DE GRUYTER
OPEN

Arch. Min. Sci., Vol. 59 (2014), No 2, p. 553–562

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2014-0040

AGNIESZKA SUROWIAK*, MARIAN BROŻEK*

METHODOLOGY OF CALCULATION THE TERMINAL SETTLING VELOCITY DISTRIBUTION OF IRREGULAR PARTICLES FOR HIGH VALUES OF THE REYNOLD'S NUMBER

METODOLOGIA WYLICZANIA ROZKŁADU GRANICZNEJ PRĘDKOŚCI OPADANIA ZIAREN NIEREGULARNYCH DLA WYSOKICH WARTOŚCI LICZB REYNOLDSA

Settling velocity of particles, which is the main parameter of jig separation, is affected by physical (density) and the geometrical properties (size and shape) of particles. The authors worked out a calculation algorithm of particles settling velocity distribution for irregular particles assuming that the density of particles, their size and shape constitute independent random variables of fixed distributions. Applying theorems of probability, concerning distributions function of random variables, the authors present general formula of probability density function of settling velocity irregular particles for the turbulent motion. The distributions of settling velocity of irregular particles were calculated utilizing industrial sample. The measurements were executed and the histograms of distributions of volume and dynamic shape coefficient, were drawn. The separation accuracy was measured by the change of process imperfection of irregular particles in relation to spherical ones, resulting from the distribution of particles settling velocity.

Keywords: spherical particles, irregular particles, settling velocity, imperfection, jigging, random variable distribution function

Na prędkość opadania ziaren będącą cechą rozdziału w procesie separacji w osadzarce wpływają właściwości fizyczne (gęstość) i geometryczne (wielkość i kształt) ziaren. Autorzy przedstawili algorytm wyliczania rozkładu prędkości opadania ziaren nieregularnych przy założeniu, że gęstość ziaren, ich wielkość i kształt są zmiennymi losowymi niezależnymi o określonych rozkładach. Wykorzystując twierdzenia rachunku prawdopodobieństwa odnoszące się do rozkładów funkcji zmiennych losowych przedstawiono ogólny wzór na funkcję gęstości rozkładu prędkości opadania ziaren nieregularnych w warunkach ruchu turbulentnego. Rozkład prędkości opadania ziaren nieregularnych wyliczono w oparciu o eksperyment przemysłowy polegający na opróbowaniu osadzarki miałowej. Na podstawie wykonanych pomiarów wykreślono histogramy rozkładów objętościowego i dynamicznego współczynnika kształtu. Korzystając z rozkładu prędkości opadania ziaren została wyliczona dokładność rozdziału jako zmiana imperfekcji procesowej ziaren nieregularnych w stosunku do ziaren sferycznych.

Słowa kluczowe: ziarna sferyczne, ziarna nieregularne prędkość opadania, im perfekcja wzbogacznie w osadzarce, rozkład zmiennej losowej.

* AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, DEPARTMENT OF MINERAL PROCESSING AND ENVIRONMENT PROTECTION, AL. MICKIEWICZA 30, 30-065 KRAKÓW, POLAND. E-mail: asur@agh.edu.pl, brozek@agh.edu.pl

1. Introduction

Beneficiation in jigs occurs during vertical pulsating motion of the medium containing particles. After some time of such motion, the particles are stratified into groups differing in physical (density) and geometrical (particle size and shape coefficient) properties. According to Mayer's potential theory (Mayer, 1964) the separation runs towards minimizing the potential energy of this unequivocal particles system. Such a stratification means that individual layers, during ideal separation, contain particles of the same value of terminal settling velocity. Thus, it can be stated that terminal settling velocity is this property (separation feature) according to which, for ideal separation, without any scattering, the stratification of the material into respective subsets of the same value of separation feature occurs.

In industrial jigging processes, as a result of particles dispersion being the effect of interactions between particles, the fouling of respective subsets with particles of neighbouring layers occurs, for which the values of settling velocity are different. This phenomenon is characterized by the probable error and imperfection, parameters which are measures the separation efficiency. Since the settling velocity of an irregular particle depends on the particle size, shape and their density, this paper presents the method of evaluation of the influence of these parameters on separation efficiency. This was done by comparing the imperfection values by irregular particles separation in ratio to its value by spherical particles separation. To achieve this, the authors calculated the distributions of settling velocities of spherical and irregular particles according to the empirical data and determined the influence of particle shape on the separation efficiency during the jigging process.

