Scientific Papers of
the Department of Electrical Power Engineering of
the Wrocław University of Science and Technology

PRESENT PROBLEMS OF POWER SYSTEM CONTROL

9

Wrocław 2018

Cover design

Piotr PIERZ

Printed in the camera ready form

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ISSN 2084-2201

Print and binding: beta-druk, www.betadruk.pl

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Present Problems of Power System Control. 9, 2018

power system protection, digital estimation,
fast dynamically corrected measurement of power

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DYNAMICALLY CORRECTED FAST ESTIMATORS OF ACTIVE AND REACTIVE POWER AND THEIR PERFORMANCE

Development of several fast estimators of active and reactive power is presented. The estimators are considered from the point of view of the fast measurement of criteria values in power system automation devices. The main idea of the method is dynamical correction of transients realized thanks to recognition of fault instant and taking into consideration of post fault samples only. In the case of reactive power it is possible to get unique transient response and to realize dynamical correction. Increasing filter window starting from fault instant allows to get better accuracy of estimation in presence of noise. Having small noise it is possible to estimate the reactive power starting from one sample and the active power from two samples of increasing data window. The functions of dynamical correction can be calculated analytically or by using a digital simulation

1. INTRODUCTION

Protection and control of power system and its elements requires information concerning many electrical quantities. Changes in the system structure, increasing number and types of sources, many nonlinear elements cause distortion of voltages and currents extremely high in case of faults i.e. during operation of protection systems. Development of digital protection system must follow the changes causing fast and reliable recognition of abnormal states of system elements and the system itself. Digital power system protection is a complex measurement and decision making device. To make decision concerning protected element measured criterion values are used. Both digital processes must be fast, safe and reliable.

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Synthesis of measurement algorithms is made assuming signal containing fundamental component of current and voltage as well as noise. The most popular algorithms are based on estimation of orthogonal components of the signals [4], [5] and Fourier technique. These can be realized using FIR filters which are especially simple in design and application.

The problem of FIR filters is such that they introduce a delay equal the data window. The longer the window the higher the delay and possible solution is either short or variable data window [3], [6], [8]. Short data window gives impaired frequency response producing sometimes high errors of estimation due to noise. Variable data window gives in general high computational burden.

The solution presented in the paper uses variable data window, however, with small computational burden thank to dynamical correction. As usual we must recognize fault instant but later we realize transient which is unique and fault independent. Estimator works very fast in few samples increasing accuracy with time until steady state is reached in time equal to filter window. During transients one coefficient only must be used to realize mentioned dynamical correction.

2. FUNDAMENTALS OF POWER ESTIMATORS

Let us assume that voltages and currents can be presented in the form of complex model 0 0:

$$\underline{x}(k) = x_c(k) + \mathrm{j}x_s(k) = X\exp\big(\mathrm{j}(k\vartheta + \beta)\big) \tag{1}$$

where: $\vartheta = \omega_1 T_s = \dfrac{2\pi}{N_1}$, ω_1 – frequency of fundamental component, T_s – sampling period, N_1 – number of samples in one period of fundamental component: $T_1 = 2\pi/\omega_1$.

Assuming that imaginary part of the signal (1) is observed, voltages and currents are given by the equations

$$\begin{aligned} v(k) &= V\sin(\vartheta k + \beta) \\ i(k) &= I\sin(\vartheta k + \beta - \varphi) \end{aligned} \tag{2}$$

Fundamental electrical quantities especially active and reactive power, voltage and current magnitudes as well as impedance components and frequency can be measured using orthogonal components voltages and currents (real and imaginary part of the signal (0). The components can be calculated processing signals by Fourier digital FIR filters.

The model (1) is useful to analysis and synthesis of mentioned above estimators of fundamental electrical quantities being also protection criterion values.

