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An efficient method for calculating system non-probabilistic reliability index

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Highlights

- A method for calculating system non-probabilistic reliability index is proposed.
- A refinement learning function is proposed to determine the best component.
- Two important factors for non-probabilistic reliability index have been considered.

Abstract

Collecting enough samples is difficult in real applications. Several interval-based non-probabilistic reliability methods have been reported. The key of these methods is to estimate system non-probabilistic reliability index. In this paper, a new method is proposed to calculate system non-probabilistic reliability index. Kriging model is used to replace time-consuming simulations, and the efficient global optimization is used to determine the new training samples. A refinement learning function is proposed to determine the best component (or performance function) during the iterative process. The proposed refinement learning function has considered two important factors: (1) the contributions of components to system non-probabilistic reliability index, and (2) the accuracy of the Kriging model at current iteration. Two stopping criteria are given to terminate the algorithm. The system non-probabilistic index is finally calculated based on the Kriging model and Monte Carlo simulation. Two numerical examples are given to show the applicability of the proposed method.

Keywords

non-probabilistic model, non-probabilistic reliability index, system reliability, implicit functions, Kriging model.

1. Introduction

Reliability analysis is performed to estimate the system probability of failure and reliability sensitivity with consideration of uncertainties[25, 19, 34]. Traditionally, uncertainties are often modeled using random variables. To assess uncertainty effects on system performance, many probabilistic-based reliability analysis methods have been reported, such as the first/second order reliability methods (FORM/SORM)[20], saddle-point approximation-based method[6], and surrogate-based method [32]. These reliability methods are generally based on probability theory, i.e., the parameter uncertainties are represented using random variables.

It is well known that sufficient samples (or data) are required to characterize a random variable. This requirement is difficult to satisfy in product early design stages. Therefore, existing probabilistic-based reliability analysis methods may encounter difficulties in this situation. To resolve the problem, non-probabilistic reliability methods are explored. Convex models were first suggested for uncertainty analysis by Ben-Haim and Elishakoff in 1990s [1, 8]; subsequently, Ben-Haim[1, 2] introduced non-probabilistic reliability principles and corresponding theories; He demonstrated that probabilistic-based methods are sensitive to probabilistic model. The small model error may lead to the large error of the result. Guo et al. [12] used interval variables to handle insufficient sample problems, and non-probabilistic reliability index and corresponding model were developed. The non-probabilistic reliability index, which is similar to the reliability index in FORM, can be used to measure the reliability of a system. Subsequently, Guo et al. [13] proposed three kinds of possible approaches to calculate non-probabilistic reliability index. Compared with probabilistic-based reliability methods, non-probabilistic reliability provides a new way to assess the reliability of a structure, and has received much attentions in past two decades. Jiang et al. [17] proposed a semi-analytic method to calculate non-probabilistic reliability index. Dong et al.[5] used interval non-probabilistic reliability method for analyzing jointed rock mass. Chen et al. [4] proposed a theoretical method for structures to conduct non-probabilistic reliability analysis. Jiang et al. [15] proposed a new method to model correlations among convex variables; then, the non-probabilistic reliability analysis was established. Xiao et al. [30] proposed a non-probabilistic reliability method for structural systems with interval variables; whereas the correlations among interval variables is determined by constraints. Nie and Li [22] proposed a direct integration method for systems with

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non-probabilistic convex model. Yang et al. [33] proposed a convex model-based non-probabilistic reliability model for the bridge crane. Jiang et al. [14, 16]proposed a non-probabilistic reliability method for systems with correlated interval variables.

In general, interval variables have obvious advantages to address insufficient information or samples because only little information is required to determine their bounds. Thus, interval-based non-probabilistic reliability methods are useful compared with existing probabilistic methods for insufficient sample problems. For interval-based non-probabilistic reliability analysis methods, the key is to estimate system non-probabilistic reliability index. Based on existing literature, there are generally two kinds of approaches that can be used, i.e., analytical approach and optimization-based approach. The former is computationally effective while its accuracy is low for highly nonlinear performance functions. The latter is extremely accurate while its compactional efficiency is low. Furthermore, performance functions, in general, are often implicit functions with time-consuming simulations in real applications. Thus, calculating non-probabilistic reliability index for systems with multiple failure modes and implicit functions is challenging, and existing approaches are generally difficult to implement. To address the problem, a new and effective method is proposed in this study to calculate non-probabilistic reliability index for systems with multiple failure modes and time-consuming simulations. A refinement learning function is proposed to determine the best component (or failure mode) during iterative process, and two stopping criteria are given to terminate the proposed algorithm. To avoid complex optimization process, the system non-probabilistic reliability index can be calculated based on the final kriging model and Monte Carlo simulation (MCS).

