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INVESTIGATIONS OF COUETTE FLOW UNSTEADY RADIATIVE CONVECTIVE HEAT TRANSFER IN A VERTICAL CHANNEL USING THE GENERALIZED METHOD OF LINES (MOL)

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This study describes a very efficient and fast numerical solution method for the non-steady free convection flow with radiation of a viscous fluid between two infinite vertical parallel walls. The method of lines (MOL) is used together with the Runge-Kutta ODE Matlab solver to investigate this problem numerically. The presence of radiation adds more stiffness and numerical complexity to the problem. A complete derivation in dimensionless form of the governing equations for momentum and energy is also included. A constant heat flux condition is applied at the left wall and a transient numerical solution is obtained for different values of the radiation parameter (R). The results are presented for dimensionless velocity, dimensionless temperature and Nusselt number for different values of the Prandtl number (Pr), Grashof number (Gr), and the radiation parameter (R). As expected, the results show that the convection heat transfer is high when the Nusselt number is high and the radiation parameter is low. It is also shown that the solution method used is simple and efficient and could be easily adopted to solve more complex problems.

Key words: Vertical channel, radiation, natural convection, Method of lines (MOL), Runge-Kutta method, Matlab ODE solvers.

1. Introduction

Natural convection flows are found in many engineering applications. For example, they are present in filtration processes, household heating, nuclear reactor cooling, solar energy collectors, etc. they involve temperature gradients that result due to the variation in fluid density, thus setting up a natural circulation of the fluid commonly known as the buoyancy effect. Several papers discuss in detail the physics of free convective flows. Interested readers may refer to references [1, 2] for gaining further insight into the flow mechanism.

A closed form solution was presented in [3, 4] for the transient free convection flow of a viscous fluid between two infinite vertical parallel plates with radiation using the Rosseland approximation. The solution of the governing partial differential equations was obtained using the Laplace-transform technique [3]. The influences of the various parameters on the velocity field, temperature distribution, fluid concentration, shear stress, rates of heat and mass transfers were presented. It was also shown that the velocity and temperature fields decrease with an increase in the value of the radiation parameter. On the other hand, the rate of heat transfer at the end of the channel decreases with an increase in either the radiation parameter or Prandtl number [4].

An exact analysis of the natural convection in an unsteady Couette flow of a viscous incompressible fluid confined between two vertical parallel plates in the presence of thermal radiation was investigated in [5-7]. The Rosseland approximation was used to describe the radiative heat flux in the energy equation. The effect of different parameters such as the thermal radiation parameter, Grashof number, Prandtl number were discussed. The momentum and the thermal boundary layer thicknesses decreased due to an increase in the value of the radiation parameter. An increase in the value of the Grashof number was found to increase the velocity of air and water while it decreased the skin-friction coefficient in these fluids.

The radiation effects on natural convection in a vertical channel with an auxiliary plate was presented in [8]. The vertical channel in this article was symmetrically heated and a uniform heat flux along the principal walls of the channel was assumed. Comparisons between experimental data carried out from different positions of the auxiliary plate and two surface emissivity values were given in terms of heated wall temperatures and radiative heat transfer. Findings of this study showed that the lowest maximum channel wall temperatures could be attained for the auxiliary plate placed at the channel exit for high channel Rayleigh numbers and heat flux values. Correlations for the average Nusselt number, global average Nusselt number and dimensionless maximum wall temperature in terms of channel Rayleigh number and auxiliary plate-to-channel height ratio were presented in the study [8].

The natural convection flow in an asymmetrically heated vertical 2D plane channel was explored in the boundary layer regime for a range of modified Rayleigh numbers (Ra) by Hemmer *et al.* [9]. They investigated the influence of different spatial arrangements of the heat sources on both heat transfer and flow dynamics. The radiation energy was neglected since water was selected to be the working fluid. The study concluded that for constant heat input and exchange surface area, two distant heat sources were more efficient to put fluid into motion and to transfer heat.

The influence of thermal stratification and surface radiation on a laminar airflow induced by natural convection in vertical, asymmetrically heated channels was investigated numerically by Ramalingom *et al.* [10]. The numerical solution was obtained by setting up the weak temperature gradient outside of the channel, and then the temperature at the bottom end of the channel was obtained as a function of time. Surface radiation was also included but it appeared to be less dominant than thermal stratification for the selected conditions. The study also observed that the local and the mean Nusselt numbers slowly increase for the investigated cases.

Rachid *et al.* [11] presented experimentally and numerically a natural convection air flow coupled with surface radiation for a vertical open-ended channel with constant heat flux at the wall. They used the radiosity method to account for the surface radiation heat transfer. The study found that surface radiation played a significant role in the evolution of the dynamical and thermal behaviour of the natural convection flow inside the channel. Another numerical study on free convection in a symmetric open channel was carried out by Bouraoui *et al.* [12] using the Ansys Fluent CFD software. The influence of the secondary ventilation of the adiabatic channel wall on flow dynamics and the heat transfer was found to be affected by the position and size of the openings.

