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Correction of location of boundaries in cadastre modernization process

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Abstract: Modernization of lands and buildings register, which is run in Poland now, assumes capturing boundaries location data by various methods. They are: direct measurement preceded by determining of boundaries in the presence of parties, photogrammetric method, cartographic method, using existing maps being in analogue form. These methods not always assure accuracy. In the paper, an idea of correcting of location of boundary points which do not fulfil accuracy requirements has been presented. These corrections have been computed by assumption of invariability of parcel area revealed in register, so far. Such way of operation from one side would avoid many misunderstandings and difficulties by applying data collected in cadastre and from the other side will not disturb used procedures, which aim is to determine boundary of parcels with demanded accuracy.

Keywords: modernization of cadastre, correction of boundary points, invariability of parcel area

1. Introduction

In lands and buildings register (real estate cadastre), coordinates of parcel boundaries, being the basis for computations an area of parcels, have important meaning. Typically parcel boundaries are identified on the ground by monuments i.e. any tangible landmark that indicates a boundary. Location of monuments should be determined earlier in the field in the presence of parties, and monuments should be measured and fixed in the ground (Bogarets et al., 2005), (Neumyvakin, 2002). Such determined boundaries should be accepted through administrative decision or through court verdict on the basis of suitable, recorded surveying documentation (Hycner, 2004). A specific determinant of such boundaries are the following attributes: “point location error” denoted by BPP = 1 and attribute “data source about boundaries” denoted by ZRD = 1, 3, 5 i 6 (MIG, 2001). These conditions should be taken into account in the process of modernization of the cadastre (Williamson et al., 2010). Still part of boundaries are those, where corners location were determined during cadastre establishing on the basis of documentation of former Austrian and Prussian cadastre or aerial photographs. These boundaries were also determined and revealed on the basis of measurements of "possession status" shown in the field by parties. Boundaries, determined in the moment of establishing lands and buildings register are usually characterized by attribute BPP value equal 2, 3 or 5 and attribute ZRD = 2, 4 or 7, most often. During lands and buildings register modernization it is possible, sometimes, in the presence of parties, location of boundaries points, which have the biggest accuracy and reliability. Nevertheless it is hard to perform always such procedure. It results mainly from financial reasons. So, big part of recorded data in cadastre does not fulfil accuracy demands suitable for details of the I[st] group of field measurement accuracy. In such case, regulations require to keep in cadastral record, existing area of parcel. Yet such demand does not lead to adjustment between area of parcel determined analytically on the basis of coordinates of its corners (measured area) and area of parcel still existing in cadastre (cadastral area).

It brings many doubts and vagueness and is the reason of various misunderstandings both in surveying, real estate turnover, taxes computation, investments projecting and in land register.

2. Cadastre modernization process

During lands and buildings modernization process, in case when coordinates of boundaries corners are acquired by methods, which do not guarantee suitable accuracy of corners, according with assumptions for points of the I[st] group of field measurement accuracy, shown in MIA (2011), it is possible to correct coordinates of these points – within the range of accuracy of their determination. Correction of coordinates of these points will cause, that area of parcel recorded in cadastre will be consistent with the area of parcel, computed by coordinates of boundary points– determined within modernization process.

Determination of corrections for coordinates should be preceded by analyses, which results would be elimination of blunders. For these activities one can apply the formula determined in Doskocz (2006) and MIA (2011), shown below.

$$dP_{\max} = m_p \sqrt{\frac{1}{8} \sum_{1=1}^{n} d_{i-1,i+1}^2}$$

where:

m_p – denotes medium error of boundary point location,

$d_{i-1,i+1}^2$ – the shortest diagonal being in front of "i" point.

After elimination of gross errors there would be possible correction of points coordinates. Correction would concern points, which had not been determined and fixed in the field earlier, and their coordinates have errors greater than 0,1m (attribute BPP from range 2, 3, 4, 5).

Conformity of parcels area would be here necessary condition for correct determination of coordinates of boundary points. These points, which have been determined earlier in the presence of parties in the field, accepted through suitable administrative decision or court verdict, would become unchangeable. Procedure of coordinates correction should concern also points of cadastral unit boundaries, if these points had not been determined earlier and/or they have not demanded accuracy.

