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Modeling of Withdrawals from Selected ATMs of the "Euronet" Network

1. Introduction

The financial sector in any country deals with large amounts of data on customers. This data is collected for use in the improvement of their services. The detection of tendencies and patterns of customer behavior reflected in the data is very important in respect for marketing, profit maximization and customer value management.

In particular a better understanding of consumer habits with respect to ATM withdrawals is crucial for the logistic management of this type of service and may assist in other studies on the topic. One of the most important tasks in the exploration of such data is to use it to manage risk by means of econometric models which are used for forecasts of the future behavior of customers.

This contribution is concerned with forecasts of the amounts of money that individuals withdraw from ATMs (automated teller machines). These forecasts may be useful in particular for ATM replenishment strategies.

Investigations focusing on payment systems have attracted scholarly attention in recent decades. This area of research combine monetary economics and banking theory (Takala, and Viren [28]). However, data on ATM transactions is far from being fully explored. Databases on ATM transactions exhibit a large coverage and are dependent on time. The reason is that there is typically become available a delay in availability, usually a few days after the reference period. However, this information is at present not being taken into account by public statistical institutions like the Central Statistical Office in Poland.

It is hard to find in the financial literature advanced econometric models for ATM withdrawal amounts especially those taking into account calendar effects.

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In existing contributions used simple linear models are usual, which take into account typical features of ATM withdrawals like calendar effects or seasonality. Although more advanced models often fit the ATM data better, they are inappropriate as forecasting tools. The reason is that estimation process of parameters of more complex models is time consuming process.

That is why in practice it is impossible to conduct fast forecasts by means of complex models in the case of thousands of ATMs. In these situations simple linear models may appear more desirable.

The new idea in this paper is the application of switch SARIMA models to ATM data. Since switch ARIMA models fit the data well, especially ex post forecasts, one can expect good performance of this kind of model in the forecasting of ATM withdrawals (forecasts ex ante).

The remaining part of the paper is organized as follows. Section 2 presents a literature overview. The following section shows the data and a description of it. In the fourth section models applied are given. In the next section empirical results are cited. The sixth section concludes the paper.

2. Literature overview

Calendar effects are an important feature of much financial and economic data. In the light of the results on ATM withdrawals presented in previous papers by Gurgul and Suder [17], [18] seasonality and calendar effects are also exhibited by this kind of data. Therefore, the literature review and the models in the empirical part of this contribution are determined by the results reported in these papers. Calendar effects being taken into account by dummies and the SARIMA model, the fitting of the mixture of normal distributions and the use of Markov switching models were the main focus in these papers. The literature overview below refers mainly to contributions dealing with these issues.

Most economic time series are directly or indirectly reflected in daily activity. They are usually recorded on a daily, monthly, quarterly basis or some other period. Thus, economic variables may be affected by daily calendar effects, which started to be noted in the late seventies e.g. Cleveland and Devlin [7] and Liu [22]. The number of working days and its link with seasonal effects are important examples. They can be considered as easily anticipated effects which affect the short-term movements of time series. These periodic fluctuations must be adequately detected and modeled in order to better analyze other non-periodic properties (Young, [29]).

The correction for calendar and seasonal effects may help suit this information for use as input to improve the performance of short-term macroeconomic forecasts (see [15]), as a tool for now casting private consumption (see [9]) or as a timely indicator of retail trade statistics (see [6]).This is why analyses concerning economic developments are adjusted for seasonal and working day effects.

Besides the number of working days, the day of the week, the week of the month, other calendar effects such as public holidays or religious events may also affect time series. For the particular series under study, calendar effects are of particular importance given that cash withdrawals vary over time and are often overlaid with additional factors such as paydays, holidays and seasonal demand, and are subject to trends and generally follow weekly, monthly and annual cycles. This type of calendar effect is mentioned in papers such as Simutis et al. [26] and is related to the logistic management of the ATM system. Institutional Change in the Payments System is addressed by Schmitz and Wood (2006).

