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RELIABILITY ANALYSIS OF COMPLEX UNCERTAINTY MULTI-STATE SYSTEM BASED ON BAYESIAN NETWORK

ZASTOSOWANIE SIECI BAYESOWSKIEJ DO ANALIZY NIEZAWODNOŚCI ZŁOŻONYCH SYSTEMÓW WIELOSTANOWYCH W WARUNKACH NIEPEWNOŚCI

Reliability analysis of complex multi-state system has uncertainty, which is caused by complex structures, limited test samples, and insufficient reliability data. By introducing fuzzy mathematics and grey system theory into the Bayesian network, the model of the grey fuzzy Bayesian network is built, and the reliability analysis method of complex uncertainty multi-state system with the non-deterministic membership function and the interval characteristic quantity is proposed in this paper. Using the trapezoidal membership function with fuzzy support radius variable to describe the fault state of the component, it can effectively avoid the influence of human subjective factors on the selection of the membership function and solve the problem that the fault states of the system and its components are difficult to define accurately. And the conditional probability table containing interval grey numbers is constructed to effectively express the uncertain fault logic relationship between the system and its components. Moreover, a parameter planning model of the system reliability characteristic quantities is constructed, and the system reliability characteristic quantities are expressed as the form of interval values. Finally, two sets of numerical experiments are conducted and discussed, and the results show that the proposed method is an effective and a promising approach to reliability analysis for complex uncertainty multi-state systems.

Keywords: *reliability analysis, Bayesian network, complex uncertainty multi-state system, fuzzy mathematics, grey system theory.*

Analiza niezawodności złożonych systemów wielostanowych obarczona jest niepewnością związaną ze złożonością ich struktury, ograniczoną liczbą próbek badawczych i niewystarczającymi danymi dotyczącymi niezawodności. W przedstawionej pracy, wprowadzenie elementów matematyki rozmytej i teorii szarych systemów do sieci bayesowskiej umożliwiło budowę modelu szarej rozmytej sieci bayesowskiej i zaproponowanie metody analizy niezawodności złożonych systemów wielostanowych w warunkach niepewności, która wykorzystuje niedeterministyczną funkcję przynależności oraz pojęcie interwałowej wielkości charakterystycznej. Zastosowanie trapezoidalnej funkcji przynależności z rozmytą zmienną promienia nośnego do opisu stanu uszkodzenia komponentu, pozwala zniwelować wpływ subiektywnego czynnika ludzkiego na wybór funkcji przynależności i eliminuje konieczność precyzyjnego definiowania stanu uszkodzenia systemu i jego elementów składowych. Opracowana tabela prawdopodobieństw warunkowych zawierająca szare liczby interwałowe pozwala wyrazić niepewne zależności logiki uszkodzeń między systemem a jego składnikami. Ponadto, w pracy skonstruowano model planowania parametrów charakterystycznych wielkości niezawodności systemu wyrażonych w postaci wartości interwałowych. W ostatniej części artykułu omówiono dwie serie eksperymentów numerycznych, których wyniki pokazują, że proponowana metoda stanowi skuteczne i obiecujące podejście do analizy niezawodności złożonych systemów wielostanowych w warunkach niepewności.

Słowa kluczowe: *analiza niezawodności, sieć bayesowska, złożony system wielostanowy, niepewność, matematyka rozmyta, teoria szarych systemów.*

1. Introduction

In modern engineering, multi-state system (MSS) is a kind of system that represents a capability allowing for more than two performance states in a system besides perfect functionality and complete fault [21]. Compared with the two-state system, MSS can define the components states of a system, and express the effect of the changes of component performance on system performance more flexibly and precisely. In the 1970s, Barlow and Wu [2] first proposed the concept of MSS and gradually established the related theory. Then, the reliability theory of MSS has been widely concerned by scholars. And the following reliability analysis methods for MSS have been developed: the extended Boolean model method [22, 26], random process theory [1, 14, 18], Monte-Carlo simulation method [23, 25], function model method [8, 16, 31], Bayesian network method [13, 29], and so on.

The uncertainty, which is caused by the insufficient information about internal structures, the scarcity of historical data and the changeability of operation environment, is one of the most crucial problems in MSS reliability analysis. Therefore, it is very difficult to define and obtain the component state performances and state probabilities. Meantime, the boundaries among component fault states fail to define and obtain with precision. So the traditional probability-based method is no longer applicable. However, non-probabilistic methods, such as evidence theory [7], grey system theory [33], probability-box [27], and fuzzy theory [15, 30], have been proposed and developed for reliability analysis of complex uncertainty MSS.