A generalized model of the unsteady natural convection heat transfer as a nanofluid flows through a rectangular vertical channel was presented in [13]. The fractional constitutive equations for the shear stress and thermal flux defined with the time-fractional Caputo derivative were used to develop the generalized mathematical model with time-nonlocality. Analytical solutions in [13] for dimensionless velocity and temperature fields were obtained by using the Laplace transform coupled with the finite sine-cosine Fourier transform. It was found that the thermal history strongly influenced the thermal transport for small values of time. It was also demonstrated that the thermal transport would enhance if the thermal fractional parameter was lowered or the volume fraction of the nanoparticles was increased.

The unsteady oscillatory Couette flow between vertical parallel plates with constant radiative heat flux was investigated in [14]. In this study, the mathematically formulated governing equations were simplified using dimensionless variables and coupled ordinary differential equations (ODEs) were obtained. The radiation energy was modelled using the Rosseland approximation and the perturbation method was used to solve the resulting governing equations for the velocity and temperature fields.

The natural convection flow of a second grade fluid with thermal radiation was analytically investigated by Nisa *et al.* [15] using the Laplace transform for the temperature and velocity fields. The energy equation was fractionalized by means of the generalized Fourier's law. It was found that the thermal history strongly influenced the thermal transport for small values of time t. It was also found that the thermal transport could be enhanced when the thermal fractional parameter was decreased or increased.

A spatially developing transitional flow in a vertical channel with one side heated uniformly and subjected to random velocity fluctuations at the inlet was studied experimentally and numerically in [16]. The

PIV experimental setup was used to obtain a completely two-dimensional velocity field in the streamwise flow direction. Numerical simulations were performed on the assumptions of a thermally stratified environment, subjected to an environmental noise, and for which the wall to wall radiations were considered. Different transition indicators were defined based on velocity and thermal quantities such as the wall temperature and velocity indicators that govern the transitional behaviour of the flow field.

The Method of Lines (MOL) is a general numerical method that converts the initial boundary value problem into a system of ordinary differential equations (ODEs). Brastos [17] used this method to convert the nonlinear hyperbolic Boussinesq equation into a first-order, nonlinear, initial-value problem. This study can be considered as a base for the application of the MOL in engineering applications since it built the basis for analysing the local truncation errors, stability and convergence. Also, many recent studies found in the literature utilized this method as a numerical tool for solving many applied engineering problems. For example, a fourth order, forced beam vibration problem subjected to axial tension was investigated using the MOL in [18].

To the best knowledge of the author, there are almost no studies in the literature that provide a numerical solution to the free convection heat transfer problems using the MOL. This study utilizes the MOL to solve the coupled non-linear thermal radiation and natural convection of the unsteady Couette flow in a channel of infinite vertical parallel walls. After this introduction, the remainder of this study is ordered as follows. The problem definition and objectives are introduced in Section 2. In Section 3, the principal governing equations and methodology are described in detail. The results and discussion are presented in Section 4, and finally, conclusions of this research work are drawn in Section 5.

2. Problem definition and objectives

The topic of free convection in vertical channels has been studied extensively in the literature due to its importance in many engineering applications such as cooling of electronic equipment, heat exchangers, nuclear reactors cooling, and geothermal systems, etc. Most of the studies in the literature does not include the effect of radiation and the Couette flow. Some studies did include the effect of radiation but with some simplified approximations such as the Rosseland approximation [19].

This paper attempts to solve and analyse the unsteady natural convection and radiation problem of an incompressible viscous fluid that is confined in a vertical channel. The two vertical walls are subjected to different boundary conditions but with the exact representation of the radiation energy term. The results will be presented for different values of the radiation parameter (R), the Nusselt number (Nu), the Prandtl number (Pr), and the Grashof number (Gr). These parameters will be defined in a subsequent section of the paper. Also, a low Prandtl number (Pr) fluid such as air is chosen to perform this study.

The other objective of this paper is to prove that a fast and efficient numerical scheme based on the method of lines (MOL) can be effectively utilized to solve such problems. The MOL method is basically based on viewing a partial differential equation as a system of ordinary differential equations. The method is adopted along with the Runge Kutta method for solving the resulting discretized state space ordinary nonlinear coupled differential equations. This is essentially a two-step process: the spatial derivatives are approximated algebraically using the finite difference method, which will result in having a system of nonlinear ODEs in the initial value variable (typically time). It is then integrated by an initial value problem solver of the Matlab ODEs solvers.

3. Governing equations and methodology

In this paper, the incompressible unsteady viscous radiating Couette flow in a vertical channel with natural convection is considered. The flow in this problem is driven mainly by natural convection and radiating walls of the considered vertical channel. The channel has two infinite vertical walls separated by a distance L. Figure 1 shows the domain of the problem and the adopted boundary conditions of this problem. The flow field in the problem is only in the horizontal direction. Also, the model includes natural convection effects in which the density of the fluid appears in both inertia (gravity and pressure), and body force (buoyancy force) terms. As a result, the Boussinesq approximation is used to relate these forces as a single component in the body force term which is related to the temperature field of the problem. The resulting governing x-momentum and energy equations of this problem are given as follows [20, 21]:

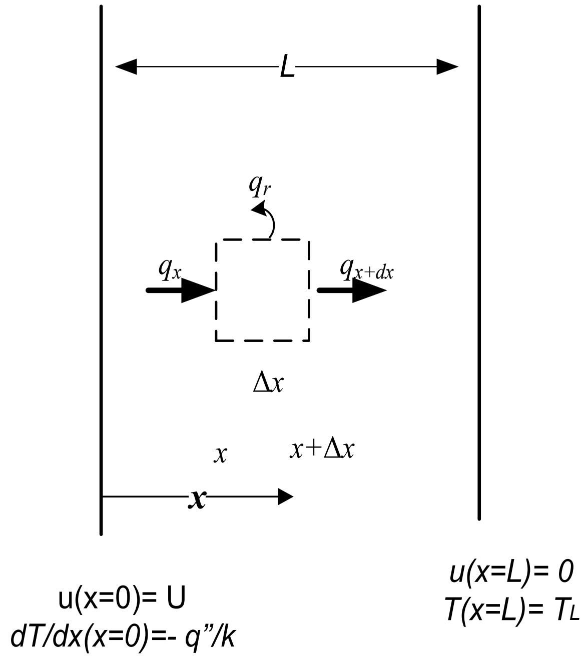

Fig.1. Vertical channel domain and boundary conditions.

$$\frac{\partial u}{\partial t} = g\beta\left(T - T_L\right) + \nu \frac{\partial^2 u}{\partial x^2} \tag{3.1}$$

where u is the fluid velocity, T is the fluid temperature, T_L is the channel wall temperature at $x = L$, β is the volume expansion coefficient, and g the gravitational acceleration.

The energy equation for the problem is derived as follows. If a control-volume (CV) of length Δx of the channel shown in Fig.1. is to be analysed, the energy equation for this CV may be written as:

$$\frac{dE_{cv}}{dt} = -A\, q''(x) \tag{3.2}$$

where E is the total energy change of the system, $q''(x)$ is rate of heat flux loss from the control volume per unit surface area, and A is the total heat transfer area of the control surface of the channel. Using the definition of the specific heat for the energy term on the left hand side of Eq.(3.2) will give:

$$\frac{dE_{cv}}{dt} = \rho\, V\, c \frac{dT}{dt} = \rho\, A\, \Delta x\, c \frac{dT}{dt} \tag{3.3}$$

where V is the volume of the channel, ρ and c are the density and the specific heat of the fluid in the channel. The energy balance with no heat generation for the control volume shown in Fig.1 can be written as:

$$\frac{dE_{cv}}{dt} = q_{cv}\,|_{in} - q_{cv}\,|_{out} \tag{3.4}$$

where

$$q_{cv}\,|_{in} = q(x), \qquad \text{and} \qquad q_{cv}\,|_{out} = q(x) + \frac{dq(x)}{dx}\Delta x + q_r\,.$$

The radiation term q_r is written as:

$$q_r = P\,\varepsilon\,\sigma\left(T^4(x) - T_L^4\right)\,\Delta x \tag{3.5}$$

where P is the perimeter of the channel, $T(x)$ is the temperature in the channel space, T_L is the wall temperature at $x = L$, ε is the surface emissivity, and σ is the Stefan-Boltzmann constant. Substitution of q_r into Eq.(3.4), will result in the following equation:

$$\rho\,A\,\Delta x\,c\,\frac{\partial T}{\partial t} = q(x) - \left[\left[q(x) + \frac{\partial q(x)}{\partial x}\Delta x\right] + \left[P\,\varepsilon\,\sigma\left(T^4(x) - T_L^4\right)\,\Delta x\right]\right]. \tag{3.6}$$

If Δx and $q(x)$ are cancelled from the above equation and $\frac{dq(x)}{dx}$ term is substituted using the Fourier's law $q(x) = -k\,A\,\frac{dT}{dx}$, then Eq.(3.6) becomes:

$$\rho\,A\,c\,\frac{\partial T}{\partial t} = k\,\frac{\partial}{\partial x}\left(A\,\frac{\partial T}{\partial x}\right) + P\,\varepsilon\,\sigma\left(T^4(x) - T_L^4\right) \tag{3.7}$$

where k is the thermal conductivity of the fluid in the channel.

It is more convenient to represent Eqs (3.1) and (3.7) in dimensionless form by introducing the following definitions for dimensionless fluid velocity, dimensionless fluid temperature, dimensionless time, dimensionless x-coordinate respectively: $\hat{u} = \frac{u}{U}$, $\theta(x,t) = \frac{T - T_L}{Lq''/\,K}$, $\hat{t} = \frac{\alpha\,t}{L^2}$, and $\hat{x} = \frac{x}{L}$. Substitution of these values of $\hat{u}$, θ, $\hat{t}$ and $\hat{x}$ into Eq.(3.1), yields the following equations for the momentum equation:

$$\frac{U\,\alpha}{L^2}\frac{\partial \hat{u}}{\partial \hat{t}} = \frac{g\,\beta\,L\,q''}{k}\,\theta + \frac{\nu\,U}{L^2}\frac{\partial^2 \hat{u}}{\partial \hat{x}^2}\,. \tag{3.8}$$

Simplifying this equation yields:

$$\frac{\partial \hat{u}}{\partial \hat{t}} = \frac{L^2}{U\,\alpha}\frac{g\,\beta\,L\,q''}{k}\,\theta + \frac{L^2}{U\,\alpha}\frac{\nu\,U}{L^2}\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} \tag{3.9}$$

or

$$\frac{\partial \hat{u}}{\partial \hat{t}} = Gr\ Pr\ \theta + Pr \frac{\partial^2 \hat{u}}{\partial \hat{x}^2} \tag{3.10}$$

where Gr is the Grashof number $\left(Gr = g\beta L^3 q'' / Uk\nu\right)$, and Pr is the Prandtl number $\left(Pr = \nu / \alpha\right)$.