For each cadastral unit one should run independently procedure of modernization lands and buildings register. As a result of such procedure, corrections to coordinates of all boundary points, which do not fulfil accuracy demands, as components of displacements of “centres” these boundary points, along with OX and OY axes, will be determined.

Determination of corrections to boundary points should run according to the rule of minimum value of components displacements for all boundary points being in discussed cadastral unit. Problem of determination corrections to boundary points coordinates is the fundamental process of estimation of model of consistency of measured and cadastral area of parcel – which should be applied during existing cadastre modernization (Pietrzak et al., 2012).

The final stage of lands and buildings register modernization should be computation of ultimate coordinates of boundary points (taking into account mentioned corrections), and computation of area of parcel together with accuracy estimation.

Such activities would lead to consistency between parcel area computed from coordinates (measured area) and parcel area existing in cadastre (cadastral area). One should underline, that attributes featured location of boundary points, as BPP or ZRD would become unchangeable. Change of parcels areas of such corrected boundaries could be done after prior delimitation or restoration boundaries in the presence of parties in the field. Lack of uniformity in the recording area is often complicated by the use of cadastral system to serve the basic tasks of the updated cadastral system (Bogarets et al., 2002; (Neumyvakin, 2002).

3. Functional conditions on boundary points coordinates

In lands and buildings register modernization one should consider conditions on determination of boundary points coordinates – in the form of function relation on areas of cadastral parcels, being the whole or part land real estate. These relations are conditions which must fulfil corrected (modelled) values of boundary points coordinates of considered cadastral parcels.

Let each value Z_i, representing existing, corrected or controlled up coordinates of boundary points, fulfils random model, that is (Czaja, 1997):

$$Z_i = \hat{Z}_i + dz_i \tag{1}$$

where $\hat{Z}_i$ denotes model value of coordinate Z_i, but dz_i is a partial of considered coordinate of boundary point, having features of random component. Randomness of value defined by formula (1) results from fulfilling the following relation: $E(Z_i) = \hat{Z}_i$

If, between values of coordinates of boundary points, functional conditions occur, so these conditions should be exactly fulfilled by model values $\hat{Z}_i$, that is:

$$S(\hat{Z}_1, \hat{Z}_2, \ldots.., \hat{Z}_n) = w \tag{2}$$

where w denotes function value S computed for model values of boundary points coordinates.

Generally conditions (2) are non linear, so by the first derivatives of Taylor series, they are reduced to linear form , that is:

$$S(\hat{Z}_1, \hat{Z}_2, \ldots.., \hat{Z}_n) + \sum_{i=1}^{n} \frac{\partial S}{\partial Z_i} dz_i = w \tag{3}$$

The best approximation of model values of coordinates of boundary points are values of these coordinates determined on the basis of source, corrected or controlled measurements, because after taking account relation $\hat{Z}_i = Z_i - dz_i$ results that differentials to model values are partial coordinates of boundary points. Thus, condition (3) one can write down in the form of:

$$\sum_{i=1}^{n} \frac{\partial S}{\partial Z_i} dz_i = w - S(\hat{Z}_1, \hat{Z}_2, \ldots.., \hat{Z}_n) \tag{4}$$

Free term in condition (4) will be denoted by t and will express difference between parcel area – recorded in cadastre and computed by coordinates, that is

$$w - S(\hat{Z}_1, \hat{Z}_2, \ldots.., \hat{Z}_n) = t \tag{5}$$

This difference should be balanced by partials dz_i of considered boundary points coordinates. Conditional equation in form (4) will take the following, ultimate form:

$$\sum_{i=1}^{n} \frac{\partial S}{\partial Z_i} dz_i = t \tag{6}$$

Denoting partial derivatives by b_i, each conditional for boundary points coordinates take the following symbolic form:

$$\sum_{i=1}^{n} b_i dz_i = t \tag{7}$$

Using Gauss formula for computation area of closed figure, of form:

$$2S = \sum_{i=1}^{n} (x_{i+1} - x_{i-1}) y_i \tag{8}$$

and applying functional notation (6), linear form of condition (8) is expressed by the following dependence:

$$\sum_{i=1}^{n} (x_{i+1} - x_{i-1}) dy_i + \sum_{i=1}^{n} (y_{i-1} - y_{i+1}) dx_i = 2S - \sum_{i=1}^{n} (x_{i+1} - x_{i-1}) y_i \tag{9}$$

Condition (9) contains determined coordinates of parcel boundary points (x_i, y_i) and corrections for coordinates of boundary points (dx_i, dy_i), which are of random character. In order to determine corrections for coordinates of boundary points, (dx_i, dy_i), one must apply the least square method, in the form of conditioned model.