Based on the probability theory and the graph theory, Bayesian network not only can effectively express the complex logical relations in the system, but also has a unique two-way reasoning mechanism, so it is particularly suitable for the reliability analysis of complex systems which have characteristics of high reliability, longevity, small samples. Bayesian network has been widely used in reliability analysis [3, 24], security analysis [9, 12], fault diagnosis [4, 17] and other fields. By introducing fuzzy set theory to Bayesian network model, a novel method of multi-state system reliability analysis is proposed by He et al.[11], which considers the multi-state, the fuzziness, and the changes of failure probability with time of the system, and its validity and practicality are verified by the flexible lifting system of a high-speed elevator. In view of the shortcomings of Bayesian network method and T-S fault tree method, Yao and Chen [28] propose the fuzzy reliability evaluation method, and conduct reliability evaluation on the hydraulic system of roadway transportation vehicles. Considering the relevant failure and incomplete coverage, Cai et al. [5] propose a reliability evaluation method for redundant systems based on Bayesian network, and evaluate the reliability of the subsea blowout preventer control system. In reference [10], a multi-state system reliability analysis method based on intuitionistic fuzzy Bayesian network is proposed, which effectively solves the problem that the accurate probability of different state of Bayesian network root node is difficult to determine. To sum up, although certain research results have been achieved based on Bayesian network, such as using the precision value for reasoning analysis, the introduction of fuzzy technology, and so on, there are still shortcomings in the existing reliability analysis methods by using Bayesian network model for complex uncertainty multi-state systems, the main problems are as follows:

(1) In the traditional reliability analysis methods, the fuzzy support radius variable of membership function of descripting fault state is a fixed value, such as in references [6, 28]. Although the traditional reliability analysis method can solve some reliability analysis problems of MSS, it is hard to avoid introducing too much subjective information in the process of constructing the membership function, which can lead to deviation and affect the accuracy of the analysis results.

(2) The traditional Bayesian network reliability method is under the precondition of the determined fault logic relationship, for example, in references [5, 10]. But due to the lack of reliability data, the limited test samples and the complicated running environment, using the exact value to describe the uncertain fault logic relationship between the system and its components cannot satisfy the requirement of reliability analysis for complex systems.

Fuzzy mathematics and grey system theory are the most active uncertain system theories which have attracted more and more attention in the field of reliability [19]. In order to solve the above problems, the membership function in fuzzy mathematics and the interval grey number in grey system theory are introduced to the Bayesian network. Then, the model of the grey fuzzy Bayesian network is built, and the reliability analysis method of complex uncertainty multi-state system with the non-deterministic membership function and the interval characteristic quantity is proposed in this paper. The proposed method uses the trapezoidal membership function with fuzzy support radius variable to describe the fault states of the component, and uses the conditional probability table containing interval grey numbers $\otimes$ to substitute for the traditional conditional probability table. Furthermore, a parameter planning model of the system reliability characteristic quantities is constructed, and the obtained reliability characteristic quantities of system are expressed in the form of interval values.

This paper is organized as follows: in Section 2, grey fuzzy Bayesian network method for system reliability modeling and analysis is introduced, and the flow chart and its specific process interpretation are given. Detailed steps of grey fuzzy Bayesian network method are introduced in Section 3. Two sets of numerical experiments are carried out and discussed to show the validity and advantages of the proposed method in Section 4. In Section 5, conclusions are drawn.

2. Grey fuzzy Bayesian network method for system reliability modeling and analysis

Bayesian network (BN) is a directed acyclic network that is composed of a directed acyclic graph (DAG) and a conditional probability table (CPT). Directed acyclic graph consists of nodes and edges. A node of DAG is used to represent the variable, which may be a unit, a failure mode, an attribute, a fault status, and so on. The edge points from the parent node to the child node, which represent the dependent relation between the parent node and the child node in the DAG. A node that does not have a parent node is called a root node which can represent a component variable. A node that does not have a child node is called a leaf node which can represent a system variable. Other nodes are called intermediate nodes which can represent the subsystem variables. Conditional probability table can quantitatively describe the causal failure logic relationship among nodes, that is, the logical relationship between the system and its components.