The dimensionless form of the energy Eq.(3.7) is achieved by substituting the dimensionless parameters into Eq.(3.7) as follows:

$$\frac{Lq''/k}{L^2/\alpha}\frac{\partial \theta}{\partial \hat{t}} = \frac{\alpha L q''}{k\,L^2}\frac{\partial^2 \theta}{\partial \hat{x}^2} + \frac{P\,\varepsilon\,\sigma}{\rho\,c}\left[\left(\frac{Lq''}{k}\theta + T_L\right)^4 - T_L^4\right]. \tag{3.11}$$

Simplifying this equation yields the following energy equation:

$$\frac{\partial \theta}{\partial \hat{t}} = \frac{\partial^2 \theta}{\partial \hat{x}^2} + R\left[\left(\theta + \hat{\theta}\right)^4 - \hat{\theta}^4\right] \tag{3.12}$$

where $\hat{\theta}$ is the temperature ratio, and R is the radiation parameter. Definitions of these parameters are as follows:

$$\hat{\theta} = \frac{T_L}{Lq''/k}\ , \qquad \text{and} \qquad R = \frac{\varepsilon\,\sigma\,L^2/\alpha}{\rho\,c}\left(\frac{Lq''}{k}\right)^3. \tag{3.13}$$

The associated initial conditions for this study are $\hat{u} = \theta = 0$ at $\hat{t} = 0$ for $0 \le \hat{x} \le 1$, and boundary conditions the flow and temperature fields are as follows:

$$\hat{u} = 1 \quad \text{at} \quad \hat{x} = 0 \quad \text{and} \quad \hat{u} = 0 \quad \text{at} \quad \hat{x} = 1\,, \tag{3.14}$$

$$\frac{d\theta}{d\hat{x}} = -1 \quad \text{at} \quad \hat{x} = 0 \quad \text{and} \quad \theta = 0 \quad \text{at} \quad \hat{x} = 1\,. \tag{3.15}$$

Equations (3.10) and (3.12) constitute a stiff nonlinear system, that needs to be solved numerically by an efficient algorithm such as the MOL. The MOL essentially is a two-step process as the spatial derivatives are approximated using the finite difference approximation, and the resulting system of ODEs in the initial value variable is then integrated by an initial value ODE solver such as the Runge-Kutta method which is embedded in the Matlab software [22]. The central differencing scheme is used for the spatial derivative. The discretized forms of the governing equations are as follows:

$$\frac{d\hat{u}_i}{d\hat{t}} = Gr\ Pr\ \theta_j + Pr\frac{\hat{u}_{i+1} - 2\,\hat{u}_i + \hat{u}_{i-1}}{\left(\Delta\hat{x}\right)^2}\ , \tag{3.16}$$

and

$$\frac{d\theta_j}{d\hat{t}} = \frac{\theta_{j+1} - 2\,\theta_j + \theta_{j-1}}{\left(\Delta\hat{x}\right)^2} + R\left[\left(\theta_j + \hat{\theta}\right)^4 - \hat{\theta}^4\right]. \tag{3.17}$$

The temporal and spatial integrations are separated in the sense that they can be treated separately in the coding procedure which adds a very attractive degree of flexibility including the use of library routines of the Matlab software. It should be noted that there are two ODEs at each grid point (i, j), where: $1 < i, j < N$. The number of grid points is N, which will consequently result in having $2N$ simultaneous coupled ordinary nonlinear ODEs. This system of ODEs is represented in the Matlab software by a vector of size $2N$. The boundary conditions are incorporated into the system of ODEs when $i, j = 1$, and $i, j = N$ for both the temperature and the flow field of the problem. On the other hand, the initial conditions are dealt with within the ODE solver.

This system of ODEs (Eqs (3.16) and (3.17)) can be solved using the fourth order Runge-Kutta method embedded in the Matlab ODE solvers. The solution procedure is based on a normalized time $\hat{t}$ that varies from *0* to *1*. In this method, the local truncation error is of the order of $O\left(\Delta\hat{x}^4\right)$. The solution method is based on solving the energy equation first, then substituting the temperature field of Eq.(3.17) into Eq.(3.16) to solve the flow field problem. In this paper, the outline for stability of the solution is based on the unconditionally stable scheme presented by Causley *et al.* [23] for solving the wave equation.