2. Methodology

2.1. Terminal settling velocity of irregular particle as a random variable

Terminal settling velocity of a particle for the large values of Reynolds number ($\mathrm{Re} > 5 \cdot 10^2$) is given by the formula (1) (Brożek & Surowiak, 2010):

$$v = \sqrt{\frac{2(\rho - \rho_o)Vg}{\psi_z \rho_o S}} \tag{1}$$

where:

v — terminal settling velocity of a particle,
V — particle volume,
ψ_z — drag coefficient for a particle,
S — particle projection area on the plane perpendicular to the motion direction,
ρ — particle density,
ρ_o — medium density,
g — acceleration due to gravity.

After expressing the volume and projection area of particle by the projective diameter (formulae (2) and (3)) (Brożek & Surowiak, 2007)

$$V = k_1 \frac{\pi d_p^3}{6} \tag{2}$$

$$S = \frac{\pi d_p^2}{4} \tag{3}$$

and including value of drag coefficient for the sphere $\psi_s = 0.46$, as well relation $\psi_z = 0.46k_2$ (Abraham, 1970; Concha & Almendra, 1979) the formula for the terminal settling velocity of irregular particle is obtained:

$$v = 5.33\sqrt{x}\sqrt{d_p}\sqrt{\left(\frac{k_1}{k_2}\right)} \tag{4}$$

where:

d_p — projective diameter of particle,
k_1 — volume shape coefficient of particle,
k_2 — dynamic shape coefficient of particle and
x — reduced particle density, given by formula (5):

$$x = \frac{\rho - \rho_o}{\rho_o} \tag{5}$$

Occurring on the right side of formula (4): reduced particle density, projective diameter, as well volumetric and dynamic particle shape coefficients are the random variables characterizing by the particular distributions. Because of that, also the value on the left side of the equation (terminal settling velocity) must be the random variable of the distribution function resulting from theorems of random variables product and quotient distribution functions (Gerstenkorn & Śródka, 1972). The next chapter presents the algorithm of calculating settling velocity distribution function in sample of polisized irregular particles. The purpose of that is to get the formula for settling velocity in form of the product of two random variables, $W = S \cdot U$ (Surowiak, 2014). In this case statistical density function of random variable W is determined by formula (Gerstenkorn & Śródka, 1972):

$$h(w) = \int_{w_{\min}}^{w_{\max}} \frac{1}{s} f_1(s) f_2\left(\frac{w}{s}\right) ds \tag{6}$$

2.2. Distribution of free settling velocity for a sample of polisized irregular particles

Let R, X, D_p, K_1, and K_2 mean random variables of particle density, reduced density, particle projective diameter, particle volumetric shape coefficient and particle dynamic shape coefficient, respectively and $Y_1 = \sqrt{X}$, $Y_2 = 5.33Y_1$, $U_1 = \sqrt{D_p}$, $Z = \frac{K_1}{K_2}$, $A = \sqrt{Z}$, $U_2 = U_1 \cdot A$. Then, on the basis of equation (4) settling velocity is the product of two random variables: variable Y_2

dependable on particle density distribution in a sample and variable U_2, which depends on the geometric properties (Niedoba, 2013a, 2013b):

$$V = Y_2 \cdot U_2 \tag{7}$$

Thereby distribution of particles settling velocity in a polisized irregular particles sample is given by the formula (Gerstenkorn & Śródka, 1972):

$$h(v) = \int_{y_{2\min}}^{y_{2\max}} f_2(y_2) g_2\left(\frac{v}{y_2}\right)\frac{1}{y_2} dy_2 \tag{8}$$

where: $y_{2\min}$ and $y_{2\max}$ are the minimal and maximal value at the random variable Y_2 respectively.