Estimators of power can be found analyzing following equations:

$$\left(P(k)+\mathrm{j}Q(k)\right)\exp(\mathrm{j}\,\vartheta m)=\frac{1}{2}\left(\underline{v}(k)\underline{i}^{*}(k-m)\right) \tag{3}$$

$$\left(P(k)+\mathrm{j}Q(k)\right)\exp(-\mathrm{j}\,\vartheta m)=\frac{1}{2}\left(\underline{v}(k-m)\underline{i}^{*}(k)\right) \tag{4}$$

where: m – number of delay samples, * – denotes conjugate value.
Voltage and current in (3), (4) have the form of (1) and their components may be easy estimated by application of non-recursive filters:

$$v_c(p)=\sum_{i=0}^{M-1}h_c(i)v(p-i),\qquad v_s(p)=\sum_{i=0}^{M-1}h_s(i)v(p-i),\qquad \text{where } M=\begin{cases}p & \text{for } p<N_1\\ N_1 & \text{for } p\geq N_1\end{cases} \tag{5}$$

for voltage and similarly for current. Impulse functions in (5) can be selected to modify the filters characteristics. In so called Fourier algorithm [6] these functions have the cosine, sine form, e.g.:

$$h_c(i)=\frac{2}{N_1}\cos\left((i+1-N_1)\vartheta\right),\quad h_s(i)=\frac{2}{N_1}\sin\left((i+1-N_1)\vartheta\right) \tag{6}$$

The constant $2/N_1$ is the scaling coefficient to determine the proper values for estimated signals (5) in steady-state (for full-period window width).
Substituting voltages and currents in the form (1), adding sides of the equations and rearranging one can get four following equations describing estimators of active and reactive power:

$$P=\frac{1}{2\cos(m\vartheta)}\left(v_s(k)i_s(k-m)+v_c(k-m)i_c(k)\right) \tag{7}$$

$$Q=\frac{1}{2\cos(m\vartheta)}\left(v_s(k)i_c(k-m)-v_c(k-m)i_s(k)\right) \tag{8}$$

$$P=\frac{1}{2\sin(m\vartheta)}\left(v_s(k)i_c(k-m)-v_s(k-m)i_c(k)\right) \tag{9}$$

$$Q=\frac{1}{2\sin(m\vartheta)}\left(v_f(k-m)i_f(k)-v_f(k)i_f(k-m)\right) \tag{10}$$

The equations above represent algorithms of power measurement assuming identical window, usually full cycle of fundamental component, of orthogonal sine, cosine filters .In the last equation such orthogonal filters are not necessary, instead we can use any identical filters including IIR. Introducing delay allows to get some variants of the algorithms. Sometimes it must differ from zero.

The algorithms, in general give the accurate value when all samples of measured signals appear inside filters window, simple it give accurate steady state value. On the other hand when these measured values are applied to control and protection systems they work during dynamical conditions. Simply because of either control action or because of faults currents and voltages change rapidly. In such cases signals include samples of voltages and currents before and after fault. Measure values are wrong until inside the window appear all samples after the fault. The solution can be shorter data window, however, it is necessary to recognize fault instant. But remain the problems of greater errors due to noises for shorter windows and orthogonality for any data window length.

These disadvantages and algorithms complexity can be avoided using dynamical correction. To realize the idea two things are necessary: to recognize fault instant, as mentioned before and to develop algorithms having measured values independent on initial phase shift of voltages and currents. The first thing allows to put into the window the signal after the fault only assuming zero signals before, i.e., simply giving always the same zero initial conditions.

On the other hand introductory calculations and tests of algorithms (7)–(10) proved that phase independent transients have both algorithms of reactive power. Both of them will be analyzed and tested below.