This paper is organized as follows. Section 2 gives a brief review of Kriging. Section 3 introduces interval-based non-probabilistic index model. The details of proposed method for calculating system non-probabilistic reliability index are presented in section 4. Two numerical examples are investigated in section 5 to demonstrate the proposed method. Section 6 presents conclusion to close the paper.

2. Kriging model

Kriging model is a Gaussian process that has been widely used in reliability engineering [21, 24, 27, 28, 29, 31, 35, 36]. In general, Kriging model has two parts, i.e., a deterministic term and a stationary Gaussian process. It is expressed as follows [7]:

$$g(\boldsymbol{x}) = \boldsymbol{f}^T(\boldsymbol{x})\boldsymbol{\beta} + z_s(\boldsymbol{x}) \tag{1}$$

where $\boldsymbol{f}(\boldsymbol{x}) = \left[f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_p(\boldsymbol{x})\right]^T$ are the regression functions, $\boldsymbol{\beta} = \left[\beta_1, \beta_2, \cdots, \beta_p\right]^T$ are the regression coefficients. $z_s(\boldsymbol{x})$ is a stochastic process with mean zero and covariance as follows:

$$\mathrm{Cov}\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right) = \sigma^2 R\left(\boldsymbol{x}_i, \boldsymbol{x}_j\right) \tag{2}$$

where $\mathrm{Cov}(\bullet)$ is the covariance, σ^2 is the process variance, and $R(\bullet)$ is the correlation function.

Given the training data, the unknown parameters $\boldsymbol{\beta}$ and σ^2 in Eqs. (1) and (2), can be estimated based on maximum likelihood estimates, respectively. Finally, for a new unobserved point, $\boldsymbol{x}$, the Kriging prediction is a normal distribution random variable as follows:

$$\tilde{g}(\boldsymbol{x}) \sim \mathbb{N}\left[\mu_{\tilde{g}}(\boldsymbol{x}), \sigma^2_{\tilde{g}}(\boldsymbol{x})\right] \tag{3}$$

where $\mu_{\tilde{g}}(\boldsymbol{x})$ and $\sigma^2_{\tilde{g}}(\boldsymbol{x}) = \mathrm{Var}_{\tilde{g}}(\boldsymbol{x})$ are Kriging mean prediction and Kriging variance, respectively. For further information of Kriging model, please see refs [3, 7, 9, 10] for details.

3. Non-probabilistic index model

Because of insufficient samples in product early design stages, probabilistic-based reliability methods are difficult to implement. Fortunately, interval variable can be used. An interval variable is defined as follows:

$$\left[Y^L, Y^U\right] = \left(Y \in \mathbb{R} \middle| Y^L \le Y \le Y^U\right) \tag{4}$$

where $\mathbb{R}$ is the real number, Y^L and Y^U are the lower and upper bounds of the interval variable, respectively. The midpoint $\bar{Y}$ and radius Y^r can be, respectively, calculated as follows.:

$$\bar{Y} = \frac{Y^L + Y^U}{2}, \; Y^r = \frac{Y^U - Y^L}{2} \tag{5}$$

Let $Z = g(\mathbf{Y})$ with $\mathbf{Y} = (Y_1, Y_2, \cdots, Y_n)$ be the system performance function. Because $\mathbf{Y} = (Y_1, Y_2, \cdots, Y_n)$ are interval variables, $Z = g(\mathbf{Y})$ must be an interval variable with the midpoint $\bar{Z}$ and radius Z^r , respectively. The non-probabilistic index η can be defined as follows[12]:

$$\eta = \frac{\bar{Z}}{Z^r} \tag{6}$$

In Eq. (6), $\eta > 1$ denotes that $\forall Y_i \in \left[Y_i^L, Y_i^U\right] (i = 1, 2, \cdots, n)$, the $g(\mathbf{Y}) > 0$. Then, the system is safe; $\eta < -1$ denotes that the system is failure; $-1 \le \eta \le 1$ denotes that the system is in uncertain state. A larger value of η means that the system is more reliable. Several studies have indicated that the non-probabilistic index η is appropriate to measure the state of structures. For more information of η , please see refs [12] and [16] for details. Based on Eqs. (5) and (6), the non-probabilistic index η can also be rewritten as follows:

$$\eta = \frac{\left(Z^U + Z^L\right)}{\left(Z^U - Z^L\right)} \tag{7}$$

According to Eq. (7), it is easy to know that the heart of calculating non-probabilistic index η is determining the lower and upper bounds of Z . In general, if all interval variables are mutually independent, the lower and upper bounds of Z can be calculated as follows:

$$\begin{cases} Z^L / Z^U = \min/\max g(\boldsymbol{Y}) \\ s.t. \\ \boldsymbol{Y}^L \le \boldsymbol{Y} \le \boldsymbol{Y}^U \end{cases} \tag{8}$$

