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AN APPLICATION OF INTUITIONISTIC FUZZY ANALYTIC HIERARCHY PROCESS IN SHIP SYSTEM RISK ESTIMATION

Hoang Nguyen

Gdynia Maritime University, Department of Engineering Sciences
Morska Street 81-87, 81-225 Gdynia, Poland
tel.:+48 586901306
e-mail: hoang@am.gdynia.pl

Abstract

In this paper, we extend the analytic hierarchy process (AHP) method and the Atanassov's intuitionistic fuzzy set (IFS) into the intuitionistic fuzzy analytic hierarchy process (IFAHP) with application in ship system risk estimation. In the safety engineering, risk estimation is in practice confronted with difficulties connected with shortage of data. In such cases, we have to rely on subjective estimations made by persons with practical knowledge in the field of interest, i.e. experts. However, in some realistic situations, the decision makers might be reluctant or unable to assign the crisp evaluation values to the comparison judgments due to his/her limited knowledge. In other words, there is a certain degree of hesitancy in human cognition and his judgment. Taking advantages of IFSs in dealing with ambiguity and uncertainty into account, the IFAHP can be used to handle with the subjective preferences of experts, who may have insufficient knowledge of the problem domain or uncertainty in assigning the evaluation values to the objects considered.

This paper also develops a new knowledge-based ranking method to derive the priority vector of the hierarchy. An illustrative example of the propulsion risk estimation of container carriers operating on the North Atlantic line is given to show the applicability and effectiveness of the proposed method.

***Keywords**: intuitionistic fuzzy sets, risk estimation, expert judgment, ship propulsion system, analytic hierarchy process, sea transport*

1. Introduction

The AHP method is widely used in multi-criteria decision-making process. The decision making process in the AHP method consists in decomposing a complex problem into a multilevel hierarchical structure of objectives, criteria, sub-criteria and so on, under which the alternatives are expressed by pairwise comparisons according to the predefined scale of relative magnitudes, e.g. natural, balanced or geometrical. Then an overall ratio scale of priorities is synthesized to rank the alternatives. In the conventional AHP model, the comparative judgments made by the decision maker are represented by crisp numbers within the 1-9 scale. However, in some realistic situations, the decision makers might be reluctant or unable to assign the crisp evaluation values to the comparison judgments due to his/her limited knowledge. Hence, the conventional AHP seems to be inadequate to explicitly capture the important assessments for deriving the priorities in these situations. To overcome this issue, Laarhoven and Pedrycz [7] introduced the fuzzy AHP (FAHP), where each pairwise comparison judgment is represented as a triangle fuzzy number with a membership function. The membership function denotes the degree to which elements considered belong to the preference set.

Since the membership function of a fuzzy set is only single-valued function, it cannot be used to express the support and objection evidences simultaneously in many practical situations. In evaluating some candidate alternatives, the decision makers may not be able to express their preferences accurately due to the fact that they may not grasp sufficient knowledge of the alternatives, or they are unable or unwilling to discriminate explicitly the degree to which the alternative is better than others. In other words, there is a certain degree of hesitation. In order to describe such situations and to model human's perception and cognition more comprehensively, Antanassov [1] extended Zadeh's fuzzy set to the intuitionistic fuzzy set (IFS), which is characterized by membership degree, non-membership degree, and hesitancy degree, which sum up to one. Afterwards, the IFS has attracted increasingly scholars' attention and has been applied to many different fields, such as decision-making [5, 18], fuzzy cognitive maps [10], medical diagnosis [4], fault diagnosis and pattern recognition [15]. Xu and Liao [20] extended the classical AHP and the FAHP to the intuitionistic fuzzy circumstances and developed the IFAHP procedure for handling comprehensive multi-criteria decision-making problems. Although there exist several measures for IFSs, many unreasonable cases made by such measures can be found as shown in [8, 12, 14]. In this paper, we present a new knowledge-based measure for IFSs, which is intuitive and reliable to rank the priorities of alternatives in decision-making procedure. Based on the amount of knowledge conveyed, the proposed measure overcomes the drawbacks of other measures and can provide reliable results in theoretical and practical computation. The performance evaluation of the proposed measure is shown in an application to the ship system risk estimation.

