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Kriging approach for local height transformations

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Abstract: In the paper a transformation between two height datums (Kronstadt'60 and Kronstadt'86, the latter being a part of the present National Spatial Reference System in Poland) with the use of geostatistical method – kriging is presented. As the height differences between the two datums reveal visible trend a natural decision is to use the kind of kriging method that takes into account nonstationarity in the average behavior of the spatial process (height differences between the two datums). Hence, two methods were applied: hybrid technique (a method combining Trend Surface Analysis with ordinary kriging on least squares residuals) and universal kriging. The background of the two methods has been presented. The two methods were compared with respect to the prediction capabilities in a process of crossvalidation and additionally they were compared to the results obtained by applying a polynomial regression transformation model. The results obtained within this study prove that the structure hidden in the residual part of the model and used in kriging methods may improve prediction capabilities of the transformation model.

Key words: kriging, polynomial regression, height transformation

1. Introduction

As geostatistical methods constantly gain attention in different fields of applied science where the need of precise forecasting is present, here the use of optimal prediction (kriging) for random fields is presented in order to transform heights from Kronstadt'60 to Kronstadt'86 height datum. This work refers to the previously published paper (Ligas and Banasik, 2012) concerning the same problem but treated with a different statistical tool – polynomial regression. From this respect the part of the paper concerning description of the study area will be limited to the necessary minimum and the reader is referred to (Ligas and Banasik, 2012) in this regard. Here, we refresh that the area under study is the Cracow with 150 homological benchmarks of the major (primary) vertical network. Table 1 summarizes the dataset

of 150 common points of the Ith class of the primary vertical network as to the height difference between Kronstadt'60 and Kronstadt'86 (dH_{86-60}).

Table 1. Summary of height differences on common benchmarks

Number of points	Min [m]	first quartile [m]	median [m]	third quartile [m]	max [m]	range [m]	average [m]	std. dev. [m]
150	-0.0514	-0.0364	-0.0344	-0.0327	-0.0281	0.0233	-0.0351	0.0037

Figure 1 presents distribution of control points of the I class as well as localization of 8 test points of the second class later used for validation purposes.

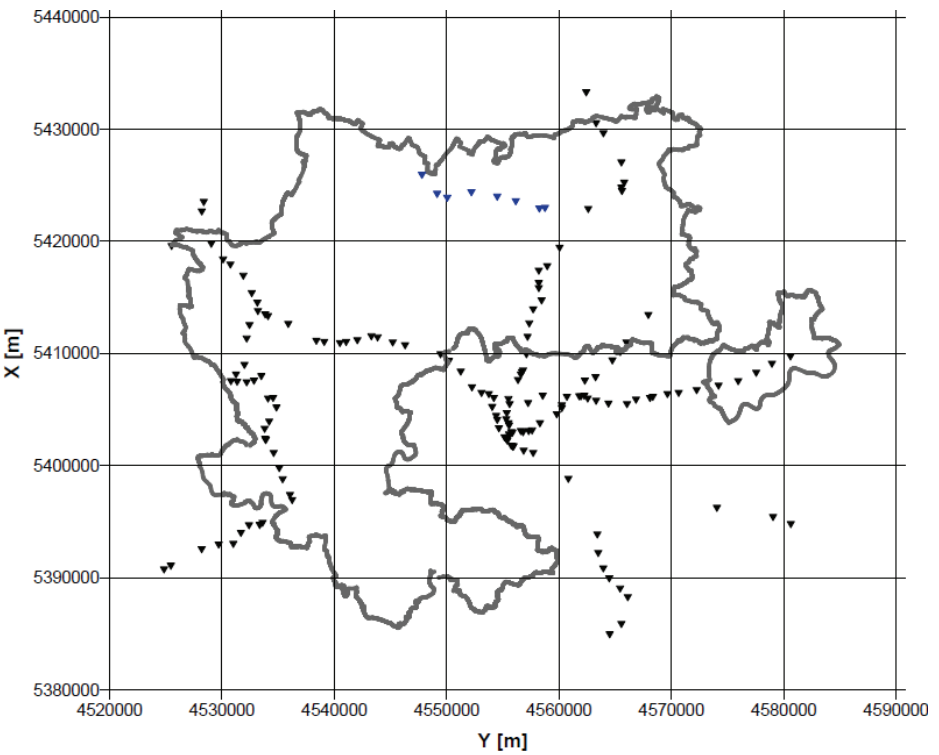

Fig. 1. The boundaries of Krakow's district (study area), black triangles – the basic vertical control network (I class), blue triangles – 8 benchmarks of the second class of the basic control network used for validation purposes