One of the earliest statistical analyses is that of credit card transactions published by Hand and Blunt [20]. In the financial literature there has been some interest in establishing the relationship between ATMs and the demand for cash. Amromin and Chakravorti [1] applied a linear regression and performed a panel data analysis in order to compare countries. The authors focused on the growth in debit card point-of-sale transactions. Boeschoten [3] and Snellman and Viren [27] pursued similar goals. However, they derived their conclusions on the basis of models of individual customer behavior. Boeschoten [3] investigated data from a survey of Dutch consumers between 1990 and 1994 by means of a deterministic inventory model. The authors assumed that individuals maintain a certain level of cash. It is used at a constant rate. They replenish their stock when it falls below a certain threshold. The contributors found that ATM users typically had a lower stock than non-users because the cost to them of obtaining cash was lower. The relationship between the cost of obtaining cash and the number of ATMs was modeled in Snellman and Viren [27] as a deterministic optimization problem, where costs were assumed to be proportional to distance from ATMs. Although these economic papers used models for customer withdrawal amounts and times, they involve simplifications to enable conclusions about macro demand for cash and can be improved when making predictions at an individual level.

Findley and Monsell [10] dealt with the necessity of taking into account different days of the week. Monthly activity can depend on the numbers of days in a particular month and the kinds of days. There can be a significant effect of the Easter season, since these holidays are mobile and scheduled from March to April. Therefore in some years these holidays are in the first quarter and some-times in the second. Different calendar effects were studied in the contribution by Simutiset al. [26]in the context of ATM logistics.

According to Findley et al. [11] taking into account calendar effects is of great importance, because it improves the features of the respective time series and leads to better results in modeling.

The literature contains different methods for the detection, estimation and adjustment of the time series which exhibit calendar effects. Cleveland and Devlin [7], [8]established, on the basis of a large sample, a distribution of withdrawal frequencies for monthly time series. They found the main frequencies for these time series. Findley and Soukup [12], [13], [14] analysed the applicability of distributions in samples in order to check the day effect. McElroy and Holland [23] suggested a nonparametric test for distribution of maxima in samples.

Withdrawals from ATMs exhibit seasonality patterns, which can be analysed by e.g. X-12-ARIMA and Tramo-Seats. This problem is complex because of mobile holidays like Easter, Ramadan or the Chinese New Year (see [19]).

Brentnall et al. [4] developed a random-effects point process model for automated teller machine withdrawals. They discussed estimation, prediction and computational issues. The authors stressed that their model may be used to forecast the behavior of an individual and to assess when changes in the pattern of individual behavior have occurred and as a description of behavior for a portfolio of accounts.

In a more recent contribution Brentnall et al. [5] took a multinomial distribution for the distribution of amounts and random effects were modeled by a Dirichlet distribution or the empirical distribution of individual maximum likelihood. The next model derived by the authors extended the multinomial distribution by incorporating a form of serial dependence and using an empirical distribution for random effects. The contributors, on the basis of a sample of 5000 UK high-street bank accounts, found that the greatest benefit from the models was for accounts with a small number of past transactions. They also found that a little information may be lost by binning and that the Dirichlet distribution might overestimate the probability of previously unobserved withdrawal amounts. The authors showed that the empirical distribution of random effects performed well because there were a large number of individual accounts.

Kufel [21]used two methods for the detection of seasonality: regression based on dummies and harmonic analysis. His results based on withdrawals time series for ATMs in Torun showed yearly, monthly, weekly and daily seasonality. The calendar effect is especially important in the case of daily withdrawals from an ATM, because this issue has not yet been dealt with sufficiently in the scientific literature (see [24]).

In the next section we will describe and conduct a preliminary analysis of the dataset properties.

3. Dataset and its properties

Our empirical analysis was based on a dataset containing daily withdrawals from 293 ATMs located in the province of Małopolska and Podkarpacie. For each ATM we examined time series covering a period no shorter than one year. In general, the dataset covers the period January 2008 – March 2012. We will also consider the issue of location when formulating final conclusions of this study. The number of examined ATMs with respect to their location are presented in Table 1:

Table 1

The number of examined ATMs with respect to their location

Location	Number
Bank Branch	84
Entertainment Centre	6
Supermarket	40
Hotel	6
Gas Station	25
Shop	49
Shopping Center	42
Transport	4
Other	37
Total	293

Source: own elaboration

The detailed results of empirical computations will be presented for four ATMs, which were chosen according to location criteria, i.e. each ATM has a different location. In Table 2 a short description of the location and some basic descriptive statistics for the ATMs chosen are presented, while Figures 1–4 present the plots of time series of daily withdrawals from these ATMs. Examining ATMs from different locations may provide some help in answering the question whether the statistical properties of the time series of withdrawals depend on the type of location considered.