2. Basic concepts

2.1. Intuitionistic fuzzy sets

In 1983, Atanassov generalized the concept of fuzzy sets given by Zadeh [21] by using membership function and non-membership function for any elements of the universe of discourse. An Atanassov's Intuitionistic Fuzzy Set (IFS) is described by:

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}, \tag{1}$$

where $\mu_A(x)$ denotes a degree of membership and $\nu_A(x)$ denotes a degree of non-membership of x to A, $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ such that:

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \forall x \in X. \tag{2}$$

To measure hesitancy of membership of an element to intuitionistic fuzzy set, Atanassov introduced a third function given by:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), \tag{3}$$

which is also called the intuitionistic fuzzy index or the hesitation margin of x to A. It is obvious that $0 \le \pi_A(x) \le 1, \forall x \in X$.

The concept of a complement of an IFS A , denoted by Ac is defined as [1]:

$$A^c = \{(x, \nu_A(x), \mu_A(x), \pi_A(x)) | x \in X\}. \tag{4}$$

2.2. Intuitionistic fuzzy preferences

Expressing preferences of alternatives in pairwise comparison is arguably convenient from viewpoint of knowledge acquisition, especially since people often find it easier to compare two alternatives than to assess single alternatives in terms of numerical values. Saaty [11] developed the 1–9 scale to describe the preferences between alternatives as extremely, very strong, strong, moderately not preferred, equally, moderately, strongly, very strongly and extremely preferred.

To each pair of alternatives (x_i, x_j) a pairwise weight r_{ij} is assigned from the set S = {1/9, 1/7, ..., 1/3, 1, 3, ..., 7, 9}, which expresses his/her individual preference of x_i over x_j as shown in Tab. 1.

The pairwise weights in such defined scale satisfy the reciprocal condition, i.e. the intensity of preference of x_i over x_j is inversely related to the intensity of preference of x_j over x_i,

$$r_{ij} > 0, r_{ij} = \frac{1}{r_{ji}}, \forall i, j = 1,2, \dots, n. \quad (5)$$

Matrix $R = (r_{ij})_{n \times n}$ of the pairwise comparison judgments represented by corresponding weights on the set of alternatives $X = \{x_1, x_2, \dots, x_n\}$, where r_{ij} $(i, j = 1,2, \dots, n)$ satisfy the reciprocal condition, is called a multiplicative preference relation.

Definition 1 [17]: Matrix of the pairwise comparison judgments $R = (r_{ij})_{n \times n}$ is called intuitionistic fuzzy relation if they are represented by IFVs, i.e. $r_{ij} = \langle \mu_{ij}, \nu_{ij} \rangle$, where μ_{ij} denotes the degree to which the alternative x_i is preferred to the alternative x_j, ν_{ij} denotes the degree to which the alternative x_i is not preferred to the alternative x_j, satisfying condition $\mu_{ij}, \nu_{ij} \in [0,1]$, $\mu_{ij} + \nu_{ij} \leq 1$, $\mu_{ij} + \nu_{ji}$, $\mu_{ij}, \nu_{ij} = 0.5$, $\mu_{ij} + \nu_{ij} + \pi_{ij} = 1$, $(i, j = 1,2, \dots, n)$, where π_{ij} denotes a hesitancy degree.

Tab. 1. Preferences of pairwise comparison and their weights in corresponding scales

Preferences	1-9 scale	0.1-0.9 scale
Extremely not preferred	1/9	0.1
Very strongly not preferred	1/7	0.2
Strongly not preferred	1/5	0.3
Moderately not preferred	1/3	0.4
Equally preferred	1	0.5
Moderately preferred	3	0.6
Strongly preferred	5	0.7
Very strongly preferred	7	0.8
Extremely preferred	9	0.9
Intermediate values	other values between 1/9 and 9	other values between 0.1 and 0.9

For any two IFVs $r_{ij} = \langle \mu_{ij}, \nu_{ij} \rangle$ and $r_{kl} = \langle \mu_{kl}, \nu_{kl} \rangle$ in the intuitionistic fuzzy relation R, Xu [17] introduced the following relations:

$$r_{ij} \oplus r_{kl} = (\mu_{ij} + \mu_{kl} - \mu_{ij}\mu_{kl}, \nu_{ij}\nu_{kl}), \quad (6)$$

$$r_{ij} \otimes r_{kl} = (\mu_{ij}\mu_{kl}, \nu_{ij} + \nu_{kl} - \nu_{ij}\nu_{kl}), \quad (7)$$

$$\lambda r_{ij} = (1 - (1 - \mu_{ij})^{\lambda}, {\nu_{ij}}^{\lambda}), (\lambda > 0), \quad (8)$$

$$r_{ij}^{\lambda} = ({\mu_{ij}}^{\lambda}, 1 - (1 - \upsilon_{ij})^{\lambda}), (\lambda > 0). \quad (9)$$