3. Experiment

To calculate the settling velocity distribution function on the basis of the algorithm presented above, the empirical particle density, projective diameter and shape coefficients distribution functions are needed, originated from the jig feed sample. These distribution functions were obtained by the following methods: density distribution $f_\rho(\rho)$ by float and sink analysis; projective diameter $g(d_p)$ and dynamic shape coefficient $w_2(k_2)$ distributions by the image analysis method and volumetric shape coefficient distribution $w_1(k_1)$ by volumetric method based on the measurement of individual particles density by picnometer and calculating its volume (Brożek & Surowiak, 2007). To calculate the dynamic shape coefficient and projective diameter, the particles photos were taken by digital camera in the most stable position. Next, by applying the image analysis computer program, the projective fields and circumferences for individual particles were calculated. The sphericity coefficients ϕ and projective diameters d_p were calculated. The dynamic shape coefficient k_2 was calculated from the statistical equation (17), given by Ganser (1993):

$$k_2 = 10^{1.8148(-\log\phi)^{0.5743}} \tag{9}$$

4. Results and discussion

4.1. Measurement results

The distribution function of particle density, according to the dispersive particle model (Brożek, 1995) is well approximated by Weibull distribution:

$$f_\rho(\rho) = \frac{354}{15.526}\rho^{2.54}\exp\left[-\left(\frac{\rho}{2.17}\right)^{3.54}\right] \tag{10}$$

The particle size distribution function of fine coal is also well approximated by Weibull (RRB) distribution function. So, the frequency function of random variable D_p is given by the formula (11).

$$g(d_p) = \frac{152}{63.01} d_p^{0.52} \exp\left[-\left(\frac{d_p}{15.27}\right)^{1.52}\right] \tag{11}$$

The Weibull distribution function of particle density and cumulative distribution functions of particles screen and projective diameters in the feed were presented in the paper Brożek and Surowiak (2008).

The investigations of other authors indicate that the distributions of shape coefficients are the gamma type ones (Hodenberg, 1998; Stark & Muller, 2005). Because of this, Rayleigh's and Weibull's distributions were fitted to these histograms. The statistical evaluation of adequacy of theoretical distributions with empirical data was the same for both distribution functions types and, consequently, Rayleigh's distributions of shape coefficients were accepted and applied in further considerations. From fitting to empirical distribution functions, which histograms are presented on figs. 1 and 2, the formulas (12) and (13) were derived for shape coefficients frequency functions:

$$w_1(k_1) = 22\, k_1 \exp(-11 k_1^2) \tag{12}$$

$$w_2(k_2) = 0.03 k_2 \exp(-0.015 k_2^2) \tag{13}$$

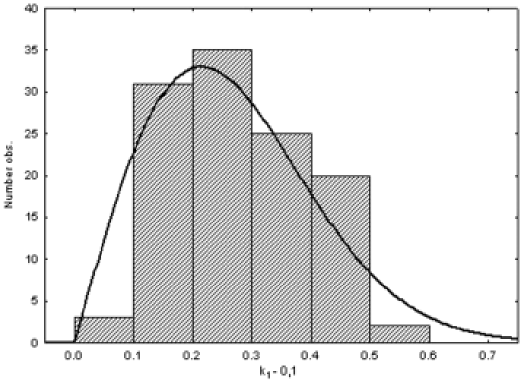

Fig. 1. Histogram of distribution of volumetric shape coefficient k_1

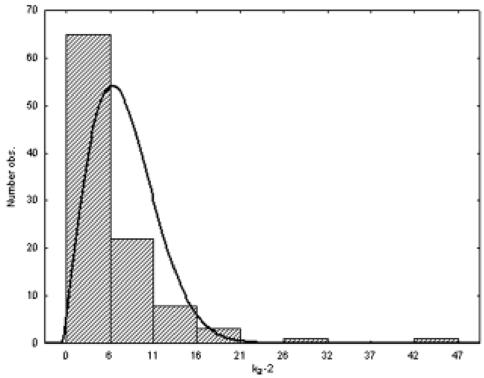

Fig. 2. Histogram of distribution of dynamic shape coefficient k_2

4.2. Distribution of irregular particles settling velocity

Having the empirical distributions of physical and geometrical features for jig feed sample, the frequency function of settling velocity was calculated on the basis of algorithm given by equation (8). The frequency functions for the individual random variables are as follows (Gerstenkorn & Śródka, 1972):

a) frequency function of variable X:

$$f(x) = 84.3(x+1)^{2.54} \exp\left[-\left(\frac{x+1}{1.5}\right)^{3.54}\right] \tag{14}$$