4. Results and discussion

Results for dimensionless velocity, dimensionless temperature and the Nusselt number are presented for a channel with unit spacing and a fixed low value of the Prandtl number. Results are presented graphically for different dimensionless parameters that include: the radiation parameter (R), the Grashof number (Gr) and the Nusselt number (Nu).

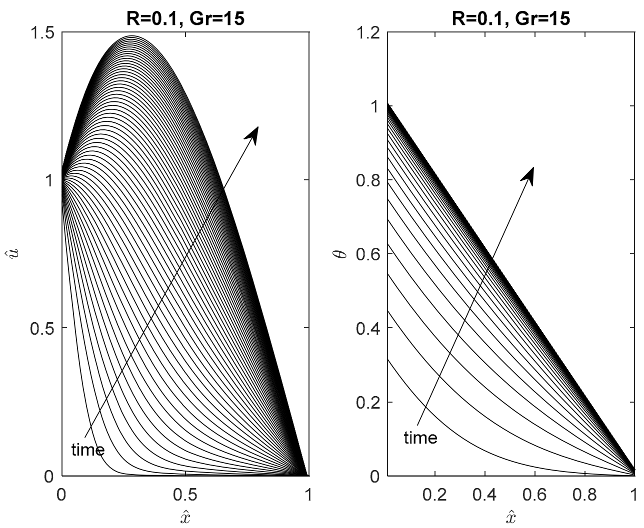

Fig.2. The non-dimensional velocity, and non-dimensional temperature transient solution $(Pr = 0.72)$.

It can be seen from Fig.2 that for fixed low values of R and Gr the non-dimensional velocity $\hat{u}$ peaks (gets to a maximum value) in time before starting to decrease again. On the other hand, no such peak is observed for the non-dimensional temperature θ which linearly decreases with the non-dimensional distance. It should be noted that in all the figures provided in this paper, the direction of the arrow represents increasing time.

The results of this study are compared with other solutions in the literature. The current solution for the dimensionless temperature and flow field dimensionless velocity is found to be in full agreement with the exact solution presented in the study of reference [7]. This comparison is shown in Fig.3. The dimensionless flow velocity $\hat{u}$ is plotted for different values of the Grashof number in Fig.4. As the Grashof number increases the peak in the transient flow velocity becomes steeper with increasing time. The dimensionless velocity is maximum along the *x*-axis of the channel while the minimum values are near or at the right wall. In Fig.3, the radiation parameter value is chosen to be *0.6*. This low value of the radiation parameter suggests that the heat transfer mechanism is driven purely by natural convection.

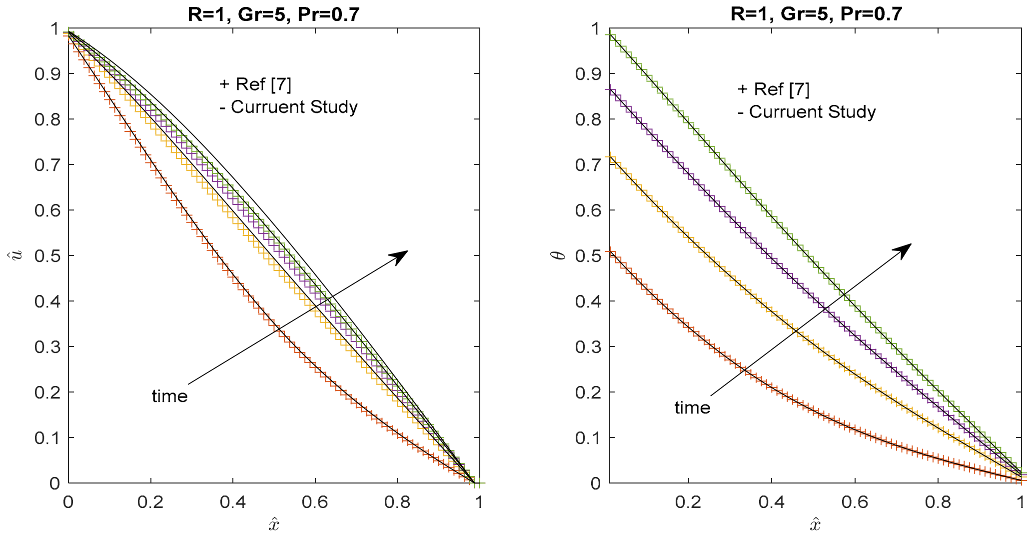

Fig.3. Comparison of the current solution with some other solution [7].