4. Gauss-Markov conditioned model

Let correlated by matrix **G** values Z_1, Z_2, …, Z_n, representing determined by measurements coordinates of boundary points, fulfil Gauss-Markov model in the simplest form (Czaja, 1997):

$$\mathbf{Z} = \mathbf{I}\hat{\mathbf{Z}} + \mathbf{dZ} \tag{10}$$

which denotes, that each value of coordinate $\mathbf{Z_i}$ of boundary point corresponds model value of this coordinate $\hat{\mathbf{Z}}_\mathbf{i}$ with random component $\mathbf{dz_i}$, which is of correction kind. From assumption (10) results, that model values are differences of coordinates $\mathbf{Z_i}$ and correction.

Assume that functional conditions are put on coordinates of boundary points in the form of (9), which will univocally be fulfilled by model values $\hat{\mathbf{Z}}_\mathbf{i}$. When functional conditions will be in linear form (7), then their form by matrix notation will be as follows:

$$\mathbf{B} \text{ x } \mathbf{dZ} = \mathbf{t} \tag{11}$$

where:

B – denotes matrix of partial derivatives of function (8), what according to dependency (9), are differences of suitable coordinates of boundary points in considered cadastral unit,

dZ – vector of estimated unknowns (differentials to points coordinates), representing corrections of suitable coordinates of boundary points in considered cadastral unit,

t – one column matrix of free terms, which contains differences of double area of parcels, recorded in cadastre and double area of parcels computed by coordinates of boundary points.

Set of conditional equations (11) recorded by apparent matrix shape, for area of four parcels determined by (n) boundary points, takes the following form:

$$\begin{Bmatrix} y_{1-n}-y_{1+n} & 0 & y_{1-n}-y_{1+n} & y_{1-n}-y_{1+n} \\ x_{1+n}-x_{1-n} & 0 & x_{1+n}-x_{1-n} & x_{1+n}-x_{1-n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ y_{i-1}-y_{i+1} & y_{i-1}-y_{i+1} & 0 & y_{i-1}-y_{i+1} \\ x_{i+1}-x_{i-1} & x_{i+1}-x_{i-1} & 0 & x_{i+1}-x_{i-1} \\ y_{n-1}-y_{n+1} & y_{n-1}-y_{n+1} & y_{n-1}-y_{n+1} & 0 \\ x_{n+1}-x_{n-1} & x_{n+1}-x_{n-1} & x_{n+1}-x_{n-1} & 0 \end{Bmatrix}^T \times \begin{Bmatrix} dx_1 \\ dy_1 \\ \ldots \\ dx_i \\ dy_i \\ dx_n \\ dy_n \end{Bmatrix} = \begin{Bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{Bmatrix} \tag{11a}$$

Estimation of differentials vector of boundary points coordinates will be made taking into account Lagrange function of the following equation (Mikhail and Ackermann, 1976):

$$\mathbf{\Psi = (Z - I\hat{Z})^T P (Z - I\hat{Z}) + 2K^T(B \times dZ - t) = dZ^T P dZ + 2K^T(B \times dZ - t) = min} \tag{12}$$

where matrix of weights **P** s defined as an inverse of matrix **G**.

Matrix **G** for considered problem contains only elements on major diagonal, which should represent variances (squares of mean errors) of individual coordinates of boundary points in considered cadastral unit. For coordinates of points determined and measured in the presence of parties, one can take an attribute BPP equals “1”, which corresponds a point location error does not exceed 10 cm. It gives variance for each point coordinate at level of 50 cm^2 and standard deviation equals 7.1 cm. For points, which coordinates were determined on the basis of digitizing process of raster cadastral map at scale of 1:2000, one can take variances of each coordinate at level of 1800 cm^2 (standard deviation equals 42.4 cm), which corresponds attribute BPP = 3 (error of point location does not exceed 60 cm) (MRRiB, 2001).