b) frequency function of variable Y_1:

$$f_1(y_1) = 172.7\, y_1 \left(y_1^2+1\right)^{2.54} \exp\left[-\left(\frac{y_1^2+1}{1.49}\right)^{3.54}\right] \tag{15}$$

c) frequency function of variable Y_2:

$$f_2\left(y_2\right) = 1.24\cdot 10^{-3}\, y_2 \left(y_2^2+28.41\right)^{2.54} \exp\left[-\left(\frac{y_2^2+28.41}{42.3}\right)^{3.54}\right] \tag{16}$$

d) frequency function of variable U_1:

$$g_1\left(u_1\right) = 4.86 u_1^{2.04} \exp\left[-\left(\frac{u_1^2}{15.2}\right)^{1.52}\right] \tag{17}$$

e) frequency function of variable Z:

$$p_1\left(z\right) = \frac{0.33\, z}{\left(0.015+11z^2\right)^2} \tag{18}$$

f) frequency function of variable A:

$$p_2\left(a\right) = \frac{0.66\, a^3}{\left(0.015+11a^4\right)^2} \tag{19}$$

g) frequency function of variable U_2:

$$g_2\left(u_2\right) = 115565.1 \int_0^{0.18} u_1^{3.04} \exp\left[-\left(\frac{u_1^2}{0.015}\right)^{1.52}\right] \frac{u_2^3\, u_1^5}{\left(0.015 u_1^4 + 11 u_2^4\right)^2}\, du_1 \tag{20}$$

h) frequency function of settling velocity of irregular particles according to the formula (8)

$$h(v) = 28.08 \int_{1.22}^{7.9} \left\{ \begin{array}{l} (y_2^2+28.41)^{2.54} \exp\left[-\left(\dfrac{y_2^2+28.41}{57,68}\right)^{3.54}\right] \times \\ \displaystyle\int_0^{0.18} u_1^{3.04} \exp\left[-\left(\frac{u_1^2}{0.015}\right)^{1.52}\right] \frac{v^3 y_2^5 u_1^5}{(0.015 y_2^4 u_1^4 + 11 v^4)^2}\, du_1 \end{array} \right\} dy_2 \tag{21}$$

Frequency function $h(v)$ was calculated numerically by means of algorithm created in C++ computer language. Figures 3 and 4 present the frequency and cumulative distribution function of settling velocity, respectively, in the polisized sample of irregular particles in the feed.

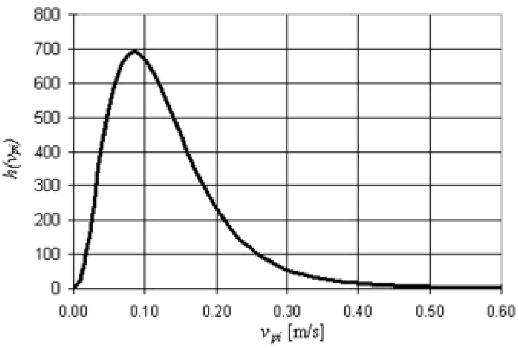

Fig. 3. Frequency function of settling velocity of irregular particles in the feed

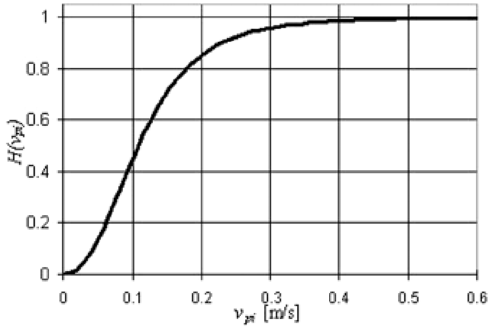

Fig. 4. Cumulative distribution function of settling velocity of irregular particles in the feed

4.3. Distribution of settling velocity of spherical particles in the feed

During calculation of settling velocity distribution function of spherical particles it was assumed that particles are spheres whose diameters are equal to the projective diameter. In other words, determining this distribution is a simulation of a possible particles settling velocity distribution in the analyzed feed sample if particles features with spherical shape. For the spherical particles the shape coefficients $k_1 = k_2 = 1$ and settling velocity is the product of two random variables Y_2 and U_1. Consequently, frequency function of settling velocity of spherical particles in the feed is as follows:

$$h(v_{ps}) = 42.56 \int_{1.2}^{7.7} (y_2^2 + 28.41)^{2.68} \exp\left[-\left(\frac{y_2^2 + 28.41}{57.68}\right)^{3.54}\right] \left(\frac{v_{ps}}{y_2}\right)^{2.04} \times \\ \times \exp\left[-\left(\frac{v_{ps}^2}{0.015 y_2^2}\right)^{1.52}\right] dy_2 \tag{22}$$