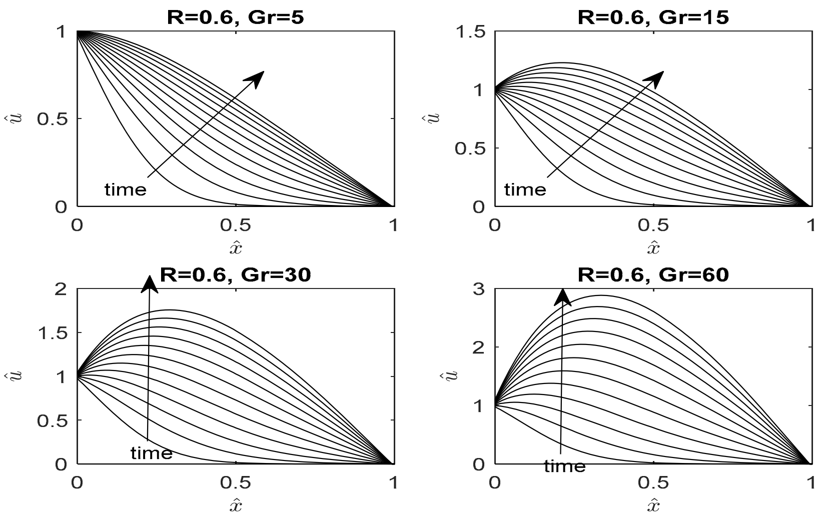

Fig.4. The non-dimensional velocity transient solution for different values of the Grashof number $(Pr = 0.72)$.

The dimensionless temperature of the fluid is displayed in Fig.5 for different values of the radiation parameter. It can be concluded from this figure that the fluid temperature decreases with the increase in the radiation parameter (R) for a low Grashof number $(Gr = 10)$. In Fig.5, both the Grashof (Gr) and Prandtl (Pr) numbers are fixed while the radiation parameter (R) is made to vary. From the figure it can be observed that as the time advances the fluid temperature tends to increase until it achieves its maximum value at the left wall of the channel when $x = 0$, then it falls smoothly to a zero value at the right wall of the channel at $x = 1$. Conversely, the dimensionless fluid temperature shows a slight increase in the same period of time as the radiation parameter is increased while keeping the Grashof number fixed to a high value as shown in Fig.6. It is further noticed that the temperature drop is non-linear, whereas in Fig.5 where the Grashof number is low, the temperature drop is non-linear for lower times and becomes linear for larger time.

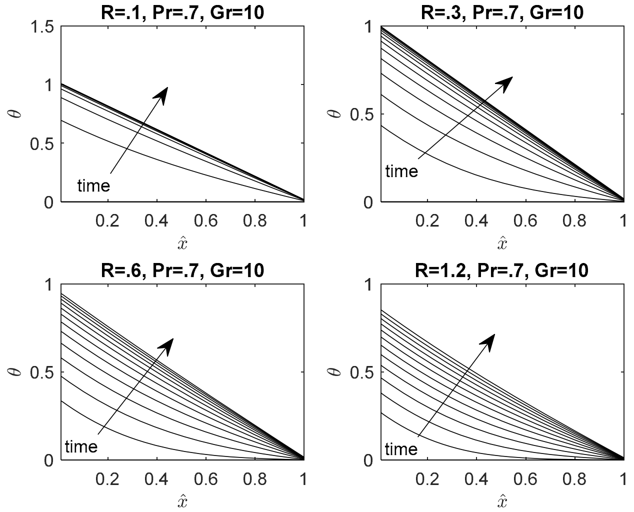

Fig.5. Effect of the radiation parameter on the non-dimensional temperature for a fixed Grashof number.

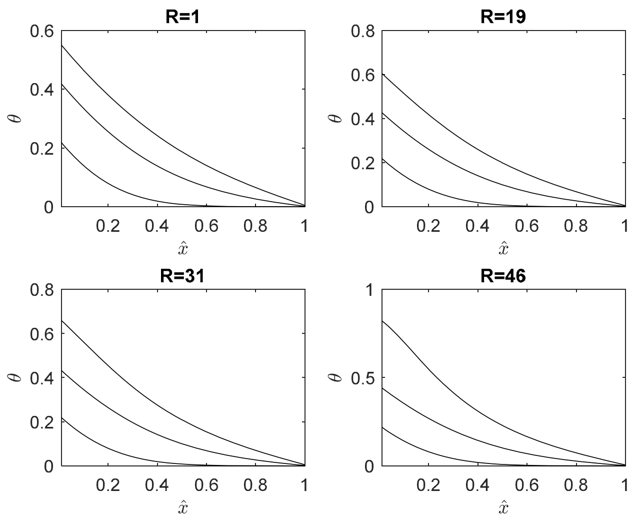

Fig.6. Effect of the radiation parameter on the non-dimensional temperature for $Gr = 30$, and $Pr = 0.72$.

The variation of Nusselt number is shown in Fig.7 for different values of the radiation parameter (R) and fixed Grashof number (Gr) and Prandtl number (Pr). The Nusselt number (Nu) is at maximum at each time step at the left wall of the channel, and decreases nearly exponentially until it reaches a zero value at the right end of the channel. As the radiation parameter (R) increases, the Nusselt number (Nu) decreases which indicates that the heat transfer is driven mainly by the radiation part of the energy. If the radiation number (R)

is increased further, one can observe that the Nusselt number (Nu) is also increased as depicted in Fig.8 in the early stages of the transient solution. This suggests that the heat transfer mechanism is governed by both radiation and convection heat transfer for the case when the radiation parameter value is high.