Necessary conditions for minimum of this function are as follows:

$$\frac{\partial \mathbf{\Psi}}{\partial \mathbf{Z}} = \mathbf{2P \times dZ + 2B^T \times K} = 0 \qquad \Rightarrow \qquad \mathbf{P \times dZ + B^T \times K} = 0 \tag{13}$$

$$\frac{\partial \mathbf{\Psi}}{\partial \mathbf{Z}} = 2\mathbf{B \times dZ} - 2\mathbf{t} = 0 \qquad \Rightarrow \qquad \mathbf{B \times dZ = t} \tag{14}$$

Set of equations (13) and (14) one can record by block form:

$$\begin{bmatrix} \mathbf{P} & \mathbf{B^T} \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{dZ} \\ \mathbf{K} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{t} \end{bmatrix} \tag{15}$$

Solution of below set of equations needs determination of inverse of block matrix, which will be recorded in the following form:

$$\begin{bmatrix} \mathbf{P} & \mathbf{B}^{\mathrm{T}} \\ \mathbf{B} & \mathbf{0} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \\ \mathbf{C}_3 & -\mathbf{C}_4 \end{bmatrix} \tag{16}$$

After multiplication, which leads to inverse of block matrix, that is:

$$\begin{bmatrix} \mathbf{P} & \mathbf{B}^{\mathrm{T}} \\ \mathbf{B} & \mathbf{0} \end{bmatrix}^{-1} \times \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \\ \mathbf{C}_3 & -\mathbf{C}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{17}$$

we gets set of 4 matrix equations which leads to following connections:

$$\begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \\ \mathbf{C}_3 & -\mathbf{C}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{-1} - \mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}}(\mathbf{B}\mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}})^{-1}\mathbf{B}\mathbf{P}^{-1} & \mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}}(\mathbf{B}\mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}})^{-1} \\ (\mathbf{B}\mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}})^{-1}\mathbf{B}\mathbf{P}^{-1} & -(\mathbf{B}\mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}})^{-1} \end{bmatrix} \tag{18}$$

After taking account (18), sought estimators for unknown vectors one can calculate according to following equation:

$$\begin{bmatrix} \mathbf{dZ} \\ \mathbf{K} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \\ \mathbf{C}_3 & -\mathbf{C}_4 \end{bmatrix} \times \begin{bmatrix} \mathbf{0} \\ \mathbf{t} \end{bmatrix} \tag{20}$$

Estimator of vector **dZ**, that is differentials of coordinates of boundary points, is expressed by following formula:

$$\mathbf{dZ} = \mathbf{C}_2\mathbf{t} = \mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}}(\mathbf{B}\mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}})^{-1}\mathbf{t} \tag{21}$$

Determined above estimator has such property, that sum of squares of differentials of coordinates of boundary points, in considered cadastral unit, takes the minimum value.

Covariance matrix, consisting of differentials of coordinates of boundary points, in considered cadastral unit, one can determine on the basis of dependency:

$$\mathbf{Cov(dZ)} = \sigma^2(\mathbf{G} - \mathbf{C}_1) = \sigma^2\mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}}(\mathbf{B}\mathbf{P}^{-1}\mathbf{B}^{\mathrm{T}})^{-1}\mathbf{B}\mathbf{P}^{-1} \tag{22}$$

Variance for estimated model $\hat{\sigma}^2$ should not be determined on the basis of value of function Ψ, because corrections to coordinates of boundary points are done with accuracy higher than their source coordinates. Thus, for this model of estimation one should take estimation at level 1, and this denotes that inaccuracy of determination of coordinates of boundary points is on level of their variances taken in matrix **G**.

On the basis of matrix (22) one can determine variances for areas of parcels in determined cadastral unit. Denoting linear formula (9) of vector of areas of determined parcels by $\mathbf{F_S}$, variance of area of parcel contained in considered cadastral unit, is expressed by formula:

$$\sigma^2(S) = \mathbf{F_S} \times \mathbf{Cov(dZ)} \times \mathbf{F_S^T} \tag{23}$$

Value of standard deviation determined on the basis of above dependency, is the foundation for computation of inaccuracy coefficient of parcel boundary regulation process in considered cadastral unit, that is:

$$\lambda_S = \frac{\sigma(S)}{S} \tag{24}$$

As permissible values of inaccuracy coefficient of parcel boundary regulation in considered cadastral unit one can suggest range, expressed by following inequality:

$$\lambda_S \leq 0{,}01 \tag{25}$$

If condition (25) are not fulfilled for considered parcel in selected cadastral unit, then boundary points of this parcel one should determine in the presence of parties. On the basis of computed coordinates of boundary points got from control measurements, the whole process of estimation of corrections, one should repeat.