2.3. Existing measures for IFSs

For convenience, throughout this paper, let $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ be an intuitionistic fuzzy value (IFV), where $\mu_\alpha, \nu_\alpha \in [0,1]$, $\mu_\alpha + \nu_\alpha \leq 1$, $\forall \alpha \in X$. In order to rank the IFVs, there have been proposed several measures for IFSs in literature. In [3] Chen and Tan proposed the score function $S(\alpha) = \mu_\alpha - \nu_\alpha$ as deviation between μ_α and ν_α. Later on, Hong and Choi [6] additionally introduced an accuracy function $H(\alpha) = \mu_\alpha + \nu_\alpha$ to evaluate the degree of accuracy of the score functions. Based on this, Xu [16] developed a procedure for ranking two IFVs α and β in case of $S(\alpha) = S(\beta)$. Szmidt and Kacprzyk [13] proposed other function taking into account hesitancy degree of an IFV as follows:

$$K(\alpha) = 1 - 0.5(E(\alpha) + \pi_\alpha), \tag{10}$$

where entropy measure $E(\alpha)$ is defined as:

$$E(\alpha) = \frac{\min(\mu_\alpha, \nu_\alpha) + \pi_\alpha}{\max(\mu_\alpha, \nu_\alpha) + \pi_\alpha}. \tag{11}$$

However, as shown in literature, many of them provide unreasonable results. In this paper, we utilize the measure proposed by H. Nguyen [9], which is intuitively appealing and simple in computation.

Definition 2 [9]: Let $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ be an IFV in finite universe of discourse X. The membership knowledge measure of $\alpha \in X$ is defined as:

$$\widetilde{K}_F(\alpha) = \begin{cases} K_F(\alpha) & \text{for } \mu_\alpha \geq \nu_\alpha, \\ -K_F(\alpha) & \text{for } \mu_\alpha < \nu_\alpha, \end{cases} \quad K_F(\alpha) = \frac{1}{\sqrt{2}} \sqrt{{\mu_\alpha}^2 + {\nu_\alpha}^2 + (\mu_\alpha + \nu_\alpha)^2}. \tag{12}$$

The membership knowledge measure $\widetilde{K}_F(\alpha)$ measures amount of information included with the plus sign for the positive information and minus one for the negative information regarding to the zero reference level of information. The larger value of $\widetilde{K}_F(\alpha)$, the greater IFV α.

3. Priority method of intuitionistic preference relation

Consistency is an important property of the preference relation. In the conventional AHP, Saaty [11] introduced a consistency index CI and a consistency ratio CR to measure the degree of consistency for a multiplicative preference relation and pointed out that if the consistency ratio CR is less than 0.1 then the multiplicative preference relation is of acceptable consistent.

Xu, Cai and Szmidt [19] defined a multiplicative consistent intuitionistic preference relation as follows:

Definition 3 [19]: The intuitionistic preference relation $R = (r_{ij})_{n\times n}$, where $r_{ij} = \langle \mu_{ij}, \nu_{ij} \rangle_{n\times n}$, $(i, j = 1, 2, \dots, n)$ is multiplicative consistent if for all $i \leq t \leq j$,

$$\mu_{ij} = \begin{cases} 0, \text{ if } \left(\mu_{it}, \mu_{tj}\right) \in \{(0,1), (1,0)\}, \\ \dfrac{\mu_{it}\,\mu_{tj}}{\mu_{it}\,\mu_{tj} + (1-\mu_{it})(1-\mu_{tj})}, \text{ otherwise} \end{cases} \quad \nu_{ij} = \begin{cases} 0, \text{ if } \left(\mu_{it}, \mu_{tj}\right) \in \{(0,1), (1,0)\}, \\ \dfrac{\nu_{it}\nu_{tj}}{\nu_{it}\nu_{tj} + (1-\nu_{it})(1-\nu_{tj})}, \text{ otherwise}. \end{cases} \tag{13}$$

An approximation of matrix R with a perfect multiplicative consistent intuitionistic matrix $\overline{R} = (\bar{r}_{ij})_{n\times n}$ is developed by Xu and Liao [20] as follows:

- for $j > i+1$, $\bar{r}_{ij} = \langle \bar{\mu}_{ij}, \bar{\nu}_{ij} \rangle$, for $j = i+1$, $\bar{r}_{ij} = r_{ij}$,
- for $j < i$, $\bar{r}_{ij} = \langle \bar{\nu}_{ij}, \bar{\mu}_{ij} \rangle$, where

$$\bar{\mu}_{ij} = \frac{\sqrt[j-i-1]{\prod_{t=1+i}^{j-1} \mu_{it}\,\mu_{tj}}}{\sqrt[j-i-1]{\prod_{t=1+i}^{j-1} \mu_{it}\,\mu_{tj}} + \sqrt[j-i-1]{\prod_{t=1+i}^{j-1} (1-\mu_{it})(1-\mu_{tj})}}, \bar{\nu}_{ij} = \frac{\sqrt[j-i-1]{\prod_{t=1+i}^{j-1} \nu_{it}\nu_{tj}}}{\sqrt[j-i-1]{\prod_{t=1+i}^{j-1} \nu_{it}\nu_{tj}} + \sqrt[j-i-1]{\prod_{t=1+i}^{j-1} (1-\nu_{it})(1-\nu_{tj})}}. \tag{14}$$

Priority vector of perfect multiplicative consistent intuitionistic matrix $\overline{R}$, where each weight is an IFV obtained from the following formula:

$$\omega_i = \left(\frac{\sum_{j=1}^{n} \bar{\mu}_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} (1-\bar{\nu}_{ij})}, 1 - \frac{\sum_{j=1}^{n} (1-\bar{\nu}_{ij})}{\sum_{i=1}^{n} \sum_{j=1}^{n} \bar{\mu}_{ij}} \right) \; (i, j = 1, 2, \dots, n). \tag{15}$$

We propose the measure of consistency of the priority method as normalized Euclidean distance between matrices R and $\overline{R}$, which is defined as follows:

$$d(R, \overline{R}) = \frac{1}{2(n-1)(n-2)} \sum_{i=1}^{n} \sum_{j=1}^{n} \sqrt{\left(\bar{\mu}_{ij} - \mu_{ij}\right)^2 + \left(\bar{\nu}_{ij} - \nu_{ij}\right)^2 + \left(\bar{\mu}_{ij} - \mu_{ij} + \bar{\nu}_{ij} - \nu_{ij}\right)^2}. \tag{16}$$

Definition 4: We call $\overline{\mathrm{R}}$ an acceptable multiplicative consistent intuitionistic preference relation of the given intuitionistic preference relation R if it fulfils the condition $\mathrm{d}(\mathrm{R},\overline{\mathrm{R}}) < 0.1$.

4. Numerical application

The risk under investigation is connected with the loss by the propulsion system (PS) of its capability to generate the driving force with a desire value and direction. In a formal model, that loss is an initiating event. It has a form of an immediate catastrophic failure (ICF) of the PS. The consequences of the PS loss by a seagoing ship are events classified by the International Maritime Organization as casualties or as incidents [23]. The probabilities of occurrence of the former events in a specific time unit constitute the propulsion risk of a ship (PR). Detailed data on losses are very difficult to obtain, particularly those related to rare events, e.g. consequences of the C1 and C2 category accidents. The data cannot be obtained from experts, as in great majority they have not experienced events where such losses occur.

Tab. 2. Categories of the ICF event consequences

	Name	Description
C1	Very serious casualty	Loss of the ship, loss of human life and/or heavy marine environment pollution.
C2	Serious casualty	Injuries or human health deterioration, ship grounding, touching a submarine object, contact with a solid object, lost seaworthiness due to defects, necessity of towing or assistance from the shore and/or marine environment pollution.
I1	Incident I	Prolonged hazard to the ship, people and environment, which can cause a sea accident. After repair by the ship crew, the ship propulsion function is not fully restored (lower propulsion system operational parameters).
I2	Incident II	As in C3, but after repair the ship propulsion function is fully restored.
I3	Incident III	Temporary hazard to the ship, people and environment, which can cause a sea accident. No repair needed.

Determination of these probabilities is in practice confronted with difficulties connected with shortage of data. In such cases, we have to rely on subjective estimations made by persons with practical knowledge in the field of interest, i.e. experts. However, in great majority they have not experienced events where such losses occur. Therefore, their practical knowledge may contain ambiguousness and uncertainty in some extent. The experts, on the other hand, prefer to formulate their opinions in the linguistic categories. This paper presents a method of the subjective estimation of propulsion risk by a seagoing ship, based on the expert judgments. It is adjusted to the knowledge of experts from ships' machinery crews and to their capability of expressing that knowledge. The method presented has been developed with an intention of using it in the decision-making procedures in risk prediction during the seagoing ship operation.