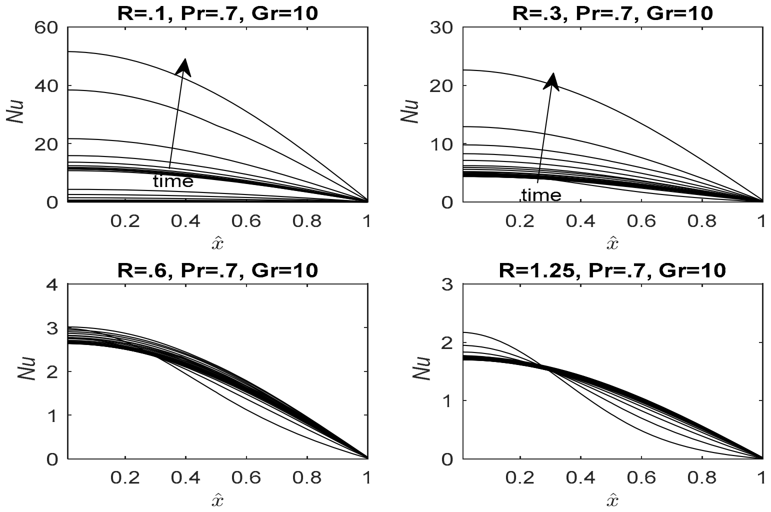

Fig.7. Effect of the radiation parameter on the Nusselt number for a fixed Grashof number.

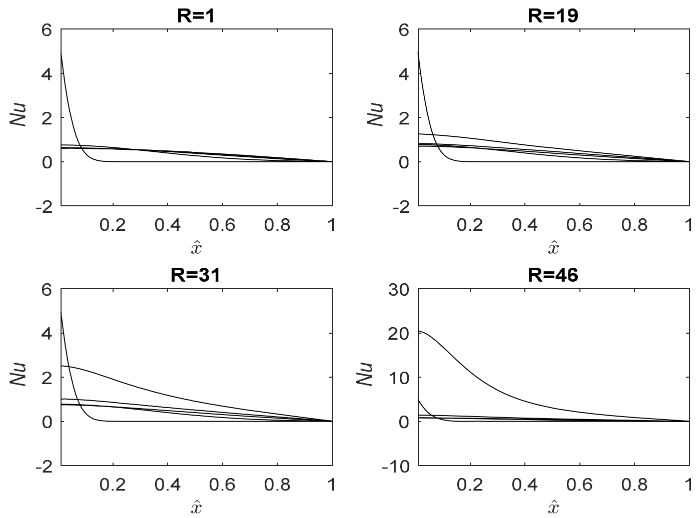

Fig.8. Effect of the radiation parameter on the Nusselt number for $Gr = 30$, and $Pr = 0.72$.

5. Conclusions

The method of lines (MOL) is used in this study to investigate the unsteady natural convective Couette flow of a viscous incompressible fluid between two parallel walls of a vertical channel in the presence of thermal radiation. The governing partial differential equations are derived in dimensionless form, and discretized using the finite central difference scheme. The resulting discretized transient coupled differential equations were solved numerically at each grid point using the Matlab software's ODE solvers. The solution method is stable, fast and efficient, therefore, it can be utilized to solve more complicated domains and geometries.

In this study, the effects of the radiation parameter and Grashof number on the flow and temperature fields have been investigated. It is shown that the radiation parameter (R) plays an essential role in the heat transfer mechanism of this problem. One of the important conclusions of this study is that when the Grashof number is increased the parabolic-shaped developing transient flow velocity profiles become more steep with time.

Also, when the radiation parameter (R) is low, the velocity of the fluid increases with the increase in the Grashof number with time. Furthermore, the Nusselt number (Nu) generally at the left moving wall is always maximum due to the dominance of the convection heat transfer in that region. The analysis in this study can be further extended to investigate the thermodynamic behaviour of fluids with a high Prandtl number (Pr). Moreover, the analysis presented in this paper can be extended easily to solve multi-dimensional fluid flow/heat transfer problems.

Acknowledgment

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Nomenclature

A – total heat transfer area $[m^2]$

c – specific heat $[kJ/kgK]$

E – total energy change of the system in the control volume $[kJ]$

Gr – Grashof number

K – thermal conductivity $[W/m^2K]$

L – horizontal length between the vertical plates $[m]$

Pr – Prandtl number

$q^{\backslash\backslash}$ – rate of heat flux $[W/m^2]$

R – radiation parameter

T – fluid temperature $[K]$

T_L – right end wall temperature $[K]$

t – time $[sec]$

$\hat{t}$ – non-dimensional time

U – left end velocity $[m/s]$

u – fluid velocity components in the x direction $[m/s]$

$\hat{u}$ – non-dimensional velocity

Greek Symbols

α – thermal diffusivity $[m^2/s]$

β – volume expansion coefficient $[1/K]$

ε – emissivity factor

θ – dimensionless temperature

ν – kinematic viscosity $\left[m^2/s\right]$

ρ – density $[kg/m^3]$

σ – Stefan-Boltzmann constant $\left(5.67\times 10-8\ W/m^2K^4\right)$

References

[1] Shirvan K. M., Mamourian M., Mirzakhanlari S., Rahimi A. B. and Ellahi R. (2017): *Numerical study of surface radiation and combined natural convection heat transfer in a solar cavity receiver*.– International Journal of Numerical Methods for Heat and Fluid Flow, vol.27. No.10, pp.2385-2399.