Geostatistical methods known under one common name – kriging – have a history dating back to fifties of the last century and were initially linked with two names: Daniel Gerhardus Krige (originator) and Georges Matheron (well suited continuator, French mathematician and geologist). But far important thing than the history is the

broad spectrum of applications which among others are: geology, hydrology, mining, remote sensing, soil science, epidemiology, geophysics; saying otherwise everywhere where the need of some form of reliable prediction is required. It is worth mentioning that geodesists developed a very similar technique called least squares collocation e.g. (Moritz, 1978; Moritz, 1980). But this has a very particular character combining observational data of different types for an optimal determination of the earth's figure and gravitational field (Moritz, 1980). Hence it is far less known outside the geodetic community. This is partially due to abovementioned particular character and also from the reason of software availability. Some (not exhaustive) theoretical comparison of the two methods may be found in the literature e.g. Dermanis (1984), Cressie (1990). Also a comparison between simple kriging and least squares prediction (less general precursor of least squares collocation) may be found in Ligas and Kulczycki (2010)

Here, the method of kriging is used for a simple problem of transformation between two height datums (Kronstadt'60 and Kronstadt'86) where the polynomial – based approach is the most common. Here, in reference to the previous paper (Ligas and Banasik, 2012) we do not use new nomenclature concerning classification of geodetic networks according to (RMAiC, 2012) as it does not have any influence on the flow of the methods involved. The irregular pattern of points within the study area (see Fig. 1) may cause potential difficulties as one always wants to have as regular pattern as possible never mind the method involved.

As the problem of spatial prediction by kriging is very broad, requires some additional knowledge from random fields theory, familiarity with specific vocabulary, and the space in the paper is limited, hence very often suitable references will be provided.

2. Theoretical background

Kriging which is synonymous to the optimal prediction for random fields (Stein 1999) has many variants. Among them one may differentiate between linear and nonlinear, univariate and multivariate kriging also within this subdivision one may have different variants of kriging depending on assumptions imposed (a firm review of kriging methods may be found in e.g. Cressie (1993), Goovaerts (1997)). The goal of kriging is the same as all other interpolation (prediction) methods: to find the optimal $Z(\mathbf{s}_0)$ (explanation of quantities after eqs. (1) and (2)) on the basis of all available data $\mathbf{Z}$ (global neighborhood) or some subset of $\mathbf{Z}$ (local neighborhood). Optimality of kriging in statistical sense is achieved by applying two fundamental principles that must be satisfied by the predictor. The first, it is unbiasedness of the predictor (1) and the second is the minimization of the mean squared error of prediction (2) (in some cases estimation e.g. kriging of the mean).

$$E\left[\hat{Z}(\mathbf{s}_0) - Z(\mathbf{s}_0)\right] = E\left[p(\mathbf{Z}, \mathbf{s}_0) - Z(\mathbf{s}_0)\right] = 0 \tag{1}$$

$$E\left[\hat{Z}(\mathbf{s}_0) - Z(\mathbf{s}_0)\right]^2 = E\left[p(\mathbf{Z}, \mathbf{s}_0) - Z(\mathbf{s}_0)\right]^2 \rightarrow \min \tag{2}$$

where:

E – expected value operator

$\mathbf{s}_0$ – spatial location for which prediction is made, here $\mathbf{s}_0 = [x_0, y_0]$

$\mathbf{Z}$ – available data (observations)

$\hat{Z}(\mathbf{s}_0) = p(\mathbf{Z}, \mathbf{s}_0)$ – predictor at $\mathbf{s}_0$ based on available data $\mathbf{Z}$