In Eq. (8), several available optimization algorithms can be used to solve it. However, dependency of interval variables should be considered in real applications. For example, functional dependency can be modeled by using both inequality and equality constraints. When functional dependency of interval variables is considered, Eq. (8) should be extended as follows:

$$Z^L/Z^U = \min/\max g(\boldsymbol{Y})$$
$$\begin{cases} s.t. \\ f_j(\boldsymbol{Y}) \le 0 \ (j=1,2,\cdots,m) \\ \boldsymbol{Y}^L \le \boldsymbol{Y} \le \boldsymbol{Y}^U \end{cases} \tag{5}$$

where $f_j(\boldsymbol{Y}) \le 0 \ (i=1,2,\cdots,m)$ are inequality or equality constraints.

In practical engineering, a system may have multiple components or failure modes. For a series system with k components (or failure modes) $g_i \ (i=1,2,\cdots,k)$, the system non-probabilistic reliability index can be expressed as follows:

$$\eta_{sys} = \min(\eta_1, \eta_2, \cdots, \eta_k) \tag{9}$$

where $\eta_i \ (i=1,2,\cdots,k)$ is the ith non-probabilistic reliability index of the corresponding component g_i.

Similarly, the system non-probabilistic reliability index for a parallel system can be given by:

$$\eta_{sys} = \max(\eta_1, \eta_2, \cdots, \eta_k) \tag{10}$$

In general, $Z_i = g_i(\mathbf{Y}) (i=1,2,\cdots,k)$ are often time-consuming implicit performance functions in real applications. Thus, using traditional optimization methods for calculating system non-probabilistic reliability index is complex and may encounter difficulties.

4. Proposed method for calculating system non-probabilistic reliability index

It is noteworthy that when $Z_i = g_i(\mathbf{Y}) (i=1,2,\cdots,k)$ are time-consuming implicit functions, it is difficult to calculate system non-probabilistic reliability index. To resolve the foregoing, the efficient global optimization (EGO)[18] based on Kriging model can be used to find the global minimum and maximum values of $g_i(\mathbf{Y})$, respectively. The following steps are summarized. (1) constructing initial Kriging model based on a small number of training samples; (2) The EGO is used to find the best added training samples at each iteration; (3) The algorithm is terminated when the given stopping criterion is met. Subsequently, the global minimum or maximum values can be approximately determined based on the final Kriging model. The best added training samples at each iteration for finding the minimum and maximum values can be determined as follows [18]:

$$\begin{cases} \boldsymbol{y}^*_{\min} = \arg\max\left[\mathrm{EI}_{\min}(\boldsymbol{y})\right] \\ \boldsymbol{y}^*_{\max} = \arg\max\left[\mathrm{EI}_{\max}(\boldsymbol{y})\right] \end{cases} \tag{11}$$

where $\mathrm{EI}_{\min}$ and $\mathrm{EI}_{\max}$ are the expected improvements with the following expressions[23]:

$$\mathrm{EI}_{\min}(\boldsymbol{y}) = \left(g_{\min} - \mu_{\tilde{g}}(\boldsymbol{y})\right)\Phi\left[\left(g_{\min} - \mu_{\tilde{g}}(\boldsymbol{y})\right)/\sigma_{\tilde{g}}(\boldsymbol{y})\right] + \sigma_{\tilde{g}}(\boldsymbol{y})\varphi\left[\left(g_{\min} - \mu_{\tilde{g}}(\boldsymbol{y})\right)/\sigma_{\tilde{g}}(\boldsymbol{y})\right] \tag{12}$$

$$\mathrm{EI}_{\max}(\boldsymbol{y}) = \left(\mu_{\tilde{g}}(\boldsymbol{y}) - g_{\max}\right)\Phi\left[\left(\mu_{\tilde{g}}(\boldsymbol{y}) - g_{\max}\right)/\sigma_{\tilde{g}}(\boldsymbol{y})\right] + \sigma_{\tilde{g}}(\boldsymbol{y})\varphi\left[\left(\mu_{\tilde{g}}(\boldsymbol{y}) - g_{\max}\right)/\sigma_{\tilde{g}}(\boldsymbol{y})\right] \tag{13}$$

where φ and Φ are the probability density function and cumulative density function of the standard normal distribution, respectively; $\mu_{\tilde{g}}(\boldsymbol{y})$ and $\sigma_{\tilde{g}}(\boldsymbol{y})$ are the Kriging prediction and the standard deviation of Kriging variance, respectively; $g_{\min}/g_{\max} = \min/\max\left(g(\boldsymbol{y}_j), j=1,\cdots,s\right)$, and s is the number of current training samples. Note that if the functional dependency of interval variables is considered, Eq. (11) should be rewritten as follows:

$$\begin{cases} \boldsymbol{y}^*_{\min} = \underset{\boldsymbol{y}\in\{\boldsymbol{Y}|f_j(\boldsymbol{Y})\le 0\}}{\arg\max}\left[\mathrm{EI}_{\min}(\boldsymbol{y})\right] \\ \boldsymbol{y}^*_{\max} = \underset{\boldsymbol{y}\in\{\boldsymbol{Y}|f_j(\boldsymbol{Y})\le 0\}}{\arg\max}\left[\mathrm{EI}_{\max}(\boldsymbol{y})\right] \end{cases} \tag{14}$$