Fuzzy mathematics studies the uncertainty problem by means of membership function. Therefore, the trapezoidal membership function with fuzzy support radius variable r is constructed to describe the fault state of the component in our study. Grey system theory studies the uncertainty problem that part of the information is known, part of the information is unknown and part of the information is scarce [19]. According to the known partial information, the range of values of some parameters can be determined, but the exact values of some parameters can not be known in system reliability analysis, so interval grey number $\otimes$ is introduced to the conditional probability table. Thus, the conditional probability table containing interval grey numbers $\otimes$ is constructed to describe the uncertain fault logic relationship between the system and its components. Taking advantages of the above two theories, the grey fuzzy Bayesian network method for system reliability modeling and analysis is shown in Fig.1.

The proposed method extends the traditional node variables to the grey fuzzy Bayesian network nodes to express the fuzzy uncertain fault state during the fault evolution process of the system and its components, and the traditional conditional probability table is extended to the conditional probability table with interval grey numbers to express the uncertain fault logic relationship between the system and its components. As can be seen from the Fig.1, the specific processes are as follows:

(1) Analyze the basic principle of the system, clarify the fault states and failure modes of the system and its components, and establish the directed acyclic graph of the system Bayesian network.

(2) According to the fault states and fault modes of the component, the trapezoidal fuzzy number (TrFN) with fuzzy support radius variable r is constructed to describe the fuzzy uncertainty of the fault state during the fault evolution process of the system and its components, as shown in Section 3.1.

(3) The conditional probability table with interval grey numbers is constructed to substitute for the traditional conditional probability, which can describe the uncertain fault logic relationship with grey system information characteristics between system and its components, as shown in Section 3.2.

(4) According to the steps (1)-(3) and the corresponding definitions of the system reliability characteristic quantities, the corresponding system reliability characteristic function is obtained, as shown in Section 3.3.

(5) Taking the system reliability characteristic function as the objective function, and taking the intervals of the interval grey numbers as the constraints, the parameter planning model of the reliability characteristic quantities is constructed. The op-

timization algorithm is used to analyze the reliability of the leaf node and the state importance measures of the root nodes. Based on analysis results, system reliability and component state importance measures can be evaluated, as shown in Section 3.4.

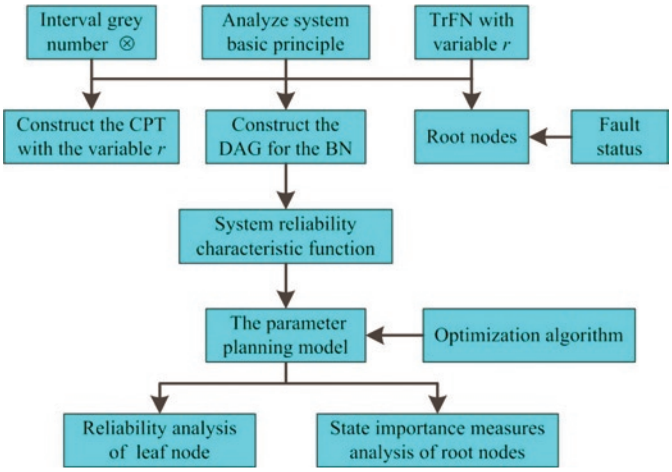


Fig. 1. Grey fuzzy Bayesian network method of reliability modeling and analysis

3. Detailed steps of grey fuzzy Bayesian network method

3.1. The construction of the TrFN with fuzzy support radius variable *r*

In engineering practice, the system and its components tend to exhibit multiple failure modes and multiple fault states during the evolution from normal operation to complete failure, and there is no strict boundary among fault states, which has certain fuzzy uncertainty. The membership functions describing the fault states of the system and its components include a triangular membership function, a trapezoidal membership function, a rectangular membership function, and so on.

The trapezoidal membership function is widely used in practical engineering and reliability analysis because of its intuitive expression and simple algebraic calculation. For ease of use, the trapezoidal membership function X is selected to describe the fault states of systems and components, and represented as:

$$X = (x_c, r, r, f, f) \tag{1}$$

In equation (1), x_c is the center of the fuzzy number support set, r is the support radius variable, f is the fuzzy area, as shown in Fig. 2.