$Z(\mathbf{s}_0)$ – true and unknown value to be predicted

From the standpoint of this work the most important kriging variants are those which take into account a trend (non – stationary expected value) in the data at hand due to the fact that the existence of strong second degree polynomial trend in height differences between the two datums was proven for this dataset (Ligas and Banasik, 2012). Hence, in order to develop kriging – based transformation model two approaches are considered. The first called here the hybrid method. Briefly, the method bases on removing systematic component from the data (detrending) then working with residual part of the prediction model and finally restoring (adding back) the systematic component. The trend is usually estimated via ordinary least squares (OLS) (or generalized least squares, GLS), residuals are "treated" with ordinary kriging (later on it will be explained why this kriging variant) and then final prediction at an unknown point is equal to the sum of the trend component and the predicted residual. The second method, universal kriging, performs the abovementioned procedure in one go. Both approaches, ordinary kriging (used in hybrid technique) and universal kriging, belong to the class of linear predictors; it means that they are linear combinations of available data, this may be written as (in case of hybrid technique $\mathbf{Z}(\mathbf{s})$ is replaced by residual part of the model i.e. $\boldsymbol{\delta}(\mathbf{s})$):

$$\hat{Z}(\mathbf{s}_0) = p(\mathbf{Z}, \mathbf{s}_0) = \boldsymbol{\lambda}^T \mathbf{Z}(\mathbf{s}) \tag{3}$$

where:

$\boldsymbol{\lambda}$ – vector of kriging weights (coefficients)

Remark on kriging weights

When applying kriging, one must be aware that the term "kriging weights" is not used in the usual meaning of a weight of an observation. Kriging weights do not constitute a convex linear combination i.e. in general they do not sum up to unity (simple kriging, no constraints on weights) and what is more important they may be negative. More on "kriging weights" and consequences in different applications may be found in e.g. Deutsch (1996), Wackernagel (1995), Armstrong (1998).

Hybrid technique (Trend Surface Analysis and ordinary kriging on residuals)

1. Set up of linear spatial model: $\mathbf{Z(s)} = \mathbf{F(s)\beta} + \mathbf{\delta(s)}$
2. Fit the model in order to estimate coefficients $\hat{\boldsymbol{\beta}}$ via OLS or GLS

Steps (1) and (2) are the stages of e.g. polynomial regression and this what is going further is just an improvement

3. Perform ordinary kriging on residuals $\hat{\boldsymbol{\delta}}(\mathbf{s})$ from the fitted model, steps (1 and 2)
4. Obtain the hybrid kriging predictor $\mathbf{Z}(\mathbf{s}_0) = \mathbf{f}(\mathbf{s}_0)^T \hat{\boldsymbol{\beta}} + p_{OK}\left[\hat{\delta}(\mathbf{s}_0) \mid \hat{\boldsymbol{\delta}}(\mathbf{s})\right]$

where:
$\mathbf{F(s)}$ – design matrix of a trend involved, $\boldsymbol{\delta}(\mathbf{s})$ – error term, $p_{OK}\left[\hat{\delta}(\mathbf{s}_0) \mid \hat{\boldsymbol{\delta}}(\mathbf{s})\right]$ – ordinary kriging predictor of the form (3) i.e.:

$$p_{OK}\left[\hat{\delta}(\mathbf{s}_0) \mid \hat{\boldsymbol{\delta}}(\mathbf{s})\right] = \boldsymbol{\lambda}^T \hat{\boldsymbol{\delta}}(\mathbf{s}) \tag{4}$$

Applying (2) subject to (1) (constrained optimization) with the predictor expressed as (4) one arrives at a system of ordinary kriging equations (5a, b) (Olea, 1999; Armstrong, 1998) solved with respect to kriging weights:

$$\mathbf{C_1}\boldsymbol{\lambda}_\kappa = \mathbf{c}_1 \tag{5a}$$

$$\begin{bmatrix} C_{11} & \cdots & C_{1n} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ C_{n1} & \cdots & C_{nn} & 1 \\ 1 & 1 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \kappa \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \\ 1 \end{bmatrix} \tag{5b}$$

where:
$\mathbf{C_1}$ – covariance matrix supplemented with a vector of "ones" resulting from constraints on ordinary kriging weights
$\boldsymbol{\lambda}_\kappa$ – vector of kriging weights supplemented with Lagrange multiplier
$\mathbf{c}_1$ – vector of covariances supplemented with "unity" resulting from constraints on ordinary kriging weights
C_{ij} – covariances between Z_i and Z_j, (between two observations from the neighborhood of actually predicted point)
c_i – covariances between Z_0 and Z_i (between the point actually predicted and every point form the neighborhood)
κ – Lagrange multiplier resulting from conditional minimization of (2)

Figure 2 shows the structure stored in $\mathbf{C_1}$ matrix and $\mathbf{c}_1$ vector of (5a or 5b) (additional row and column of "ones" as well as Lagrange multiplier concerns constraints on kriging weights, in ordinary kriging weights must sum up to "unity"). It may be seen that all available information contained in data is used. Each row in $\mathbf{C_1}$ matrix (except the last row and the last column) contains covariances between each pair of observations from the neighborhood (Fig. 2 shows this structure for the first row; blue dashed lines linking observation no. 1 and all other observations from the neighborhood). Vector $\mathbf{c}_1$ contains information on covariances between a point actually predicted (unknown value) and all data values from the neighborhood – red dashed line linking point no. "0" with points numbered 1..*n*.