Table 2

Description of location and basic descriptive statistics for four chosen ATMs

ATM number	1	2	3	4
province	Małopolska	Podkarpacie	Podkarpacie	Małopolska
City	Zakopane	Rzeszów	Przemyśl	Kraków
Location type	Bank branch	Shopping center	Gas station	Supermarket
Sample size	1552	1552	1552	1552
Mean	39261.9	84856.5	31996.9	91076.2
Median	40350	88050	31375	90850
Standard deviation	17599.0	34510.3	16181.5	33314.4
Coefficient of variation	44.82%	40.67%	50.57%	36.58%
Minimum	200	0	100	0
Maximum	108250	174450	88800	235350
Skewness	0.0754	–0.1457	0.3911	0.1045
Kurtosis	–0.1042	–0.5691	–0.1040	0.3197

Source: own elaboration

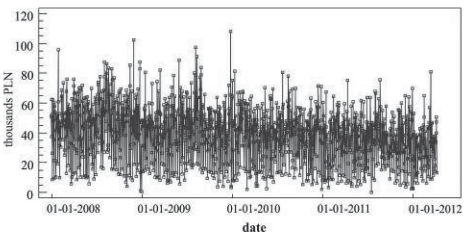

Figure 1. Accumulated daily withdrawals from ATM 1

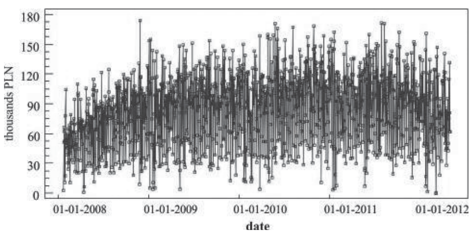

Figure 2. Accumulated daily withdrawals from ATM 2

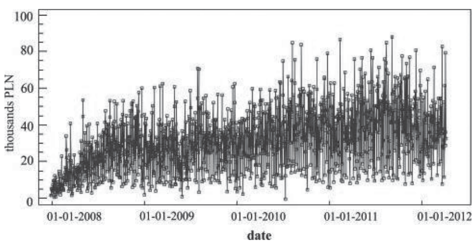

Figure 3. Accumulated daily withdrawals from ATM 3

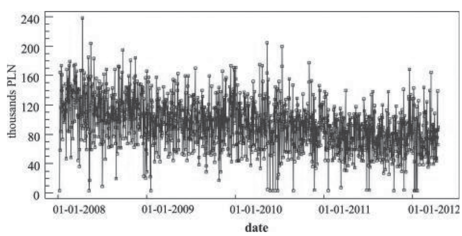

Figure 4. Accumulated daily withdrawals from ATM 4

Figures 1–4 and table 2 provide evidence to claim that the behavior of these time series of daily withdrawals as well as the values of computed descriptive statistics are significantly varied.

In order to choose a suitable econometric model to fit the time series presented, one should analyze both the statistical properties of the data as well as the properties of a possible theoretical distribution.

In the next part of this paper we will present the results of the analysis of the basic econometric properties of the data, including seasonality, stationarity and autocorrelation.

In order to examine the issue of seasonality a spectral density estimator (periodogram) was applied. Figures 5 and 6 present periodograms for ATM 1 and ATM 4.