The consequence of an ICF event may be only one of the five consequence categories listed in Tab. 2. The subset of C1 and C1 consequences, denoted by $C = (C1 \cup C2)$ is defined as hazardous events in the ship risk model. The other events are only incidents, which can cause some loss of operational time and some PS repair expenditures. Let us note that those events can occur only once in a given time interval. Their occurrence causes break in a normal ship operation as the ship is sunk or loses its seaworthiness and must undergo repairs. The events may occur after each subsequent ICF type PS failure. The occurrence takes place with a given conditional probability, the condition being the presence of failure. We also assume that the probability does not depend on the ICF event serial number – it is the same for all the ICF events within a specific time interval t.

The PR is a probability of occurrence of the subset C consequences under condition of ICF type PS failure occurrence. It is determined by the following expression:

$$\mathrm{R}\big(\mathrm{C}, (\mathrm{P_{ICF}(t), k})\big) = \mathrm{P(C|ICF)} \prod_{\mathrm{k=1}}^{\mathrm{K}} (\mathrm{P_{ICF}(t), k})(1 - \mathrm{P(C|ICF)})^{\mathrm{k-1}}, \tag{17}$$

where $(P_{ICF}(t), k), k = 1, 2, \ldots, K$ denotes probability of the ICF type PS failures, in a number k, in time t; $\mathrm{P(C|ICF)}$ probability of the subset C consequences after an ICF type failure.

The example adopted from [2], discusses investigation of a PS consisting of a low speed piston combustion engine driving a fixed pitch propeller and auxiliary subsystems (including the electrical subsystem), installed in a container carrier ship, operating in the North Atlantic region. Experts were marine engineers with long experience (50 ship officers with chief engineer or second engineer diploma). Special questionnaire was prepared for them containing definition of the investigated object, schematic diagrams of subsystems and sets, precisely formulated questions and tables for answers. It was clearly stated in the questionnaire that an ICF type failure might be caused by a device failure or by a crew action. In [2], the probabilities of the number of ICF type events occurred in time t = 1 [year] are depicted as follows:

$$\{(\mathrm{P_{ICF}(t), k})\} = \{(0.0821, 0), (0.2052, 1), (0.2565, 2), (0.2124, 3), (0.1336, 4), (0.0668, 5), \ldots\}.$$

The maximum probability is 0.2565, which corresponds to 2 ICF type events during 1 year, and the probability that such event will not occur amounts to 0.0821.

The data for probability estimation of the consequence of the ship propulsion loss were obtained from a group of 30 experts (ship engineers) using a specially prepared questionnaire [2]. The experts revealed their opinions on the chances of consequence occurrence under condition of the seagoing ship propulsion system ICF failure in the form of linguistic values: extremely not preferred, very strongly not preferred, strongly not preferred, moderately not preferred, equally preferred, moderately preferred, strongly preferred, very strongly preferred, extremely preferred. These data were transferred using the assumed 9-stage scale to construct the intuitionistic fuzzy preference matrices (Tab. 3). We take into account the practical experience of experts as a factor of their hesitancy degree in judgments as follows: $\pi_\alpha = 1/\mathrm{ex}$ where ex denotes the expert practical experience in years. The more experience, the less uncertainty he/she has. A sample matrix, which was created by the expert and transferred into the intuitionistic fuzzy preference relation, is shown as below.

Tab. 3. Intuitionistic preference relation of consequence category of expert with ex=10 [years]

	C1	C2	I1	I2	I3
C1	<0.5, 0.5>	<0.45, 0.45>	<0.2, 0.7>	<0.1, 0.8>	<0.0, 0.9>
C2	<0.45, 0.45>	<0.5, 0.5>	<0.2, 0.7>	<0.1, 0.8>	<0.0, 0.9>
I1	<0.7, 0.2>	<0.7, 0.2>	<0.5, 0.5>	<0.3, 0.6>	<0.2, 0.7>
I2	<0.8, 0.1>	<0.8, 0.1>	<0.6, 0.3>	<0.5, 0.5>	<0.3, 0.6>
I3	<0.9, 0.0>	<0.9, 0.0>	<0.7, 0.2>	<0.6, 0.3>	<0.5, 0.5>

According to Eq. (14), we construct the perfect multiplicative consistent intuitionistic preference relation $\overline{\mathrm{R}}$ of the intuitionistic preference relation R.