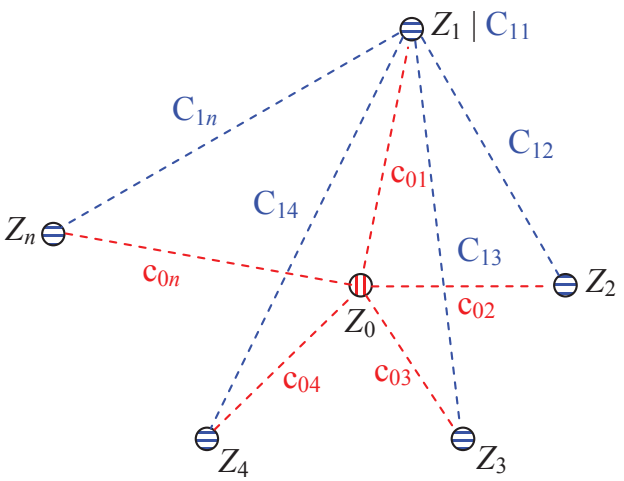

Fig. 2. Structure stored in the matrix $\mathbf{C_1}$ (the first row) and the vector $\mathbf{c}_1$; blue dots with horizontal bars (1..*n*) – observations from the neighborhood; red dot with vertical bars (0) – value to be predicted

The above formulas (5a) and (5b) are expressed in terms of the covariance function but in case of ordinary kriging they may be expressed in terms of more general structure function i.e. semivariogram. The two functions, covariance function and semivariogram, under certain conditions are equivalent. In this place it is worth introducing a quotation from (Heiskanen and Moritz, 1967) on the usefulness of the covariance function: *The remarkable thing is that the only function needed to derive optimum predictors in the mean square sense is a covariance function*. More on the covariance function and semivariogram may be found in e.g. (Olea, 2006; Schabenberger and Gotway, 2005; Cressie, 1993; Goovaerts, 1997; Chiles and Delfiner, 1999) and from geodetic ground in Moritz (1978) and lectures "Statistical analysis of environmental data" by F. Sanso available online at http://geomatica.como.polimi.it/corsi/.

Of course, one may argue that the choice of simple kriging for the hybrid technique would be definitely better due to the fact that residuals form the OLS (GLS) satisfy $E(\boldsymbol{\delta}) = \mathbf{0}$ thus the assumption of known and constant mean of the random field (necessary for simple kriging) is satisfied. But, kriging from practical point of view is usually performed with some quantity of data (local neighborhood) rather than with all available data (global neighborhood) in order to avoid solving large systems of equations of the form (5b). Thus, for some subregions (determined by local neighborhood) of a spatial domain this assumption (zero mean) may not hold at all and thus provide unreliable results of prediction. Hence, the ordinary kriging assuming constant and unknown mean value of a random field is applied. This method adds some robustness to the hybrid technique.

Universal kriging

Universal kriging (or kriging with a trend model) is a step further beyond ordinary kriging. It drops the assumption of constant (stationary) mean value (as it is in ordinary kriging $E[Z(\mathbf{s})] = \mu$). Decomposition of the process into the mean value and a disturbance still holds here ($Z(\mathbf{s}) = \mu(\mathbf{s}) + \delta(\mathbf{s})$), but the mean value of the process is modeled by a function of location in space expressed by coordinates of observations i.e. $E[Z(\mathbf{s})] = \mu(\mathbf{s})$ is no longer constant but is an unknown linear combination of known functions $\{f_0(\mathbf{s}),\ldots,f_p(\mathbf{s})\}$, $\mathbf{s} \in D$ (Cressie, 1993) (D – spatial domain or simply "study area"). Trend, being a model of low frequency, characterizes the average behavior of the phenomenon in the spatial domain D (study area), whereas the disturbance, being a high frequency component, describes fluctuations of the phenomenon. Trend is usually modeled in polynomial form (linear, quadratic or seldom higher order fashion) i.e.:

$$\mu(\mathbf{s}) = \sum_{i=0}^{p} f_i(\mathbf{s})\beta_i \tag{6}$$

where: $\{f_0(\mathbf{s}) = 1, f_1(\mathbf{s}) = x, f_2(\mathbf{s}) = y, f_3(\mathbf{s}) = xy,\ldots\}$.