Frequency function $h(v_{ps})$ was calculated numerically by means of algorithm created in C++ computer language. Figures 5 and 6 present simulated frequency and cumulative distribution function of settling velocity, respectively, in the sample of spherical particles.

Comparing the distributions of irregular particles (Fig. 3) and spherical particles settling velocity in the feed (Fig. 5) it can be concluded that irregular particles shape reduced three times the maximum value of settling velocity in comparison with spherical particles.

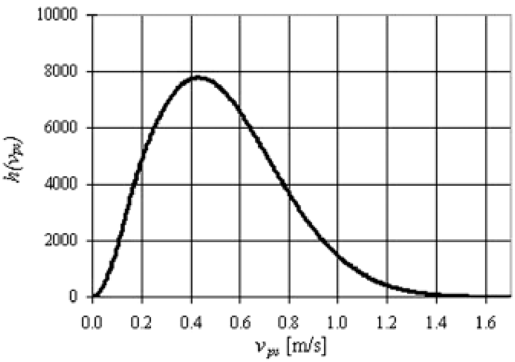

Fig. 5. Frequency function of spherical particles settling velocity in the feed

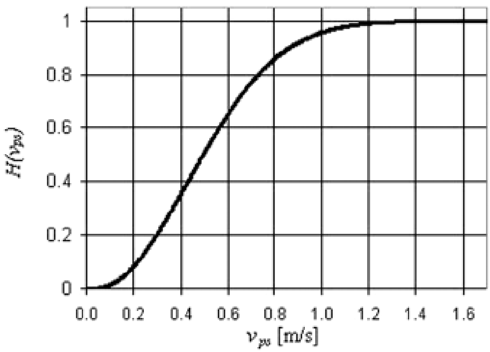

Fig. 6. Cumulative distribution function of spherical particles settling velocity in the feed

4.4. Influence of particles irregularity on separation efficiency

For evaluating the influence of the difference in settling velocity distributions, occurring from the irregular shape of particles, on separation efficiency, the following dependencies were applied for irregular and spherical particles:

$$\Delta I_i = I_i - I_o = c\,\mathrm{tg}\alpha_n \tag{23}$$

$$\Delta I_s = I_s - I_o = c\,\mathrm{tg}\alpha_s \tag{24}$$

where:

- ΔI_i and ΔI_s — change of process imperfection by beneficiation of irregular and spherical particles, respectively;
- I_i and I_s — process imperfection for irregular and spherical particles;
- α_i and α_s — inclination angle of tangent to particles settling velocity distribution function for irregular and spherical particles, respectively with the settling velocity equal to partition velocity calculated from partition curve;
- I_o and c — constants characteristic for the certain type of the jig.

The dependencies (23) and (24) are analogical to the dependency being applied for density as the separation parameter (Samylin et al., 1976). The quotient of the formulae (23) and (24) gives:

$$\frac{\Delta I_i}{\Delta I_s} = \frac{\mathrm{tg}\alpha_i}{\mathrm{tg}\alpha_s} \tag{25}$$

For the division of the tested coal sample in two-product jig manufactured by Allmineral, the partition curve is given by the formula (26) (Surowiak, 2007).

$$T(v) = \phi_o\left(\frac{v - 0.142}{0.045}\right) \tag{26}$$

where: ϕ_o — Laplace function.

The value of the partition velocity is equal $v_r = 0.142$ m/s. For this settling velocity value, the inclination of the curve $H(v)$ in the point $v = 0.142$ m/s is equal tg$\alpha_n = 437.5$ and $H_r(v_r = 0.142) = 66.87\%$ (Fig. 4). For the spherical particles (Fig. 6), for $H_s = 66.87\%$ the velocity $v_s = 0.614$ m/s (Surowiak & Brożek, 2014). The inclination of distribution function in this point is equal to tg$\alpha_s = 140$. From the equation (25), the ratio of imperfection changes for irregular particles and spherical ones separations is equal to:

$$\frac{\Delta I_i}{\Delta I_s} = \frac{437.5}{140} = 3.4 \tag{27}$$

Such significant difference between imperfection changes for irregular particles and spherical ones occurs from nearly three times narrower variation range for irregular particles settling velocity comparing to spherical ones.