Intuitively, Kriging model can be used to approximately determine the minimum and maximum values for each component (or failure mode), i.e., $\mathrm{EI}_{\min}$ strategy is used to construct Kriging model $\tilde{g}_\Delta$ for determining the global minimum value $g_{\min} \approx \min(\tilde{g}_\Delta(\boldsymbol{y}))$, and $\mathrm{EI}_{\max}$ strategy is used to construct the other Kriging model $\tilde{g}_\nabla$ for the global maximum value $g_{\max} \approx \max(\tilde{g}_\nabla(\boldsymbol{y}))$. Based on the available values of $g_{\min}$ and $g_{\max}$, the non-probabilistic reliability index can be calculated. However, this manner is not effective if a system involving k components (or failure modes). The reasons are as follows: (1) it is computationally expensive because all Kriging models are required to accurately construct for calculating system non-probabilistic reliability index; (2) it is not effective because single training sample is added at each iteration. To address these issues, we proposed an efficient method for calculating system non-probabilistic reliability index.

For a series system with k components (or failure modes), the constructed Kriging model and non-probabilistic reliability index of the ith component are denoted as $\tilde{g}_i$ and $\tilde{\eta}_i$, respectively. Based on Eq. (9), the system non-probabilistic reliability index can be calculated as $\eta_{sys} \approx \min(\tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_k)$. It is easy to know that the system non-probabilistic reliability index is mainly dependent on the minimum value of $(\tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_k)$, the other indexes have no contribution to the system non-probabilistic reliability index. Thus, the component (or failure mode) with the smaller non-probabilistic index is more important than others for a series system in the iterative process, and more training samples should be selected for it. Furthermore, more training samples should be selected for the component (or failure mode) with inaccurate Kriging model to yield accurate non-probabilistic reliability index. Thus, we propose a strategy for finding the best component at each iteration, which is called as refinement learning function. The above-mentioned two cases are needed to combine in the proposed refinement learning function. Thus, the proposed refinement learning function for series systems is defined as follows:

$$i^* = \underset{i=1,2,\cdots,k}{\arg\min}\left(\tilde{\eta}_i / SD(\tilde{g}_i)\right) \tag{15}$$

where $i^* \in \{1,2,\cdots,k\}$ is the best component (or failure mode) that is needed to refine at current iteration, $\tilde{\eta}_i$ is the non-probabilistic reliability index of the ith component at current iteration, and $SD(\tilde{g}_i)$ is used to measure the variance of $\tilde{\eta}_i$ caused by $\tilde{g}_i$. Furthermore, the optimization models in Eqs. (11) and (14) are difficult to directly solve. Thus, the EGO combined with the MCS are used to improve computational efficiency. Suppose that a MC candidate sample set with n_s samples is denoted by $\{\boldsymbol{y}_c\}$, the $SD(\tilde{g}_i)$ is defined as follows:

$$SD(\tilde{g}_i) = \left(\sigma_{\tilde{g}_i}\left(\boldsymbol{y}^{(i)}_{\min *}\right) + \sigma_{\tilde{g}_i}\left(\boldsymbol{y}^{(i)}_{\max *}\right)\right) \Big/ \sqrt{\mathrm{E}\left[\mathrm{Var}_{\tilde{g}_i}\left(\{\boldsymbol{y}_c\}\right)\right]} \tag{16}$$

where $\boldsymbol{y}_{\min*}^{(i)} = \arg\min \tilde{g}_i\left(\{\boldsymbol{y}_c\}\right)$, $\boldsymbol{y}_{\max*}^{(i)} = \arg\max \tilde{g}_i\left(\{\boldsymbol{y}_c\}\right)$, $\mathrm{E}(\bullet)$ is the expectation operator, $\mathrm{Var}_{\tilde{g}_i}$ is the Kriging variance of the Kriging model $\tilde{g}_i$ with $\sqrt{\mathrm{Var}_{\tilde{g}_i}\left(\{\boldsymbol{y}_c\}\right)} = \sigma_{\tilde{g}_i}\left(\{\boldsymbol{y}_c\}\right)$, and $\boldsymbol{y}_{\min*}^{(i)}, \boldsymbol{y}_{\max*}^{(i)} \in \{\boldsymbol{y}_c\}$.