Applying optimality criteria (1) and (2) (constrained optimization) to the predictor expressed as (3) and having in mind no longer constant mean one arrives at a system of universal kriging equations (7a, b) (Olea, 1999; Goovaerts, 1997) solved with respect to kriging weights.

$$\mathbf{C_F}\boldsymbol{\lambda}_{\kappa} = \mathbf{c_f} \tag{7a}$$

$$\begin{bmatrix} C_{11} & \cdots & C_{1n} & 1 & f_1(\boldsymbol{s}_1) & f_2(\boldsymbol{s}_1) & \cdots & f_p(\boldsymbol{s}_1) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ C_{n1} & \cdots & C_{nn} & 1 & f_1(\boldsymbol{s}_n) & f_2(\boldsymbol{s}_n) & \cdots & f_p(\boldsymbol{s}_n) \\ 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \\ f_1(\boldsymbol{s}_1) & \cdots & f_1(\boldsymbol{s}_n) & 0 & 0 & 0 & \cdots & 0 \\ f_2(\boldsymbol{s}_1) & \cdots & f_2(\boldsymbol{s}_n) & 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ f_p(\boldsymbol{s}_1) & \cdots & f_p(\boldsymbol{s}_n) & 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \cdots \\ \lambda_n \\ \kappa_1 \\ \kappa_2 \\ \cdots \\ \cdots \\ \kappa_p \end{bmatrix} = \begin{bmatrix} c_1 \\ \cdots \\ c_n \\ 1 \\ f_1(\boldsymbol{s}_o) \\ f_2(\boldsymbol{s}_o) \\ \cdots \\ f_p(\boldsymbol{s}_o) \end{bmatrix} \tag{7b}$$

where:

$\mathbf{C_F}$ – covariance matrix supplemented with a matrix of trend components resulting from constraints on universal kriging weights

$\boldsymbol{\lambda_\kappa}$ – vector of kriging weights supplemented with a vector of Lagrange multipliers

$\mathbf{c_f}$ – vector of covariances supplemented with a vector of trend components resulting from constraints on universal kriging weights

Detailed derivation of the universal kriging predictor may be found e.g. in Cressie (1993), Schabenberger and Gotway (2005), Olea (1999) and also in Chauvet and Galli (1982) which is entirely devoted to the universal kriging method. One may easily notice that universal kriging reduces to ordinary kriging for $\mu(\mathbf{s}) = f_0(\mathbf{s})\beta_0 = \beta_0 = \mu$ (unknown, constant mean, see (5b))

Universal kriging predictor may also be presented in a different form. Dividing (7b) into block matrices and performing suitable operations one arrives at the expression of the following form:

$$p_{UK}\left[Z(\mathbf{s}_0) \mid \mathbf{Z}(\mathbf{s})\right] = \hat{Z}(\mathbf{s}_0) = \boldsymbol{\lambda}^T \mathbf{Z}(\mathbf{s}) = \mathbf{f}(\mathbf{s}_0)^T \hat{\boldsymbol{\beta}} + \mathbf{c}^T \mathbf{C}^{-1}\left[\mathbf{Z}(\mathbf{s}) - \mathbf{F}(\mathbf{s})\hat{\boldsymbol{\beta}}\right] \tag{8}$$

where: $\hat{\boldsymbol{\beta}} = \left(\mathbf{F}(\mathbf{s})^T \mathbf{C}^{-1} \mathbf{F}(\mathbf{s})\right)^{-1} \mathbf{F}(\mathbf{s})^T \mathbf{C}^{-1} \mathbf{Z}(\mathbf{s})$

This is nothing but Goldberger's best linear unbiased predictor (Goldberger, 1962) only in a spatial framework, see also work by Dermanis (1984) comparing kriging with least squares collocation.

