$$L(U,h) = (889U^{0.02582} + 22.31)\cdot$$
$$\cdot(0.2599e^{-0.06631h} + 0.8681) \qquad (\mu H) \tag{7}$$

In order to investigate the accuracy of the proposed model, the values obtained from Equation (7) are compared against those of experiments. Figure 6 shows the results. As it can be seen, the error in this model is very low except at the fundamental frequency. Since, inductors are normally used at a frequency other than the fundamental frequency; this error is of no concern.

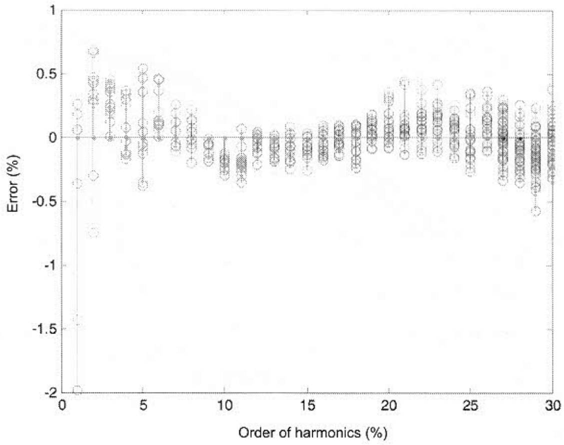

Fig. 6. Error percentage for Equation 7

4. CONCLUSIONS

In this paper, a new model is developed for non-linear frequency-dependent inductors. The model is based on experimental results. A formula is proposed to determine the inductance at different voltage levels and different frequencies. The results show that the proposed model is very accurate at frequencies other than the fundamental frequency. The error of the model is lower compared to other proposed models in the literature.

REFERENCES

1. Heydt G.T.: *ELECTRIC POWER QUALITY: Stars in a Circle Publications*, 2nd edition, 1994.
2. Hatziadoniu C.J.: *Modeling of Linear Component for Harmonic Studies*. IEEE Conf Nov 2003.
3. Kelley W, Edward S.W, Rhode J.P.: *Transformer Derating for Harmonic Currents: A Wide-Band Measurement Approach for Energized Transformer*. IEEE Transaction on Industrial Application, 35, No.6: 1450-1457, 1999.
4. Wang Y.J., Wang J.H.: *Modeling of Frequency-Dependent Impedance of the Third Rail Used in Traction Power Systems*. IEEE Transaction on Power Delivery, 15, No.2: 750-755, 2000.
5. Merrikhi J, Moghani J.S, Fallah E.: *Harmonic Mitigation Potential of Inductive Shunt Harmonic Impedances*. 2nd International Conference on Power Electronics Systems and Applications, 266-268, 2006.

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APPENDIX

Harmonic generator data

Harmonic generator data

Model	Chroma 61505
Nominal Output	
Rated power	4000 w
Effective voltage	300 v
Max current	96 A
Nominal frequency	50 or 60 Hz
Max order of harmonics	40

Power quality analyzer data

Model	Qualy Star 2,00
Software version	5.01
Max voltage	600 v
Max current	600 A
Frequency range	0-3000 Hz