[2] Foroushani S., Naylor D. and Wright J.L. (2020): *Heat transfer correlations for laminar free convection in vertical channels with asymmetrically heated isothermal walls*.– Heat Transfer Engineering, vol.41, No.5, pp.418-432.

[3] Chaudhary R.C. and Jain P. (2008): *Exact solutions of incompressible Couette flow with constant temperature and constant heat flux on walls in the presence of radiation*.– Turkish Journal of Engineering and Environmental Sciences, vol.31, No.5, pp.297-304.

[4] Mandal H. K., Das S., and Jana R. N. (2014): *Transient free convection in a vertical channel with variable temperature and mass diffusion*.– Chemical and Process Engineering Research, vol.23, pp.38-54.

[5] Narahari M. (2009): *Natural convection in unsteady Couette flow between two vertical parallel plates in the presence of constant heat flux and radiation*.– In MACMESE'09 Proceedings of the 11th WSEAS International Conference on Mathematical and Computational Methods in Science And Engineering, pp.73-78.

[6] Narahari M. (2010): *Effects of thermal radiation and free convection currents on the unsteady Couette flow between two vertical parallel plates with constant heat flux at one boundary*.– WSEAS Transactions on Heat and Mass Transfer, vol.5, No.1, pp.21-30.

[7] Narahari M. and Pendyala R. (2013): *Exact solution of the unsteady natural convective radiating gas flow in a vertical channel*.– AIP Conference Proceedings, vol.1557, pp.121-124.

[8] Andreozzi A. and Manca O. (2015): *Radiation effects on natural convection in a vertical channel with an auxiliary plate*.– International Journal of Thermal Sciences, vol.97, pp.41-55.

[9] Hemmer C., Popa C.V., Sergent A. and Polidori G. (2016): *Heat and fluid flow in an uneven heated chimney*.– International Journal of Thermal Sciences, vol.107, pp.220-229.

[10] Ramalingom D., Cocquet P.-H. and Bastide A. (2017): *Numerical study of natural convection in asymmetrically heated channel considering thermal stratification and surface radiation*.– Numerical Heat Transfer; Part A: Applications, vol.72, No.9, pp.681-696.

[11] Rachid A., Amine Z., Imad K., Khalid S. and Miloud R. (2017): *Numerical study of natural convection in an open-ended channel with surface radiation: comparison with experimental data*.– International Journal of Mechanical and Mechatronics Engineering, vol.17, No.1, pp.23-29.

[12] Bouraoui M., Rouabah M.S., Abidi-Saadb A., Korichi A., Pop C. and Polidori G. (2017): *Numerical simulation of a double skin with secondary ventilation flow on adiabatic wall*.– Frontiers in Heat and Mass Transfer, vol.8, Article No.18, pp.1-6.

[13] Ahmed N., Vieru D., Fetecau C. and Shah N. A. (2018): *Convective flows of generalized time-nonlocal nanofluids through a vertical rectangular channel*.– Physics of Fluids, vol.30, No.5, Article No.052002, https://doi.org/10.1063/1.5032165.

[14] Bunonyo K. W., Amos E. and Eli I. C. (2018): *Unsteady oscillatory Couette flow between vertical parallel plates with constant radiative heat flux*.– Asian Research Journal of Mathematics, vol.11, No.2, pp.1-11.

[15] Nisa Z.U., Shah N.A., Tlili I., Ullah S. and Nazar M. (2019): *Natural convection flow of second grade fluid with thermal radiation and damped thermal flux between vertical channels*.– Alexandria Engineering Journal, vol.58, No.4, pp.1119-1125.

[16] Thebault M., Giroux-Julien S., Timchenko V., Ménézo C. and Reizes J. (2019): *Detailed flow development and indicators of transition in a natural convection flow in a vertical channel*.– International Journal of Heat and Mass Transfer, vol.143, Article No.118502, https://doi.org/10.1016/j.ijheatmasstransfer.2019.118502.

[17] Bratsos A.G. (1998): *The solution of the Boussinesq equation using the method of lines*.– Computer Methods in Applied Mechanics and Engineering, vol.157, No.1-2, pp.33-44.

[18] Sarker P. and Chakravarty U.K. (2020): *A generalization of the method of lines for the numerical solution of coupled, forced vibration of beams*.– Mathematics and Computers in Simulation, vol.170, pp.115-142.

[19] Howell J. R., Menguc M. P. and Siegel R. (2015): *Thermal radiation heat transfer*.– CRC Press.

[20] Hibbeler R. C. (2020): *Fluid Mechanics*.– Pearson Education Limited, United Kingdom.

[21] Bejan A. (2013): *Convection Heat Transfer*– Fourth Edition, John Wiley & Sons.

[22] MATLAB, Software Package (2017): *Mathworks*.– Version R2017a, Massachusetts, Natick.

[23] Causley M., Christlieb A. and Wolf E. (2017): *Method of lines transpose: an efficient unconditionally stable solver for wave propagation*.– Journal of Scientific Computing, vol.70, No.2, pp.896-921.

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