3. Results

Results of the two approaches presented in the paper have been compared with the use of a crossvalidation technique (leave-one-out) and also confronted with the polynomial regression model described in (Ligas and Banasik, 2012). Crossvalidation

is a very useful technique of choosing the model of best prediction performance by dropping some data from the dataset and trying to forecast their values on the basis of remaining data (more on crossvalidation the reader may find in e.g. Efron and Tibshirani (1993), Maddala (1992), Hastie et al (2009)). This approach prevents against accepting an overfitted model with poor prediction capabilities as it is known that one may get a perfect fit with almost null prediction capabilities. In fact, crossvalidation technique was also used to select optimal local search neighborhood for universal kriging (6 sectors with 8 neighboring observations in each) and for the hybrid method (6 sectors with 4 neighboring observations in each). The following table (Table 2) presents the results of the crossvalidation for two methods considered in the paper

Table 2. Results of crossvalidation for polynomial regression, universal kriging and hybrid method

Crossvalidation						
Quadratic function	Minimum error					-1.2 [cm]
	Maximum error					0.5 [cm]
	Averaged crossvalidation error					-0.1 [cm]
	Mean squared crossvalidation error					0.4 [cm]
	Summary of crossvalidation errors					
Interval [mm]	[0, 1]	(1, 2]	(2, 3]	(3, 4]	(4, 5]	> 5
No of points	29	39	31	20	13	18
Universal kriging	Minimum error					-0.5 [cm]
	Maximum error					0.6 [cm]
	Averaged crossvalidation error					0.0 [cm]
	Mean squared crossvalidation error					0.2 [cm]
	Summary of crossvalidation errors					
Interval [mm]	[0, 1]	(1, 2]	(2, 3]	(3, 4]	(4, 5]	> 5
No of points	90	22	17	14	5	2
Hybrid method	Minimum error					-0.5 [cm]
	Maximum error					0.6 [cm]
	Averaged crossvalidation error					0.0 [cm]
	Mean squared crossvalidation error					0.2 [cm]
	Summary of crossvalidation errors					
Interval [mm]	[0, 1]	(1, 2]	(2, 3]	(3, 4]	(4, 5]	> 5
No of points	93	23	17	12	2	3

and the ones from polynomial regression model. All the crossvalidation statistics are identical with those used in Ligas and Banasik (2012), hence there should be no confusion in comparison the results.

As may be seen both kriging based approaches (with quadratic trend) to the local transformation problem reveal much better crossvalidation characteristics than the pure polynomial regression model (quadratic function). Minimum crossvalidation error from kriging approaches decreased roughly by half comparing to the polynomial model on the other hand the maximum crossvalidation error rests on the same level. Zero averaged crossvalidation error for the two methods presented in the paper proves that the prediction was made without an apparent bias (no under/over estimation present in the model). This is a slight improvement over the polynomial approach where this error is on the level of -0.1 cm. Also, mean squared crossvalidation error has been reduced by half. One may also confront the number of the crossvalidated errors within given intervals from all the methods presented; here the advantage of kriging methods over the polynomial solution is clearly visible. These all crossvalidation statistics confirm that two kriging based approaches reproduce the data in a much better way. As far as two kriging based approaches are concerned they reveal very comparable prediction capabilities. It would be hard to decide upon one of the two definitely. Similarly, as in Ligas and Banasik (2012) the true prediction capabilities of the methods have been verified on 8 test points of the II*nd* class of the primary vertical network (old nomenclature). Table 3 reveals the results.

Table 3. Comparison of the "true" prediction errors on eight test points of the II*nd* class of the primary vertical network for polynomial regression, universal kriging and hybrid technique (minimum, maximum and average computed from absolute values; PR – polynomial regression, UK – universal kriging, H – hybrid method, (T – P) – True "minus" Predicted)

Point No.	True dH [cm]	Predicted dH (PR) [cm]	Predicted dH (UK) [cm]	Predicted dH (H) [cm]	T – P (PR) [cm]	T – P (UK) [cm]	T – P (H) [cm]
1	-3.69	-4.04	-4.02	-4.01	0.35	0.33•	0.32•
2	-3.93	-3.98	-3.91	-4.02	0.05	-0.02•	0.09
3	-4.27	-3.99	-3.99	-4.04	-0.28	-0.28e	-0.23•
4	-3.81	-4.06	-4.04	-4.06	0.25	0.23•	0.25e
5	-3.97	-4.09	-4.03	-4.15	0.12	0.06•	0.18
6	-4.07	-4.11	-4.20	-4.15	0.04	0.13	0.08
7	-4.21	-4.12	-4.25	-4.21	-0.09	0.04•	0.00•
8	-4.27	-4.14	-4.27	-4.22	-0.13	0.00•	-0.05•
Minimum					0.04	0.00	0.00
Maximum					0.35	0.33	0.32
Average					0.16	0.14	0.15