3. FAST DYNAMICALLY CORRECTED POWER ESTIMATORS

3.1. REACTIVE POWER ESTIMATORS

As it was written the two reactive power estimators are initial phase independent assuming zero initial conditions. Simpler, in general seems to be the forth one from (7)–(10). This requires two identical filters, one to current and the second to voltage and two simple samples delay. The filters can be arbitrary chosen: sin, cos, low pass, no filter as well as any arbitrary IIR filter. On the other hand the second algorithm requires four orthogonal filters so has greater computational burden but signal delay can be avoided so could be one sample faster.

$$Q = \frac{1}{2\sin(m\vartheta)}\left(v_f(k-m)i_f(k) - v_f(k)i_f(k-m)\right) \tag{11}$$

The estimators operate in dynamical conditions when measured criterion values of signals parameters change rapidly. To find dynamical features of the estimator let us assume that at the instant p equal to zero the input signals of the filter F are voltage and current given by (0. Signals $v(p)$ and $i(p)$ can then be described as follows:

$$v(p) = 1(p)v(p);\ i(p) = 1(p)i(p) \tag{12}$$

where: $1(p) = \begin{cases} 1 & \text{for } p \geq 0 \\ 0 & \text{for } p < 0 \end{cases}$.

If the filters had zero initial conditions then their output signals are given by the equations:

$$\begin{aligned} v_f(p) &= V\left(a(p)\sin(\vartheta p + \beta) - b(p)\cos(\vartheta p + \beta)\right) \\ i_f(p) &= I\left(a(p)\sin(\vartheta p + \beta - \varphi) - b(p)\cos(\vartheta p + \beta - \varphi)\right) \end{aligned} \tag{13}$$

where: $a(p) = \sum_{i=0}^{p-1} h(i)\cos(\vartheta i)$, $b(p) = \sum_{i=0}^{p-1} h(i)\sin(\vartheta i)$, $h(i)$ – filter impulse function.

Substituting (13) to (11) one can get:

$$Q = Q(p)T(p) \tag{14}$$

where $Q = \frac{1}{2}VI\sin(\varphi)$.

$$\begin{aligned} T(p) &= a(p)a(p-m) + b(p)b(p-m) \\ &\quad + \left(a(p)b(p-m) - a(p-m)b(p)\right)\cos(m\vartheta)/\sin(m\vartheta) \end{aligned} \tag{15}$$

Function $T(p)$ does not depend on initial phase shift of the signals. Functions $a(p)$, $b(p)$ can be determined analytically for a given filter impulse function. Assuming $h(i) = h_c(i)$ (6) one obtains from (13):

$$\begin{aligned} a(p) &= \frac{\sin(2p\vartheta) + p\sin 2\vartheta}{2N_1\sin\vartheta} \\ b(p) &= \frac{\sin^2(p\vartheta) - p\sin^2\vartheta}{N_1\sin\vartheta} \end{aligned} \tag{16}$$

Substitution (16) into (15) after some simplification yields the following relation:

$$T(p)=\frac{1}{N_1^2}\Bigg(k(j-m)+\frac{1}{\sin\vartheta\sin(m\vartheta)}\Big(k\sin\big((j-m)\vartheta\big)\sin\big((j+1)\vartheta\big) \\ -(j-m)\sin(k\vartheta)\sin\big((k-m+1)\vartheta\big)\Big)\Bigg), \tag{17}$$

where:

$$k=\begin{cases} p & \text{for } p<N, \\ N & \text{otherwise,} \end{cases} \quad j=\begin{cases} p & \text{for } p<(N+m), \\ N & \text{otherwise,} \end{cases} \quad T(p)=0 \quad \text{for } p\le m,$$

Since the function is unique it is possible to make dynamical correction of the estimator (11).

Combining equations (11) and (14) one can get dynamically corrected estimator of reactive power:

$$Q(p)=\frac{1}{2T(p)}\big(v_f(p-m)i_f(p)-v_f(p)i_f(p-m)\big) \tag{18}$$

The function $T(p)$ depends on the filters used only and for any arbitrary filter can be either calculated according to (17) (or similar relation obtained for other applied filter) or found by numerical simulation. The structure of reactive power estimator is shown in Fig. 1 and responses of reactive power estimators for full-cycle non-recursive cosine filters: uncorrected and corrected according to (18) (with applied delay $m = 5$ and $N_1 = 64$ samples/period) are shown in Fig. 2. It can be seen that in fact (17) represents two-parts function with division point for $p = N_1$.