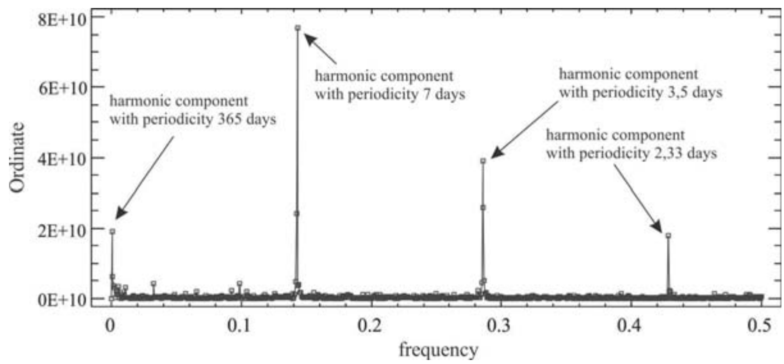

Figure 5. Periodogram based on ATM 1 data

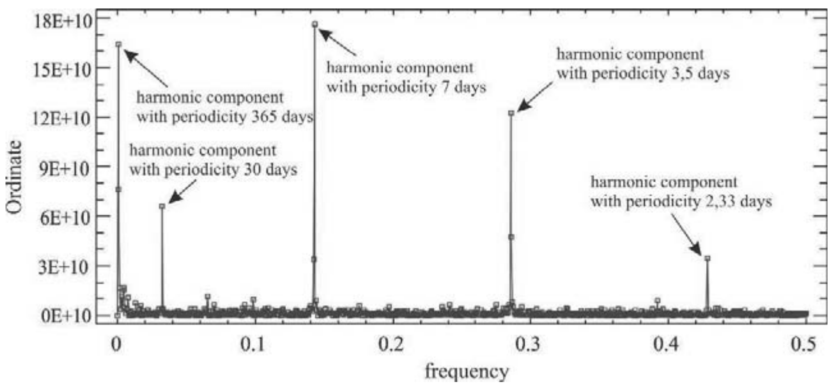

Figure 6. Periodogram based on ATM 4 data

One can easily see that in the case of both ATMs the periodograms provide evidence which supports the existence of annual and one-week cycles. For ATM 4, we can also see a rise in spectral density for the frequency of 0,033 which corresponds to a 30-day cycle, i.e. a monthly cycle. On both periodograms one can also see shocks for frequency 0.285 and 0.428, which are called harmonic shocks and support the existence of 3.5 and 2.33-day cycles respectively. The one-week cycle is a multiple of these two cycles.

One-week and annual cycles were found for all 293 ATMs examined. In addition, for 130 ATMs a monthly cycle was also found. In general, these 130 ATMs were usually located in shopping centers and supermarkets. Thus, seasonality should be taken into consideration when building an econometric model of daily withdrawals.

Another important feature of this data are the characteristics of the autocorrelation (ACF) and partial autocorrelation (PACF) functions. In order to build a model (in particular to establish the lag parameter in the case of autoregressive models), which would fit well to the daily data on ATM withdrawals, one should analyze the plots of ACF and PACF presented in Figures 7–8 (ACF for ATM 2 and ATM3) and Figures 9–10 (PACF for ATM 2 and ATM3). In each figure a suitable confidence interval was also marked.

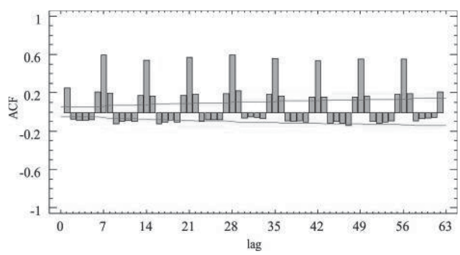

Figure 7. Plot of ACF for daily withdrawals from ATM 2

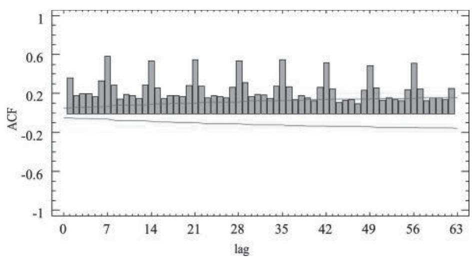

Figure 8. Plot of ACF for daily withdrawals from ATM 3

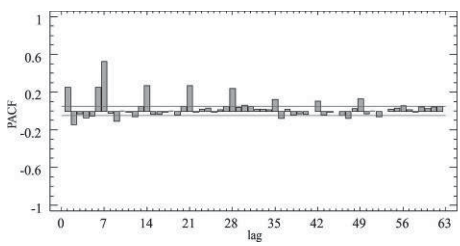

Figure 9. Plot of PACF for daily withdrawals from ATM 2

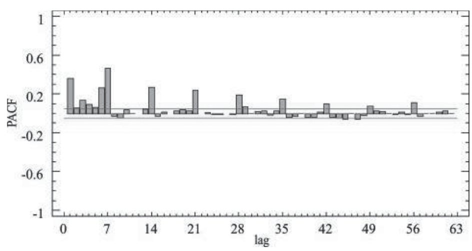

Figure 10. Plot of PACF for daily withdrawals from ATM 3

The plots of ACF and PACF for both ATMs confirm the existence of one-week cycles (ACF and PACF reach their highest values at multiples of 7). In addition the fact that both ACF and PACF are slowly decreasing suggests that these time series may be driven by e.g. an ARIMA process with specific lags. The plots of ACF and PACF for the remaining ATMs are very similar.