Based on the proposed refinement learning function, the best component (or failure mode) i^* can be determined at each iteration. Two important factors having a major effect on the system non-probabilistic reliability index are considered, i.e., (1) the component (or failure mode) contribution to the system non-probabilistic reliability index, and (2) the accuracy of the Kriging models. The selected component (or failure mode) at each iteration has the following features, i.e., (1) it has generally the smaller non-probabilistic index than most components, (2) its Kriging model is generally less accurate than others, and (3) for both. Since non-probabilistic reliability index is dependent on the global minimum and maximum values of performance function, the best two added training samples are selected at each iteration based on the EGO. These two samples have the maximum expected improvement on current minimum value and maximum value, respectively. These two training samples for the i^*th component can be selected as follows:

$$\begin{cases} \boldsymbol{y}_{\min}^{*\left(i^*\right)} = \arg\max\left[\mathrm{EI}_{\min}^{\left(i^*\right)}\left(\boldsymbol{y}_c\right)\right] \\ \boldsymbol{y}_{\max}^{*\left(i^*\right)} = \arg\max\left[\mathrm{EI}_{\max}^{\left(i^*\right)}\left(\boldsymbol{y}_c\right)\right] \end{cases} \tag{17}$$

where

$$\mathrm{EI}_{\min}^{\left(i^*\right)}\left(\boldsymbol{y}_c\right) = \left(g_{\min}^{\left(i^*\right)} - \mu_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\right)\Phi\left[\left(g_{\min}^{\left(i^*\right)} - \mu_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\right)\Big/\sigma_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\right] + \sigma_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\varphi\left[\left(g_{\min}^{\left(i^*\right)} - \mu_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\right)\Big/\sigma_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\right],$$

$$g_{\min}^{\left(i^*\right)} = \min\left(g_{i^*}\left(\boldsymbol{y}_j\right), j = 1,2,\cdots,s_{i^*}\right),$$

$$\mathrm{EI}_{\max}^{\left(i^*\right)}\left(\boldsymbol{y}_c\right) = \left(\mu_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right) - g_{\max}^{\left(i^*\right)}\right)\Phi\left[\left(\mu_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right) - g_{\max}^{\left(i^*\right)}\right)\Big/\sigma_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\right] + \sigma_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\varphi\left[\left(\mu_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right) - g_{\max}^{\left(i^*\right)}\right)\Big/\sigma_{\tilde{g}_{i^*}}\left(\boldsymbol{y}_c\right)\right],$$

$g_{\max}^{\left(i^*\right)} = \max\left(g_{i^*}\left(\boldsymbol{y}_j\right), j = 1,2,\cdots,s_{i^*}\right)$, and s_{i^*} is the number of available training samples of the i^*th component.

With the added two training samples at each iteration, the overall computational time is reduced compared with adding single sample point, and the Kriging model $\tilde{g}_{i^*}$ is also refined. When the given stopping criteria are met, the proposed algorithm is terminated. The stopping criteria are given by:

$$\max\left(\mathrm{EI}_{\min}^{\left(i^*\right)}\left(\{\boldsymbol{y}_c\}\right)\right) \le \varepsilon_1 \text{ and } \max\left(\mathrm{EI}_{\max}^{\left(i^*\right)}\left(\{\boldsymbol{y}_c\}\right)\right) \le \varepsilon_2 \tag{18}$$

where $\varepsilon_1, \varepsilon_2$ are two given small positive numbers such as $\varepsilon_1 = \varepsilon_2 = 10^{-2}$. Note that if the functional dependency of interval variables is considered, Eq. (11) should be rewritten as follows:

$$\begin{cases} \boldsymbol{y}_{\min}^{*\left(i^*\right)} = \underset{\boldsymbol{y}_m \in \{\boldsymbol{y}_c | f_j(\boldsymbol{y}_c) \le 0\}}{\arg\max}\left[\mathrm{EI}_{\min}^{\left(i^*\right)}\left(\{\boldsymbol{y}_m\}\right)\right] \\ \boldsymbol{y}_{\max}^{*\left(i^*\right)} = \underset{\boldsymbol{y}_m \in \{\boldsymbol{y}_c | f_j(\boldsymbol{y}_c) \le 0\}}{\arg\max}\left[\mathrm{EI}_{\max}^{\left(i^*\right)}\left(\{\boldsymbol{y}_m\}\right)\right] \end{cases} \tag{19}$$

where $\{\boldsymbol{y}_m\}$ is the MC candidate set that satisfies the constraints, and $\{\boldsymbol{y}_m\} \in \{\boldsymbol{y}_c\}$.