5. Conclusions

Assuming that the settling velocity is the random variable being the function of random variables representing physical and geometrical particle properties and applying the probability theorems connected with random variables functions, the equations for distribution functions of terminal settling velocity for irregular and spherical particles were derived for turbulent motion occurring during jigging process.

The character of shape coefficient and particle size distribution functions influences on irregular particles separation efficiency. The irregular particles shape makes narrower the range of variation of settling velocity lowering the difference between settling velocities in comparison with spherical particles. The result of this is lowering of irregular particles separation adequacy, what is determined by the higher value of imperfection.

Acknowledgements

This work was performed as a part of the University of Science and Technology Research Program No.11.11.100.276 project.

References

Abraham F., 1970. *Functional dependence of drag coefficient of a sphere on Reynolds number*. Phys. Fluids, 13, p. 2194-2195.

Brożek M., 1995. *The distribution of selected physical properties in the crushed material*. Arch. Min. Sci., Vol. 40, No 1, p. 83-100.

Brożek M., Surowiak, A., 2007*, Effect of particle shape on jig separation efficiency*. Physicochemical Problems of Mineral Processing, 41, p. 397-413.

Brożek M., Surowiak A., 2008. *Distribution of terminal settling velocity of spherical particles for the condition of turbulent motion*. XXIV International Mineral Processing Congress, Beijing, China, p. 310-315.

Brożek M., Surowiak A., 2010. *Argument of distribution at enrichment in the jig*, Arch. Min. Sci., Vol. 55, No 1, p. 21-40.

Concha F., Almendra E.R., 1979. *Settling velocities of particulate systems, 1 Settling velocities of individual spherical particles*. Int. J. Mineral Processing, 5, p. 349-367.

Ganser G.H., 1993. *A rational approach to drag prediction of spherical and nonspherical particles*, Powder Technology, 77, p. 143-152.

Gerstenkorn T., Srodka T., 1972. *Theory of combinations and probability calculus*. PWN, Warsaw, (in Polish).

Hodenberg M., 1998. *Gravimetric and optical particle analysis of mixed particle samples*. Aufbereitungs Technik, 39, p. 461-466.

Mayer F.W., 1964. *Fundamentals of a potential theory of the jigging process*. in Proc. VII IMPC, New York, pp. 75-86.

Niedoba T., 2013a. *Statistical analysis of the relationship between particle size and particle density of raw coal*. *Physicochemical Problems of Mineral Processing,* 49 (1), p. 175-188.

Niedoba T., 2013b. *Elementy metodologii stosowania dwu- i wielowymiarowych rozkładów właściwości materiałów uziarnionych do opisu wzbogacania węgli*. (*Methological elements of applying two- and multi-dimensional distributions of grained materials properties to coal beneficiation*). Gospodarka Surowcami Mineralnymi – Mineral Resources Management 29(2), s. 155-172 (in Polish).

Samylin N.A., Zolotko A.A., Poczinok V.V., 1976. *Jigging*. Izd. Nedra, Moscow, (in Russian).

Stark U., Muller A., 2005. *Effective methods for measurement of particle size and shape*. Aufbereitungs Technik, 45, p. 6-16.

Surowiak A., 2007. *Influence of particles physical and geometrical properties distribution on the separation efficiency of jigging process on the example of coal*. Ph, D Thesis, Main Library of University of Science and Technology, Cracow, Poland, (in Polish).

Surowiak A., Brozek M., 2014. *Methodology of calculation the terminal settling velocity distribution of spherical for high values of the Reynold's number*. Arch. Min. Sci., Vol. 59, No 1, p. 267-280.

Surowiak A., 2014. *Influence of particle density distributions of their settling velocity for narrow size fractions*. Mineral Resources Management, 30, (1), p. 105-122 (in Polish).

Received: 18 September 2013