In the next step of our analysis we checked the stationarity of the time series. We used unit root tests available in Gretl (ADF, ADF-GLS and KPSS). Table 3 presents the results of unit root testing for all 4 ATMs:

Table 3

Results of stationarity analysis

Test type	**ADF**		**ADF-GLS**	**KPSS**
ATM \ number	**with constant**	**with constant and periodical dummies**		
ATM 1	−28.13 (7.26e−051)	−8.56 (1.00e−014)	−19.87 (3.66e−040)	7.68
ATM 2	−7.571 (8.64e−012)	−8.39 (3.27e−014)	−7.522 (8.29e−013)	5.77
ATM 3	−5.546 (1.36e−006)	−5.83 (2.88e−007)	−7.883 (9.57e−014)	12.43
ATM 4	−22.90 (2.99e−051)	−17.92 (4.59e−043)	−5.918 (7.14e−09)	11.00

Source: own elaboration

The results presented in table 3 provide evidence that all four time series are stationary. A similar analysis performed for the remaining 289 ATMs confirmed that the time series of daily withdrawals of all but two of the ATMs are nonstationary. The nonstationary (and integrated in order one) time series were the withdrawal series of the ATM in the entertainment center in Oswiecim and the series of the ATM in the shopping center in Rzeszów.

Besides the properties examined so far, previous papers also dealt with the issues of fitting a specific distribution to the empirical data on daily withdrawals and testing the significance of calendar effects.

In [18] the authors proved that among all the distributions tested the mixture of three normal distributions provides the best fit to the empirical data. These results may suggest that there exist three states in the structure of withdrawals: high withdrawals, medium withdrawals and low withdrawals. This may be a consequence of the presence of the calendar effect in the time series of ATM withdrawals. In [17] the authors proved that specific days in the year, e.g. church holidays, national holidays and other red-letter days and long weekends, have a significant impact in this context.

The basic statistical and econometric analysis proved that although the time series are related to different locations, their basic properties are quite similar. Thus, one may claim that it is reasonable to use the same econometric model for the all time series under study. This would significantly simplify the procedure of building forecast models and support their practical applications.

4. Modeling of withdrawals from ATM

As mentioned in section 1, papers dealing with the issue of modeling ATM withdrawals are relatively rare, especially studies dealing with an analysis of a single ATM. Taking into account the properties of the dataset, the results presented in section 3, and the suggestions of [17] and [18], in this paper we applied a SARIMA model. This kind of model was, however, applied only to certain subseries, not the full time series available. Using the suggestions related to the issue of fitting a theoretical distribution to the empirical data which are presented in [18], from each time series two or three subseries were selected according to data division in the process of distribution fitting. For each of these subseries, an individual SARIMA model was estimated.

In addition, in this paper the results of analyzing traditional ARIMA and SARIMA models with dummy variables are also presented. Although these models are quite often used for forecasting purposes in time series analysis, their application in the case of the time series of daily withdrawals has not been described in the literature so far. Thus, it may turn out that these models provide a relatively good fit to the data, which could predispose them to various forecasting issues. On the other hand, if these models do not fit well to the empirical data it may suggest that some other directions should be followed in further research.

All examined models were estimated via maximum likelihood. The lag length was established based on information criteria, mainly the Akaike criterion. When this criterion reached similar values for different lag length, the mean absolute percentage error (MAPE) was compared in order to establish the final lag length. Moreover, for each model traditional diagnostic tests were conducted to test for the normality of distribution of the error term (Jarque–Bera test),the autocorrelation of the error term (Durbin-Watson test) and the presence of ARCH structures in the residuals (Engle ARCH test).