Similarly, the system non-probabilistic reliability index for a parallel system is calculated as $\eta_{sys} \approx \max\left(\tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_k\right)$; thus, the refinement learning function is defined as follows:

$$i^* = \underset{i=1,2,\cdots,k}{\arg\max}\left(\tilde{\eta}_i \times SD\left(\tilde{g}_i\right)\right) \tag{20}$$

Based on proposed refinement learning functions in Eqs. (15) and (20), the component (or failure mode) with the smaller/larger non-probabilistic index and inaccurate kriging model will be generally selected at the current iteration. It is noteworthy that the best component i^* is changeable instead of a determined value. The proposed method for calculating system non-probabilistic reliability index is summarized as follows:

Step 1: A large number of uniform distribution samples are generated within the interval variables using the MCS, and the generated MC sample set with n_s samples is denoted as $\{\boldsymbol{y}_c\}$;

Step 2: Finding the samples satisfy the constraints $f_j\left(\boldsymbol{y}\right) \le 0\ \left(j = 1,2,\cdots,m\right)$. The sample set is denoted as $\{\boldsymbol{y}_m\}$, and $\{\boldsymbol{y}_m\} \in \{\boldsymbol{y}_c\}$;

Step 3: Selecting a small number of initial training samples $\{\boldsymbol{y}_s\}$ from $\{\boldsymbol{y}_m\}$, and calculating corresponding responses $\{\boldsymbol{z}_s\} = \boldsymbol{g}\left(\{\boldsymbol{y}_s\}\right)$, where $\boldsymbol{g} = \left(g_1, g_2, \cdots, g_k\right)$. Subsequently, constructing initial Kriging models $\tilde{Z}_i = \tilde{g}_i\left(\boldsymbol{Y}\right)\left(i = 1,2,\cdots,k\right)$ based on $\left(\{\boldsymbol{y}_s\}, \{\boldsymbol{z}_s\}\right)$ individually.

Step 4: Using the proposed refinement learning functions in Eq.(15) or Eq.(20) to find the best component (or failure mode). For a series system, Eq. (15) is used; for a parallel system, Eq. (20) is used.

Step 5: Using the Eqs. (17) or (19) to find the best two added training samples for the i^*th component (or failure mode), and the number of training samples of the i^*th component is updated as $s_{i^*} = s_{i^*} + 2$; then, the corresponding Kriging model of the i^*th component (or failure mode), $\tilde{g}_{i^*}$, is refined with the added two training samples.

Step 6: Checking the stopping criteria in Eq. (18). If they are met, proceed to Step 7; otherwise, goes back to step 4.

Step 7: Calculating non-probabilistic reliability index of each component (or failure mode) based on the final constructed Kriging models, these indexes are $\{\tilde{\eta}_i, i = 1,2,\cdots,k\}$.

Step 8: The system non-probabilistic reliability index is estimated as $\eta_{sys} \approx \min\left(\tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_k\right)$ and $\eta_{sys} \approx \max\left(\tilde{\eta}_1, \tilde{\eta}_2, \cdots, \tilde{\eta}_k\right)$ for series and parallel systems, respectively.

4. Numerical examples

In this section, two numeral examples are investigated to show the applicability of the proposed method. This first is a parallel system; the second is a cantilever with three failure modes and is a series system. For each example, the system non-probabilistic reliability index calculated based on the true performance functions is reported as the benchmark for accuracy comparisons. It is noteworthy that all per-

formance functions in the numerical examples are viewed as implicit functions for the proposed method.

Example 1–a mathematical problem

Suppose that a parallel system has two performance functions as follows [11]:

$$\begin{cases} g_1(Y_1,Y_2) = 2 - Y_2 + \exp\left(-0.1Y_1^2\right) + \left(0.2Y_1\right)^4 \\ g_2(Y_1,Y_2) = \alpha - Y_1Y_2 \end{cases} \tag{21}$$

Y_1 and Y_2 are two independent interval variables with $Y_1 \in [-2,2]$ and $Y_2 \in [-2,2]$. Two cases are respectively considered: (1) the functional dependency is not involved, and (2) the functional dependency is modeled as $Y_1 + Y_2 - 0.5 \geq 0$. The results of system non-probabilistic reliability indexes under different α are shown in Table 1 and Fig. 1, respectively.