5. Example of estimation process of correction of coordinates of boundary points

To illustrate process of estimation of correction of coordinates of boundary points, part of cadastral unit, consisting of 6 cadastral parcels, will be considered. Coordinates of four boundary points will be corrected, but the rest eight points represent outer boundary of the unit. It shows Figure 1.

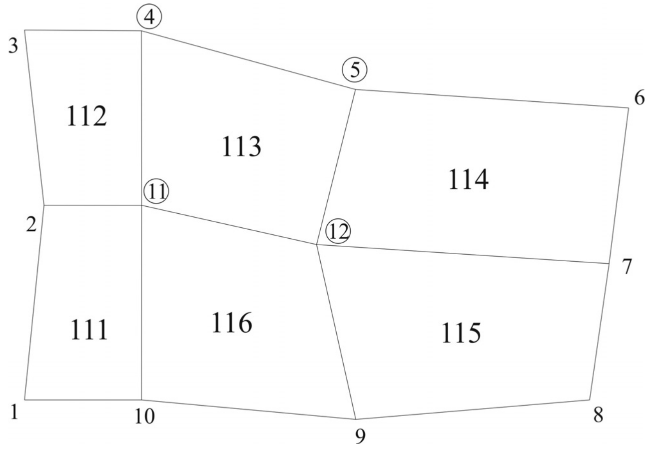

Fig. 1. Analysed case. In the circle are the points where corrections are computed.

Determined coordinates of considered cadastral parcels and their standard deviations (mean errors) resulting from measurement technology, have been inserted in Table 1.

Table 1. Coordinates of boundary points of considered cadastral parcels

Point No	X [m]	Y [m]	BPP	σ(X) [m]	σ(Y) [m]
1	200.00	100.00	1 = 0.1m	0.071	0.071
2	300.00	110.00	1 = 0.1m	0.071	0.071
3	390.00	100.00	1 = 0.1m	0.071	0.071
4	390.00	160.00	2 = 0.3m	0.212	0.212
5	360.00	270.00	2 = 0.3m	0.212	0.212
6	350.00	410.00	1 = 0,1m	0.071	0.071
7	270.00	400.00	1 = 0.1m	0.071	0.071
8	200.00	390.00	1 = 0.1m	0.071	0.071
9	190.00	270.00	1 = 0.1m	0.071	0.071
10	200.00	160.00	1 = 0.1m	0.071	0.071
11	300.00	160.00	3 = 0.6m	0.424	0.424
12	280.00	250.00	3 = 0.6m	0.424	0.424

Let points denoted by numbers 1, 2, 3, 6, 7, 8, 9 and 10 represent boundary points of selected cadastral unit; thus points denoted as 4, 5, 11 and 12 should be corrected. For estimation of corrections of coordinates of points No 4, 5, 11 and 12 conditional equations for six parcels denoted by numbers 111, 112, 113, 114, 115 and 116 will be expressed. On the basis of coordinates of boundary points inserted in Table 1, areas of considered parcels have been computed, and their values have been presented in Table 2. In the last column of the Table 2, areas coming from existing cadastral document, have been shown.

Table 2. List of areas of cadastral parcels

Parcel number	Area of parcel computed by coordinates after modernization process [m^2]	Area of parcel taken from existing cadastral document [m^2]
111	5 500	5 503
112	4 950	4 940
113	8 750	8 701
114	11 750	11 730
115	10 800	10 792
116	9 350	9 330

Matrix **B** for set of six conditional equations of form (11), containing 8 corrections to coordinates of four points, takes the following elements:

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & -50 & -100 & 0 & 0 \\ -60 & -90 & 0 & 0 & 50 & -90 & 0 & 0 \\ -110 & 60 & -90 & -110 & 90 & 110 & 110 & -60 \\ 0 & 0 & -160 & 70 & 0 & 0 & 130 & 90 \\ 0 & 0 & 0 & 0 & 0 & 0 & -130 & 80 \\ 0 & 0 & 0 & 0 & -90 & 80 & -110 & -110 \end{bmatrix}$$

Inverse of weight matrix $\mathbf{P}^{-1} = \mathbf{G}$ is variances for individual coordinates of considered boundary points, thus after taking into account standard deviations contained in Table 1, it makes the following form.