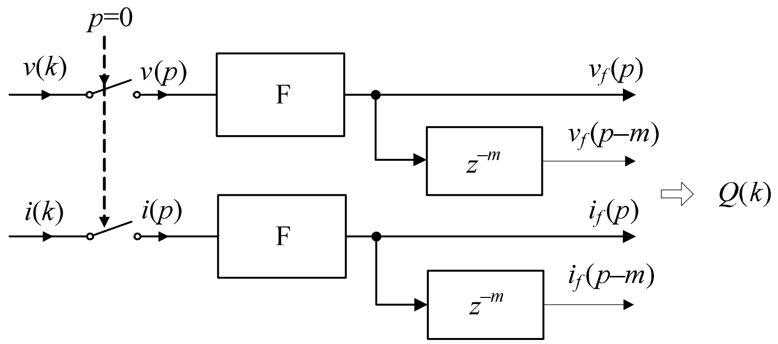

Fig. 1. A structure of reactive power estimator

Similar to described above dynamically corrected reactive power estimator can be obtained using sine, cosine orthogonal components and (8). Assuming for instance delay m equal to zero one can get:

$$Q = \frac{1}{2}\left(v_s(k)i_c(k) - v_c(k)i_s(k)\right) \tag{19}$$

In this case instead of any filter we use a pair of orthogonal filters, however, a delay can be avoided.

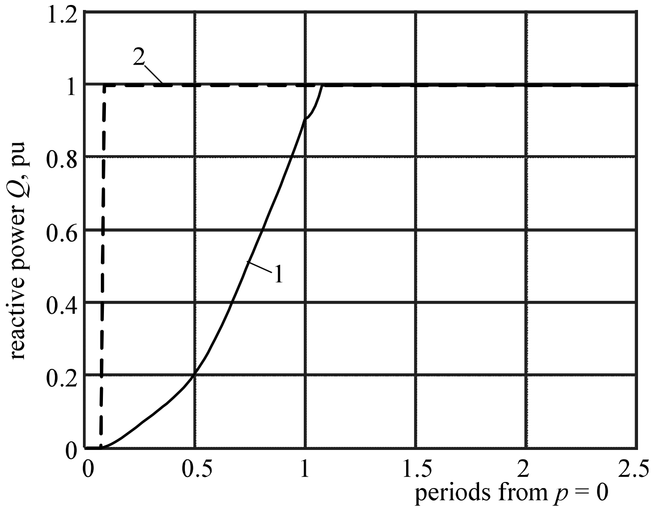

Fig. 2. Time responses of reactive power estimator: 1) typical uncorrected, 2) dynamically corrected; delay $m = 5$, $N_1 = 64$ samples/period

The structure of this reactive power estimator is shown in Fig. 3. It is seen that we must use zero initial condition of a pair of orthogonal filters identical for voltage and current and having the same increasing window. The procedure starts when fault has been recognized – see (12). Now the signals can be written similarly to (13) in the form:

$$\begin{aligned}
v_c(p) &= V\left(a_c(p)\sin(\vartheta p + \beta) - b_c(p)\cos(\vartheta p + \beta)\right) \\
v_s(p) &= V\left(a_s(p)\sin(\vartheta p + \beta) - b_s(p)\cos(\vartheta p + \beta)\right) \\
i_c(p) &= I\left(a_c(p)\sin(\vartheta p + \beta - \varphi) - b_c(p)\cos(\vartheta p + \beta - \varphi)\right) \\
i_s(p) &= I\left(a_s(p)\sin(\vartheta p + \beta - \varphi) - b_s(p)\cos(\vartheta p + \beta - \varphi)\right)
\end{aligned} \tag{20}$$

where: $a_n(p) = \sum_{i=0}^{p-1} h_n(i)\cos(\vartheta i)$, $b_n(p) = \sum_{i=0}^{p-1} h_n(i)\sin(\vartheta i)$, n – means c or s, adequately, and $h_c(i) = A\cos(\vartheta i)$, $h_s(i) = A\sin(\vartheta i)$, $A = 2 / N_1$.