Table 1. System non-probabilistic reliability index under different α

Non-probability reliability index		$\alpha = 4$	$\alpha = 5$	$\alpha = 6$	$\alpha = 7$	$\alpha = 8$
Proposed method	η_{sys}	1.325	1.325	1.504	1.753	2.007
	η^*_{sys}	1.401	1.404	1.589	1.866	2.154
True value	η_{sys}	1.324	1.324	1.500	1.750	2.000
	η^*_{sys}	1.398	1.398	1.571	1.857	2.143

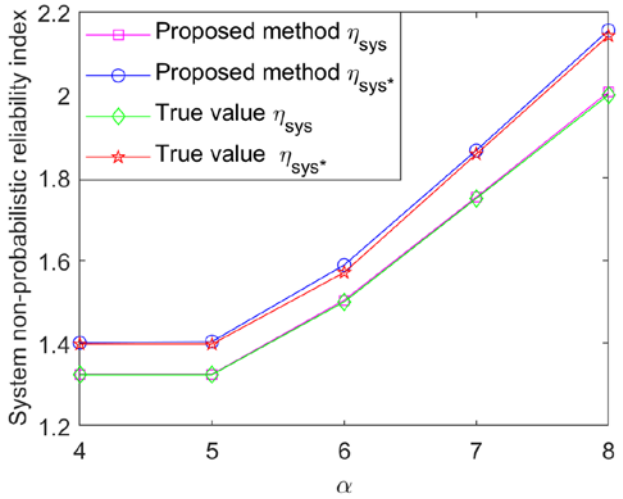

Fig. 1 System non-probabilistic reliability index under different α

In this example, the number of initial training samples and MC candidate samples are 6 and 3×10^5, respectively; the parameters in stopping criteria are set as $\varepsilon_1 = \varepsilon_2 = 10^{-2}$. In Table 1, η_{sys} and η^*_{sys} denote the system non-probabilistic reliability indexes without or with considering functional dependency, respectively. The results based on the real performance functions and existing optimization algorithms are viewed as the benchmark for accuracy comparisons. From Table 1, it shows that the proposed method yields acceptable accuracy level results compared with the true results. The errors come from two aspects: (1) the constructed final Kriging models, and (2) using MC candidate samples to approximately determined the lower and upper bounds of responses. The proposed method is effective and easy to implement because it does not involve complex optimization process.

Note that all performance functions are viewed as implicit functions and replaced using Kriging models in the proposed method. Furthermore, it is observed in Table 1 that the system non-probabilistic reliability index with/without considering functional dependency is different. Thus, functional dependency has obvious effect on system non-probabilistic reliability index.

Table 2. Detailed information of one iterative process α *=8*

No. of iterations	0	1	2	3	4
η_1^*	1.4047	1.4047	1.4045	1.4045	1.4045
η_2^*	3.1082	2.1549	2.1549	2.1548	2.1538
η_{sys}^*	3.1082	2.5149	2.1549	2.1548	2.1538

The details of one iterative process with considering functional dependency under α=8 are shown in Table 2 and Fig. 2, respectively. From Table 2, it is easy to know that the system non-probabilistic reliability index is controlled by component (i.e., failure mode) 2, whereas component (i.e., failure mode) 1 has almost no contribution to it. The proposed method is terminated after four iterations, one time of iteration is for component (failure mode) 1, and three times are for component (failure mode) 2. Thus, the proposed refinement learning function can properly identify key component (failure mode) that has important contribution to system non-probabilistic reliability index. It combines two important factors to properly improve computational efficiency.

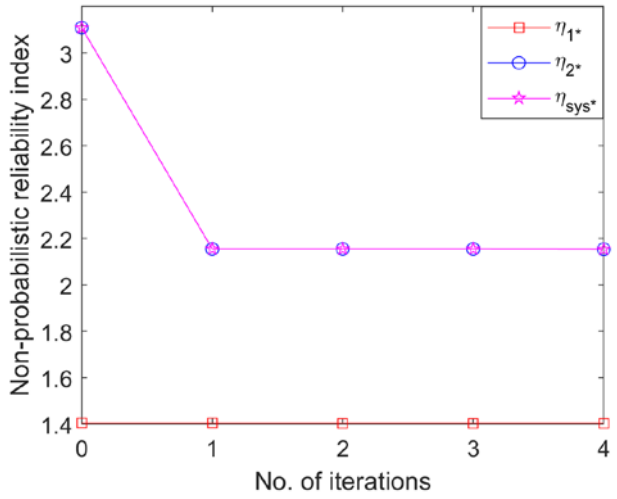

Fig. 2. Details of one iterative process under α *=8*

Example 2–a cantilever system

A cantilever beam with an external load is shown in Fig. 3. This is a series system with the following three performance functions [26]:

$$\begin{aligned} g_{Disp}(L,P,E,B,H) &= 4.0 - \frac{4PL^3}{EBH^3} \\ g_{Stress}(L,P,B,H) &= 4000.0 - \frac{6PL}{BH^2} \\ g_{Moment}(L,P) &= 25000.0 - PL \end{aligned} \tag{22}$$

where H is the cross-sectional height, B is the cross-sectional width, L is the length of the beam, E is the young's modulus with

the precise value 1×10^7, and P is the applied load. The details of these interval variables are listed in Table 3.