4.1. Empirical results

In this section we present the empirical results obtained during the estimation of individual time series models. For each of the four ATMs examined, a detailed estimation result will be presented. We will also choose the model which fits the data best. The columns in Tables 4–7 contain information on model type, AIC criterion and the mean absolute percentage error. The last three columns contain the results of diagnostic tests. In the case of models estimated on the basis of subseries, diagnostic tests of the error term were performed for each subseries separately. In addition, in Figures 11–14 the results of fitting the optimal model to the empirical data in the September 2011–November 2011 period are also presented. In some models variables E1,E2,E3,E4,E5 are related to particular calendar effects like:

1) E1 no-trading days, e.g. New Year, Easter, Christmas,
2) E2 trading days during a long weekend,
3) E3 trading days directly before a long weekend or holidays,
4) E4 trading days directly after a long weekend or holidays,
5) E5 various occasional holidays, e.g. Grandmother's Day, Grandfather's Day, Valentine's Day, Women's Day.

Table 4

Results of modeling the structure of withdrawals from ATM 1

Model type	AIC	MAPE	Error term autocorrelation	Normality test (p-value)	ARCH effect test
G1 – SARIMA(1,0,0)x(0,0,0)$_7$ G2 – SARIMA(0,0,0)x(5,0,0)$_7$ G3 – SARIMA(2,0,0)x(0,0,0)$_7$	24987.8	12.82%	no	0.120	0.1814
			no	0.778	0.4572
			no	0.329	0.0723
SARIMA(1,0,0)x(5,0,0)$_7$	33784.5	62.13%	yes	8.62E-26	0.0180
SARIMA(3,0,4)x(6,0,0)$_7$ with E1,E2,E3,E4,E5 variables	30463.1	25.94%	yes	8.78E-05	0.9304
ARIMA(4,0,2) with day-of-the-week-specific variables	33498.3	58.98%	yes	3.18E-42	0.0345
ARIMA(3,0,1)with E1,E2,E3,E4,E5variables and day-of-the-week-specific variables	33397.8	56.37%	no	4.71E-25	0.0139

Source: own elaboration

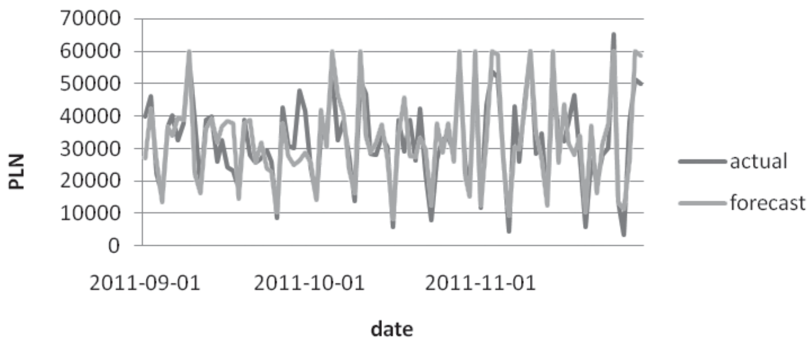

Figure 11. Results of modeling withdrawals from ATM 1 in the September 2011–November 2011 period

Table 5

Results of modeling withdrawals from ATM 2

Model type	AIC	MAPE	Error term autocor-relation	Normality test (p-value)	ARCH effect test
G1 – SARIMA(2,0,0)x(1,0,0)$_7$	31051.3	20.33%	no	0.092	0.2671
G2 – SARIMA(2,0,0)x(0,0,0)$_7$			no	0.564	0.6372
G3 – SARIMA(0,0,0)x(0,0,0)$_7$			no	0.200	0.2014
SARIMA(1,0,0)x(6,0,0)$_7$	35745.8	50.34%	yes	8.41E-25	3.90E-08
SARIMA(1,0,0)x(7,0,0)$_7$with variables E1,E2,E3,E4,E5	32184.7	27.54%	yes	7.41E-13	0.0238
ARIMA(2,0,1)with day-of-the-week-specific variables	35480.4	50.66%	yes	5.79E-34	4.27E-11
ARIMA(2,0,2) with E1, E2, E3, E4, E5 variables and day-of-the-week-specific variables	35209.8	45.39%	yes	2.95E-32	2.03E-05