$$\bar{R} = \begin{bmatrix} < 0.5, 0.5 > & < 0.45,0.45 > & < 0.17, 0.66 > & < 0.09,0.77 > & < 0.0,0.86 > \\ < 0.45,0.45 > & < 0.5, 0.5 > & < 0.2,0.7 > & < 0.01,0.78 > & < 0.05,0.85 > \\ < 0.66,0.17 > & < 0.7,0.2 > & < 0.5, 0.5 > & < 0.3,0.6 > & < 0.16,0.69 > \\ < 0.77,0.09) & < 0.78,0.10 > & < 0.6,0.3 > & < 0.5, 0.5 > & < 0.3,0.6 > \\ < 0.86,0.0 > & < 0.85,0.05 > & < 0.69,0.16 > & < 0.6,0.3 > & < 0.5, 0.5 > \end{bmatrix}.$$

Then, using Eq. (16) we check the consistency of the perfect intuitionistic preference relation, $\mathrm{d}(\mathrm{R}, \overline{\mathrm{R}}) = 0.047 < 0.1$. According to Eq. (15), we obtain priority vector of consequence category occurrence under condition of ICF event as follows: $\omega_1 = \langle 0.08, 0.83 \rangle$, $\omega_2 = \langle 0.09, 0.84 \rangle$, $\omega_3 = \langle 0.17, 0.75 \rangle$, $\omega_4 = \langle 0.21, 0.69 \rangle$ and $\omega_5 = \langle 0.25, 0.64 \rangle$.

Using Eq. (12), we compute the membership knowledge measures of these IFVs as follows:

$\widetilde{K}_F(\omega_1) = -0.891$, $\widetilde{K}_F(\omega_2) = -0.890$, $\widetilde{K}_F(\omega_3) = -0.845$, $\widetilde{K}_F(\omega_4) = -0.826$ and $\widetilde{K}_F(\omega_5) = -0.804$. Since $\widetilde{K}_F(\omega_1) < \widetilde{K}_F(\omega_2) < \widetilde{K}_F(\omega_3) < \widetilde{K}_F(\omega_4) < \widetilde{K}_F(\omega_5)$,

then the ranking of the consequence category occurrence is $C1 \prec C2 \prec I1 \prec I2 \prec I3$ and C1 is the least probably consequence category following the ICF failure of PS.

Taking into account the membership degrees of the priority vector, i.e. the first part of the IFVs, as intensity degrees of consequence categories to the complete set of mutually exclusive consequences, we derive the normalized weight vector of consequence categories as follows:

$$p_{C/ICF} = (p_{C1}, p_{c2}, \dots, p_{C5})_{ICF} = (0.107, 0.115, 0.205, 0.262, 0.311).$$

These weights satisfy the Kolmogorov's axioms [2] and are interpreted as the probability of the consequence categories. The propulsion risk as probability of ICF event occurrence with consequence related to a very serious casualty or serious casualty, in one-year operational time of a container carrier on the Europe – North America line is derived from Eq. (17) as follows:

$$R\big(C, (P_{ICF}(t), k)\big) = \{(0.0493, 1), (0.0081, 2), (0.00113, 3), (162E-6, 4), (272E-7, 5)\}.$$

The PR depends on the annual number, from 1 to 5 of ICF events in a year, and reaches the maximum value $R_{max} = 0.0493$ in the case of the first such event. The PR value decreases along with the increasing annual number of ICF failures as the conditional probability of subsequent ICF event occurrence substantially decreases. The total risk – the alternative of all risks connected with five numbers of ICF events during a year – amounts to 0.059.

The obtained results of propulsion risk estimation seem to be intuitive in terms of the order of magnitude. According to the International Union of Marine Insurance [22], the subset C events occur now in approx. 0.2% cases of the world fleet vessels and contribute to about 30% of the total number of incidents. This pertains to ships of gross register tonnage above 500 GT. The results are also qualitatively adequate – the identified trends of changes of the investigated quantities are in line with logic of the respective events.

5. Conclusions

In this paper, the IF-AHP method has been proposed for the risk estimation of the ship propulsion loss consequences, which is based exclusively on the judgments elicited by experts – experienced marine engineers. The obtained results show that the proposed method is powerful and useful in dealing with imprecise and uncertain data, which are available in such cases. Combining IF and AHP methods allow incorporating the hesitancy and limited knowledge of expert judgments in the multi-criteria decision making problems. The proposed method is particularly useful in the expert investigations. It is worth noticing that subjective investigation results may (but not necessarily) be charged with greater error than objective results acquired in real operational process. Therefore, the further researches should be focused on validation of the proposed method by the objective results.

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