Table 3. Detailed information of interval variables

Interval variables	L	B	H	P
Lower bound	180	3.6	2.7	90
Upper bound	220	4.4	3.3	110

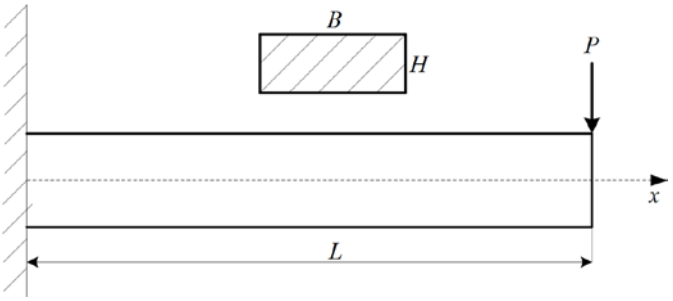

Fig. 3. A cantilever beam

Table 4. Detailed information of one iterative process (example 2)

No. of iterations	0	1	2	3	4	5
η_1	0.355	0.070	0.058	0.058	0.058	0.053
η_2	0.350	0.350	0.350	0.169	0.169	0.169
η_3	1.201	1.201	1.201	1.201	1.203	1.203
η_{sys}	0.350	0.070	0.058	0.058	0.058	0.053

In this example, the number of initial training samples and MC candidate samples are 12 and 5×10^5, respectively; the parameters of stopping criteria are set as $\varepsilon_1=\varepsilon_2=10^{-2}$. The benchmark result of system non-probabilistic reliability index from the real performance functions is about 0.011. The result from the proposed method is 0.053. Considering the definition of non-probabilistic reliability index, the error between the proposed method and the benchmark can be ignored for making decision.

One iterative process of example 2 is shown in Table 4 and Fig. 4, respectively. Based on the Table 4 and Fig. 4, both indicate that the system non-probabilistic reliability index is mainly dependent on component 1, i.e., the first performance function, whereas the other two components (failure modes) have no contribution to it. The proposed method is terminated after five times of iterations, one is for components (failure modes) 2 and 3, respectively, and three are for component (failure mode) 1. Thus, the proposed refinement learning function has properly identified the key component (failure mode) that has important contribution to system non-probabilistic reliability index. Furthermore, the components (failure modes) with inaccurate Kriging models have also considered in the proposed refinement learning function. Thus, it provides a useful manner to balance above-mentioned two factors to some extent.

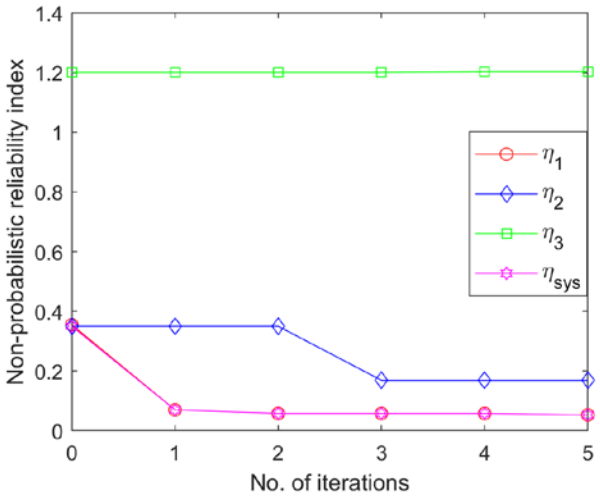

Fig. 4. Details of one iterative process (example 2)

5. Conclusions

In practical engineering, it may be difficult to collect sufficient samples for all variables, especially for a product in its early design stage. Thus, probabilistic-based reliability analysis methods may not be applicable because they require sufficient samples to characterize random variables. Interval variable has obvious advantages for addressing insufficient sample problems. In this paper, an efficient method is proposed for calculating system non-probabilistic reliability index. The refinement learning functions are developed to determine the best component (failure mode) for series and parallel systems, respectively. Two important factors that have a major effect on the system non-probabilistic reliability index have been considered, i.e., (1) the component (failure mode) contribution to the system non-probabilistic reliability index, and (2) the accuracy of the Kriging models. When the best component (failure mode) has been identified at each iteration, two training samples are selected to refine the corresponding Kriging model, which can reduce overall computational time. The EGO combined with MCS can be used to improve computational efficiency. Two examples show that the proposed method can yield accurate results and is generally effective for systems with multiple failure models and implicit functions. Based on the proposed method, it does not require to accurately construct Kriging model for each component (failure mode). In general, a larger system non-probabilistic index indicates that the system is more reliable.

It should be noted that the functional dependency is considered in the study. It is different from the correlation among interval variables. Furthermore, because the proposed method is based on the Kriging model, it is difficult to use for high-dimensional problems due to the "curse of dimensionality". The future work will be focused on addressing these problems.

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