Source: own elaboration

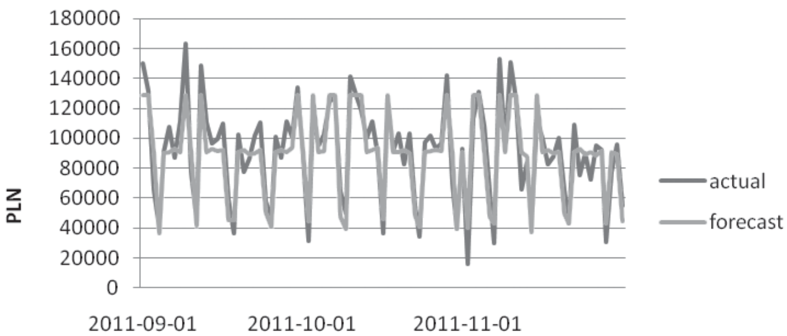

Figure 12. Results of modeling withdrawals from ATM2 in the September 2011–November 2011 period

Table 6

Results of modeling of withdrawals from ATM 3

Model type	**AIC**	**MAPE**	**Error term autocorrelation**	**Normality test (p-value)**	**ARCH effect test**
G1 – SARIMA(1,0,0)x(4,0,0)$_7$	29326.0	19.89%	no	0.370	0.0789
G2 – SARIMA(1,0,0)x(0,0,0)$_7$			no	0.841	0.4321
G3 – SARIMA(2,0,0)x(0,0,0)$_7$			no	0.120	0.3210
SARIMA(1,0,0)x(6,0,0)$_7$	33467.6	63.02%	yes	9.25E-15	1.98E-01
SARIMA(1,0,0)x(6,0,0)$_7$with E1, E2, E3 variables	30162.9	60.38%	yes	2.52E-06	0.3964
ARIMA(5,0,1)with day-of-the-week-specific variables	33188.8	67.94%	yes	1.70E-15	1.12E-02
ARIMA(3,0,1) with E1, E2, E3 variables and day-of-the-week-specific variables	33185.5	66.11%	yes	5.84E-16	3.12E-02

Source: own elaboration

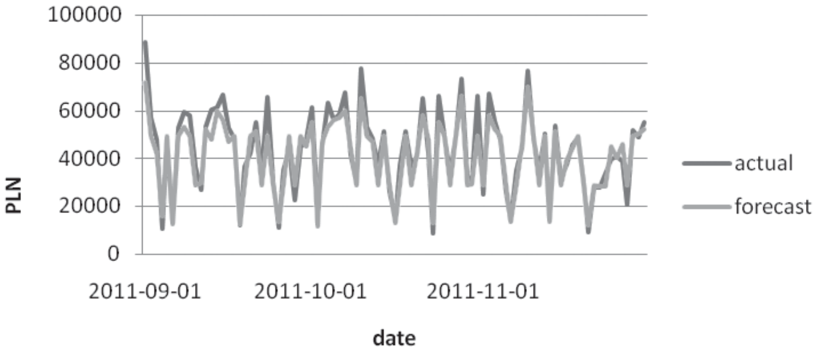

Figure 13. Results of modeling withdrawals from ATM 3 in the September 2011–November 2011 period

Table 7

Results of modeling of withdrawals from ATM 4

Model type	**AIC**	**MAPE**	**Auto-correla-tion of residu-als**	**Test of normality of residu-als (p-value)**	**test of ARCH effect (p-value)**
G1 – SARIMA(0,0,0)x(0,1,0)$_7$	30793.0	8.12%	no	0.445	0.2109
G2 – SARIMA(1,0,0)x(2,0,0)$_7$			no	0.792	0.4129
G3 – SARIMA(0,0,0)x(0,0,0)$_7$			no	0.239	0.1093
SARIMA(1,0,0)x(2,0,2)$_7$	34492.8	13.80%	yes	2.87E-78	1.12E-08
SARIMA(2,0,0)x(3,0,1)$_7$with E1, E2, E3 variables	31177.1	10.36%	yes	5.35E-55	9.94E-05
ARIMA(2,0,2)with day-of-the-week-specific variables	35791.4	16.45%	yes	5.54E-79	1.05E-10
ARIMA(3,0,1) with E1, E2, E3, E4, E5 variables and day-of-the-week-specific variables	35909.2	14.78%	yes	8.22E-28	1.49E-09