As may be seen in Table 3, the improvement in prediction capabilities over polynomial regression model in the true validation process is visible but not so "spectacular" as in the case of crossvalidation presented in Table 2. These less "spectacular" results may result from the fact that benchmarks of the II*nd* class (old nomenclature) were determined with slightly less demanding accuracy criteria in comparison to benchmarks of the I*th* class on which the transformation models were built. For the hybrid technique a half of points (marked with dots in Table 3) has lower values of the differences between the true height difference and that predicted (true prediction error) and one equal (marked with the letter "e") comparing to the polynomial solution. In case of universal kriging the situation is slightly better because six out of eight points (marked with dots in Table 3) have lower true prediction errors than those obtained from polynomial regression model and similarly as before one is equal (marked with the letter "e"). The simple summary statistics of true prediction errors (minimum, maximum and average) are slightly lower for two kriging based transformation models. It is a pity that all eight test points are concentrated in one region (the northern part of the study area, see Figure 4 in Ligas and Banasik, 2012) because it gives only a selective image of the true prediction abilities. But, taking into account the overall results from tables (2) and (3) one may expect a better performance of the two kriging – based methods with a certain level of certainty.

Transformation of new points basing on universal kriging may be carried out in two ways either by constructing a suitable GRID model representing height differences between two height datums dH and reading their values whenever it is necessary or by constructing and solving a suitable universal kriging system of equations with the knowledge of semivariogram (or covariance function) parameters and the parameters concerning the neighborhood for every new transformed point. For the case of hybrid technique presented here it is very similar to this written above and the difference is that the GRID model (or the solution to the ordinary kriging system) represents the residual part of the model this time. In order to obtain the total height difference the residual part must be added to the previously established trend model.

As it may be seen the use of kriging either in compact form (universal kriging) or hybrid technique is more time consuming than the use of polynomial regression models. In the latter, the knowledge of parameters of correctly built model is the only thing required. In the kriging based approach the polynomial regression (trend surface analysis) is a preliminary step which is then used to extract additional information hidden in a residual part of the model.

4. Conclusions

In the paper, the use of kriging – based local transformation between two height datums – Kronstadt'60 and Kronstadt'86 has been presented. Due to the existence of strong polynomial trend in height differences between the two datums a natural decision was to use kriging methods that take into account such regularity in the dataset. One of them

is universal kriging and the other is a hybrid method combining trend surface analysis and kriging on residuals. The two kriging – based approaches have been compared to each other as to the prediction capabilities and confronted with the results from the polynomial regression fit presented in (Ligas and Banasik, 2012). Comparison of the three methods was done through the leave-one-out crossvalidation technique and on test points of the II*nd* class (old nomenclature) of the major vertical network. The results obtained in this study both in crossvalidation and true validation prove that the kriging based approach to local height transformation turned out to be more effective than the polynomial regression model in respect of predictive capabilities and that the structure hidden in the residual part of the model may improve prediction capabilities of the height transformation model. These very promising results encourage wider research on this problem to fully exploit the abilities of spatial prediction methods.

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Zastosowanie metody krigingu do lokalnej transformacji wysokości

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Streszczenie

W artykule przedstawiono lokalną transformację między dwoma układami wysokości (Kronsztadt'60 oraz Kronsztadt'86, ostatni z nich będący obecnie częścią Państwowego Systemu Odniesień Przestrzennych w Polsce) z wykorzystaniem metod geostatystycznych – kriging. Ze względu na fakt, iż różnice wysokości między dwoma układami na punktach dostosowania wykazywały silny trend pod uwagę wzięto tylko te metody, które uwzględniają tego typu niestacjonarność procesu. Zastosowano dwie metody: hybrydową (Analiza Trendu Powierzchniowego z interpolacją reszt do modelu za pomocą krigingu zwyczajnego) oraz kriging uniwersalny. Przedstawiono rys teoretyczny obydwu metod. Dokonano porównania wyżej wymienionych metod pod względem ich zdolności predykcyjnych w procesie kroswalidacji modeli a zarazem otrzymane wyniki skonfrontowano z wynikami otrzymanymi z regresji wielomianowej. Otrzymane wyniki dowodzą, iż struktura ukryta w rezydualnej części modelu używana przez kriging może podnieść zdolności predykcyjne modelu transformacji.