$$\mathbf{G} = \begin{bmatrix} 0.045 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.045 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.045 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.045 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.18 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.18 \end{bmatrix}$$

Elements of matrix **t** are differences of double areas of individual parcels, listed in Table 2, that is

$$\mathbf{t} = \begin{bmatrix} 6 \\ -20 \\ -98 \\ -40 \\ -16 \\ -40 \end{bmatrix}$$

Solution of set of conditional equations, according to formula (21), together with minimization of function (12), leads to the following results matrices:

$$
\mathbf{C_2} = \begin{bmatrix}
-0.002 & -0.002 & -0.003 & -8.327\cdot 10^{-4} & -0.002 & -0.001 \\
0.005 & -0.005 & 0.002 & -0.002 & 0.002 & -0.002 \\
-0.002 & -0.002 & -0.002 & -0.004 & -0.002 & -0.003 \\
-0.005 & 0.002 & -0.003 & 0.002 & -0.002 & 0.001 \\
-0.006 & 0.004 & 7.439\cdot 10^{-4} & -0.002 & 3.31\cdot 10^{-4} & -0.003 \\
-0.007 & -0.002 & -3.719\cdot 10^{-4} & 0.001 & -1.655\cdot 10^{-4} & 0.002 \\
1.058\cdot 10^{-5} & -0.002 & -3.349\cdot 10^{-4} & 0.001 & -0.005 & -0.002 \\
1.719\cdot 10^{-5} & -0.003 & -5.442\cdot 10^{-4} & 0.002 & 0.005 & -0.003
\end{bmatrix}
$$

$$
\mathbf{dZ} = \begin{bmatrix}
0.401 \\ 0.030 \\ 0.533 \\ 0.146 \\ 0.014 \\ -0.067 \\ 0.192 \\ 0.112
\end{bmatrix}
$$

Resulted values of corrections should be inserted to boundary points with opposite signs, and should represent „centre" of boundary point. Model coordinates of boundary points are determined as differences of coordinates of these points, inserted in Table 1, and computed corrections, inserted in Table 3.

Table 3. Corrected coordinates of boundary points for considered cadastral parcels

Number	*x* [m]	*y* [m]	*-dx* [m]	*-dy* [m]	$\hat{x}$ [m]	$\hat{y}$ [m]
1	200.00	100.00	0	0	200.00	100.00
2	300.00	110.00	0	0	300.00	110.00
3	390.00	100,00	0	0	390.00	100.00
4	**390.00**	**160.00**	0.401	0.03	**389.599**	**159.97**
5	**360.00**	**270.00**	0.533	0.146	**359.467**	**269.854**
6	350.00	410.00	0	0	350.00	410.00
7	270.00	400.00	0	0	270.00	400.00
8	200.00	390.00	0	0	200.00	390.00

Table 3.

Number	x [m]	y [m]	$-dx$ [m]	$-dy$ [m]	$\hat{x}$ [m]	$\hat{y}$ [m]
9	190.00	270.00	0	0	190.00	270.00
10	200.00	160.00	0	0	200.00	160.00
11	**300.00**	**160.00**	0.014	-0.067	**299.986**	**160.067**
12	**280.00**	**250.00**	0.192	0.112	**279.808**	**249.888**

Area of considered parcels, computed on the basis of corrected coordinates contained in Table 3, correspond to the area of these parcels contained in existing cadastral documents.

Variance of estimated model $\hat{\sigma}^2$ should not be determined on the basis of values of function Ψ, because corrections to coordinates of boundary points are set out in the field; thus for this model of estimation one should take variance at level 1, and this denotes that inaccuracy of determination of coordinates of boundary points is at level of their variances taken in matrix **G**.