Source: own elaboration

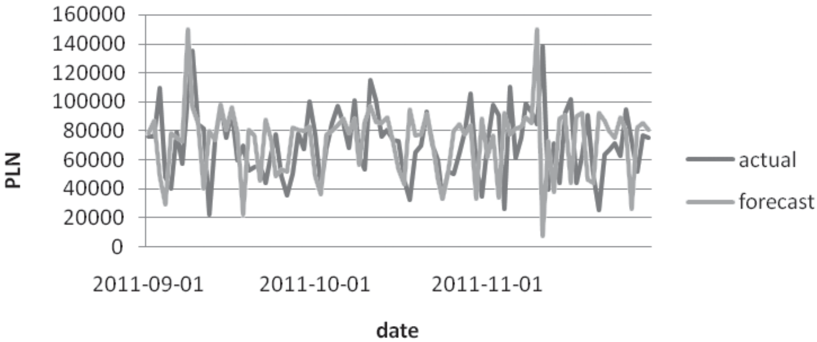

Figure 14. Results of modeling withdrawals from ATM 4 in the September 2011–November 2011 period

An analysis of the empirical outcomes presented in Tables 4–7 leads to the conclusion that modeling each subseries (constructed for each ATM separately) leads to the best results. Both AIC and MAPE indicate that the model based on forecasting subseries related to different ATM states is the most promising one of all alternatives examined. The results suggest that such models may be successfully applied in forecasting ATM withdrawals. The advantage of these models in comparison to other ones is especially visible with respect to the values of MAPE (for ATMs 1-3 the MAPE is much smaller in model 1). In addition, one should note that in all cases SARIMA models provide a better fit to the data than ARIMA models with seasonal dummies. This implies that the seasonality present in the time series of daily withdrawals is of a stochastic (not deterministic) nature. Model 1 is also superior to other alternatives in terms of diagnostic tests. These tests suggest that for the majority of other models, the desired modeling assumptions regarding the error term are unfulfilled.

The results of the analysis of the remaining 289 ATMs are in line with the formulated conclusions. For all ATMs subseries-based modeling led to the best results. Moreover, in most of the cases the residuals of SARIMA models fulfilled all basic assumptions. In Table 8 the basic statistics of MAPE calculated for ATMs with and without respect to location type are presented. MAPE is one of the most important issues in the management of ATM networks.

As can be seen in Table 8 average MAPE reached a value of almost 17%. The best fit was achieved for ATMs located in the neighborhood of transport hubs (12.55%) while the worst fit was found for bank branches (19.97%). However, the differences in average MAPE for all ATMs are small. The biggest differences in average MAPE were found for ATMs located in supermarkets (66.68%) while for gas stations these differences were smallest (25.70%) .

Table 8

Location	Average	Median	Standard Deviation	Coeffi-cient of variation	Mini-mum	Maxi-mum
Bank Branch	19.89	18.46	8.16	41.03%	10.64	62.7
Entertainment center	19.44	18.38	8.2	42.33%	10.91	33.40
Supermarket	16.71	14.23	11.14	66.68%	6.24	68.72
Hotel	19.50	19.45	5.12	26.25%	11.54	26.70
Gas Station	14.63	14.57	3.76	25.70%	7.50	24.21
Shop	15.03	13.53	4.29	28.59%	8.05	30.24
Shopping Center	14.18	13.59	5.35	37.75%	5.45	28.66
Transport	12.55	12.39	3.60	28.69%	8.37	17.07
Other	15.56	15.19	4.70	30.21%	6.43	26.13
Total	16.71	15.06	7.32	43.80%	5.45	68.72

Source: own elaboration

5. Conclusions

The modeling of time series is directly related to the issue of forecasting, which is one crucial aspect of the management process.

In this paper we aimed to fit basic econometric models and their modifications to a dataset of daily withdrawals from ATMs.The empirical results suggest that SARIMA models applied to specific subseries may be useful tools in describing the structure of daily withdrawals regardless of the location of the ATM. Moreover, the empirical outcomes suggest that the model discussed is superior to other modeling alternatives and provides the best fit to the empirical data. The advantage of using models built separately for individual subseries is twofold. First, in such an approach the error term was found to have the desired properties. Secondly, the mean absolute percentage error was smallest for this type of model.

From a practical point of view, the value of MAPE (17%)obtained by our approach provides a basis for claming that it may be used in the forecasting and management of ATM networks.

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