Substituting (20) to (18) one can obtain:

$$Q = Q(p)T(p) \tag{21}$$

where: $Q = \frac{1}{2}VI\sin(\varphi)$, $T(p) = a_c(p)b_s(p) - a_s(p)b_c(p)$.

Function $T(p)$ does depend on initial phase shift of signals just allowing for dynamical correction as before:

$$Q(p) = \frac{Q}{T(p)} = \frac{v_s(p)i_c(p) - v_c(p)i_s(p)}{2T(p)} \tag{22}$$

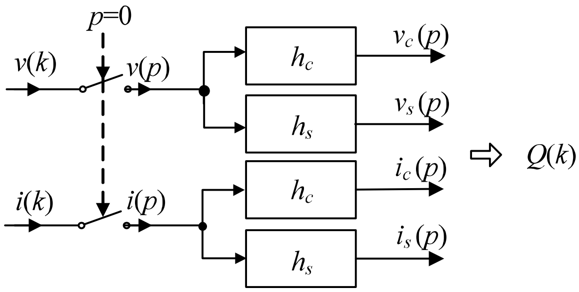

Fig. 3. A structure of second type of reactive power estimator

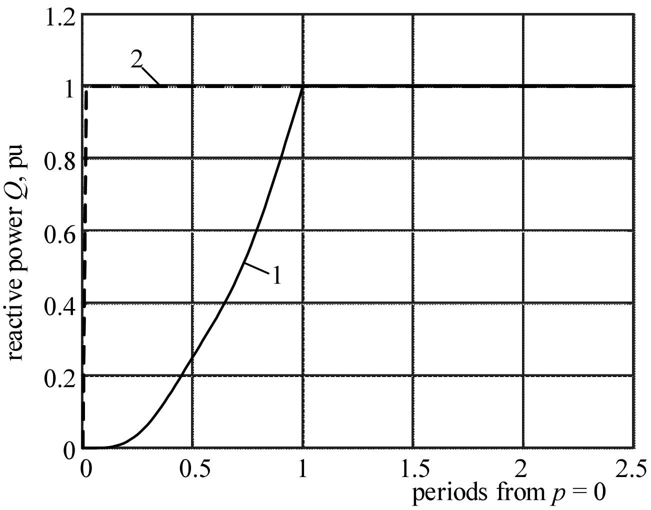

Fig.4. Time response of second type of reactive power estimator:
1) uncorrected, 2) dynamically corrected;
standard full cycle Fourier (cosine/sine) filters in both cases, $N_1 = 64$

The function $T(p)$ depends on the orthogonal filters used and for any filters used can either be calculated or found by numerical simulation. Responses of estimators with dynamical correction and without it applying non-recursive full cycle sine-cosine filters are shown in Fig. 4.

3.2. ACTIVE POWER ESTIMATORS

Such simple results cannot be obtained for remaining estimators of protection criterion values. Time response always depend on initial phase shifts and the response is not unique and it is impossible to find a function similar to $T(p)$.

However, the worked out estimators can be used to estimate the active power very simple. To do that one of the signals of reactive estimator (voltage or current) must be changed. It is enough for instance to introduce the phase shift of $\pi/2$ to current. Such shifting means of course the delay of certain number of samples and estimation time increases, however, that time can be limited to single sample. The phase changing on $\pi/2$ can be obtained if, for instance, the signal is processed according to the equation [2], [7]:

$$i_d(k) = \frac{i(k-n) - i(k)\cos(\vartheta n)}{\sin(\vartheta n)} \tag{23}$$

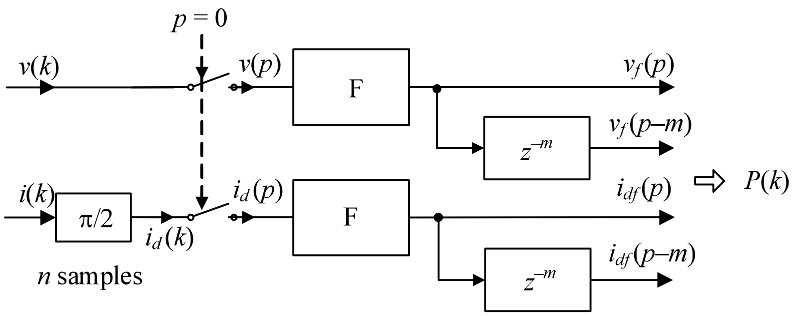

Fig. 5. A structure of active power estimator

A structure of active power estimator is presented in Fig. 5 where Q estimator would be anyone of two described before. Time response of active power estimators using dynamical correction with delay m = 5 samples and n = 10 samples for introducing the phase shifting according to (23) is presented in Fig. 6b). The applied correction function is the same as in (17). Selection of the introduced delay samples: m and n will influence on the frequency response of the final algorithm, however, it is seen that it is possible to get one sample of additional delay only. For comparison with the standard algorithm (7) Fig 6a) presents result of active power estimation without correction during transient period. Shaded area shows changing of the response for different initial phase.

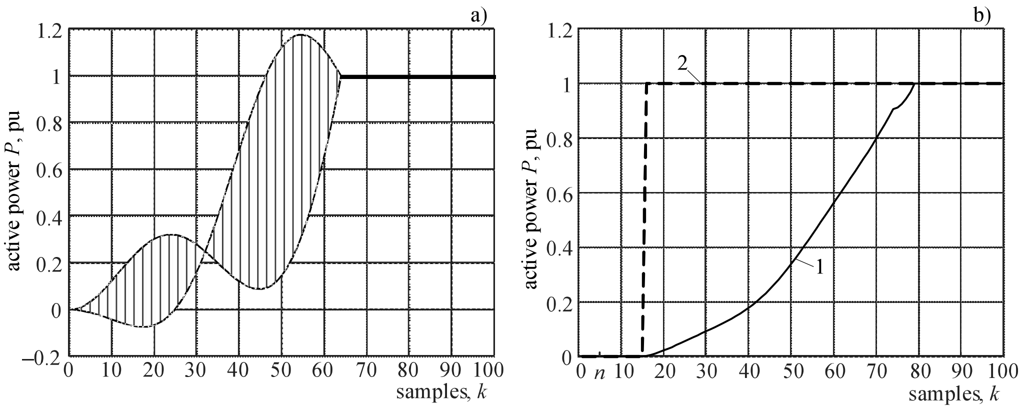

Fig. 6. Time responses of active power estimators: a) applied the first algorithm of active power estimation, b) applied the second algorithm; 1 – typical uncorrected, 2 – dynamically corrected; N_1 = 64 samples/period, n = 5 samples, m = 10 samples

5. CONCLUSIONS

– Several methods of fast estimation of reactive and active power have been presented in the paper.
– Thanks to recognition of fault instant it is possible to take into account post fault samples of signals only.
– Such separation of pre fault and post fault samples allows to obtain unique, phase independent transients of estimation process i.e. before steady state of filters and estimator.
– These unique transients have each, any, reactive power estimator. Some of them apply orthogonal filters the other arbitrary FIR or IIR filters.
– These leads to the idea of dynamical correction using function which can be either calculated or obtained from digital simulation.
– Since the function depends of sampling frequency it is possible to keep in memory pre calculated or pre simulated values.
– Active power estimator can be realized using adequate delay of current signal to get phase shift of minus π/2.
– If noise level is small it is possible to get active or reactive power in two samples.
– In presence of noise it is possible to match responses of the filters to get given accuracy of estimation in given time.
– Development of estimators of the other protection criterion values and their performance tests will be realized soon.

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