After applying formula (23), matrix of covariance for areas of considered parcels, of the form given below, have been resulted:

$$\mathbf{cov(S)} = \mathbf{BGB}^{\mathrm{T}} =$$

$$= \begin{bmatrix} 2.247\cdot10^3 & 1.169\cdot10^3 & -2.787\cdot10^3 & 0 & 0 & -629.216 \\ 1.169\cdot10^3 & 2.431\cdot10^3 & -916.858 & 0 & 0 & -2.103\cdot10^3 \\ -2.787\cdot10^3 & -916.858 & 8.067\cdot10^3 & 1.901\cdot10^3 & -3.434\cdot10^3 & -862.925 \\ 0 & 0 & 1.901\cdot10^3 & 5.865\cdot10^3 & -1.744\cdot10^3 & -4.351\cdot10^3 \\ 0 & 0 & -3.434\cdot10^3 & -1.744\cdot10^3 & 4.189\cdot10^3 & 988.768 \\ -629.216 & -2.103\cdot10^3 & -862.925 & -4.351\cdot10^3 & 988.768 & 6.957\cdot10^3 \end{bmatrix}$$

On the basis of diagonal elements of above matrix, one can determine standard deviations of area of cadastral parcels after correcting coordinates of boundary points of these parcels. Results of these computations contain Table 4.

On the basis of above list one can see, that the biggest inaccuracy characterised the parcels which got the biggest corrections, e.g. parcels No 113, 114 and 116.

For all of considered parcels an inequality (25) is fulfilled, and it means that areas of parcels revealed in descriptive cadastral documents after modernization process have not been changed and that they are consistent with area computed by coordinates.

Table 4. List of parcel areas and their standard deviations after correcting coordinates of boundary points

Parcel number	Area of parcel computed by coordinates, corrected after modernization process $[m^2]$	Area of parcel taken from existing cadastral document $[m^2]$	Standard deviation of parcel area $\sigma(S_i)$ $[m^2]$
111	5 503	5 503	47.4
112	4 910	4 940	49.3
113	8 701	8 701	89.8
114	11 730	11 730	76.6
115	10 792	10 792	64.7
116	9 330	9 330	83.4

6. Conclusions

Modernization of existing lands and buildings register, which has been run for over 10 years, assumes that register files should be stored and managed by pertinent DBMS. Unfortunately, legal regulations on revealing an area of parcels when a parcel area is computed on the basis of coordinates – in case when these coordinates are not determined with accuracy demanded for the Ist group of details – is not conformant with parcel area shown in cadastral documents.

It causes a number of problems, both in parcel owners as well as surveyors during register updating.

Solution presented in the paper makes, that within accepted range of accuracy, location of boundary points depends of an area of parcel stored in a cadastral record. Inserted corrections are within the range of accuracy of determination of boundary point, assuming that limiting error of such coordinate is determined at level of confidence P = 0,99 (3*σ – assuming σ = BPP), and area by coordinates and cadastral area are equal.

Conclusions are as follows.

Correction to boundary points of accuracy less than required by regulations causes:

- conformity of parcel area computed by coordinates with parcel area recorded in cadastre,
- reducing quantity of derivative errors in modernized cadastre resulting from discrepancy of parcels area,
- improving reliability of cadastral data – within real estate description.

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Korekta granic w procesie modernizacji ewidencji gruntów i budynków

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Streszczenie

Modernizacja ewidencji gruntów i budynków, która obecnie jest przeprowadzana w Polsce, zakłada pozyskiwanie współrzędnych punktów granicznych różnymi metodami. Są to: metoda pomiaru bezposredniego, poprzedzona ustaleniem granic w terenie w obecności stron, metoda fotogrametryczna, metoda kartometryczna, wykorzystująca istniejące mapy analogowe. Metody te nie zawsze zapewniają dokładność właściwą dla szczegółów I grupy dokładnościowej. Zgodnie z obowiązującymi przepisami, gdy dokładność określenia położenia punktu załamania granicy nie spełnia standardów technicznych, w ewidencji gruntów i budynków zachowuje się dotychczasową powierzchnię działki. Utrzymana, zatem zostaje rozbieżność pomiędzy powierzchniami geodezyjną a ewidencyjną działki.

W artykule przedstawiono koncepcje wprowadzenia poprawek do współrzędnych punktów granicznych, które nie spełniają wymogów dokładnościowych. Poprawki te obliczone zostały przy założeniu warunku niezmienności pola powierzchni działki ewidencyjnej dotychczas ujawnionego w rejestrze gruntów. Taki sposób postępowania pozwoliłby uniknąć wielu nieporozumień i trudności przy wykorzystywaniu danych zgromadzonych w ewidencji gruntów i budynków, a z drugiej strony nie zaburzałby dotychczas stosowanych procedur, których celem jest ustalenie granic z dokładnością właściwą dla tego typu szczegółów terenowych.