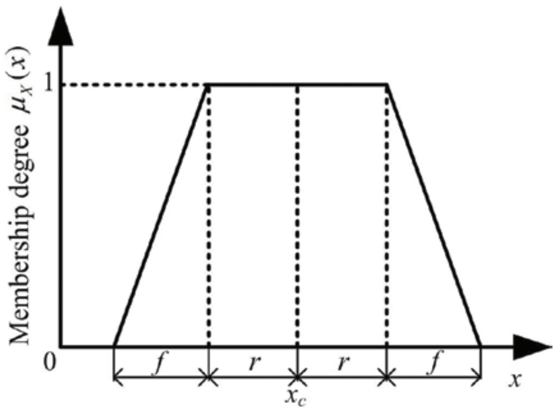


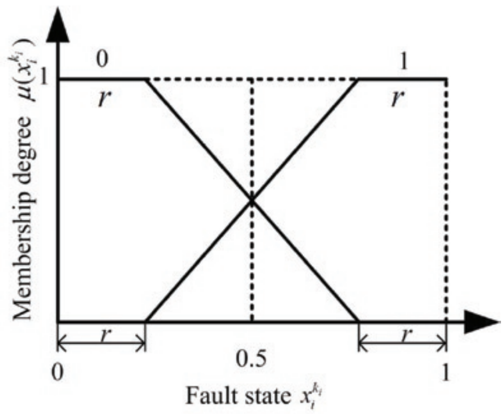
Fig. 2. Trapezoidal membership function

It is assumed that the system and its components have two fault states (fault-free and fault) or three fault states (fault-free, semi-fault and fault), represented by fuzzy numbers 0, and 1 or 0, 0.5 and 1 respectively, and the fault state of the node x_i (i=1, 2,⋯, n) is $x_i^{k_i}$ (k_i=1, 2,⋯, α_i). Then the sum of the membership degrees of the components' current fault state must be 1. In other words, the components for the two fault states must satisfy equation (2), and the components for the three fault states must satisfy equation (3):

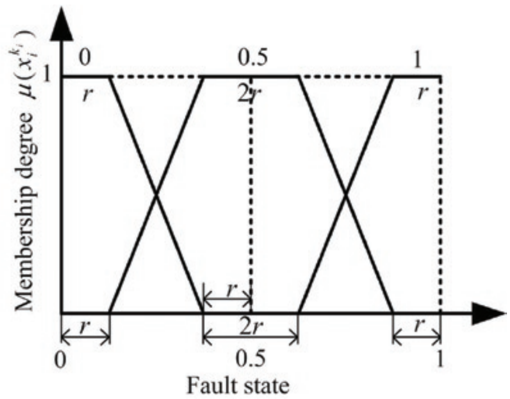
$$\mu_{\tilde{0}}(x_i^{k_i}) + \mu_{\tilde{1}}(x_i^{k_i}) = 1 \tag{2}$$

$$\mu_{\tilde{0}}(x_i^{k_i}) + \mu_{\widetilde{0.5}}(x_i^{k_i}) + \mu_{\tilde{1}}(x_i^{k_i}) = 1 \tag{3}$$

If the fault state of the node x_i (i=1, 2,⋯, n) is $x_i^{k_i}$ (k_i=1, 2,…, α_i) in grey fuzzy Bayesian network, making use of the trapezoidal membership number function shown in Fig.2, combining equation (2) and equation (3) at the same time, then the trapezoidal membership function with the fuzzy support radius variable r is constructed, and shown in Fig.3. The variable r ($0 \le r \le 0.25$) is the fuzzy support radius of the trapezoidal membership function.



(a) Two fault states $x_i^{k_i}$



(b) Three fault states $x_i^{k_i}$

Fig. 3. Membership function of the component fault states $x_i^{k_i}$ *with variable r*

According to Fig.3, the deterministic region and the uncertain region of the trapezoidal membership function vary with the value of the fuzzy support radius variable r. Take Fig.3 (b) for an example, when $r = 0$, the trapezoidal membership function is transformed into the triangular membership function, as shown in Fig.4, and when $r = 0.25$, the trapezoidal membership function is transformed into the rectangle membership function, as shown in Fig.5. From Fig.3 (a) and (b), by the calculation of the equation (2) and equation (3), the membership degree of each fault state can be obtained, as shown in Table 1 and Table 2.

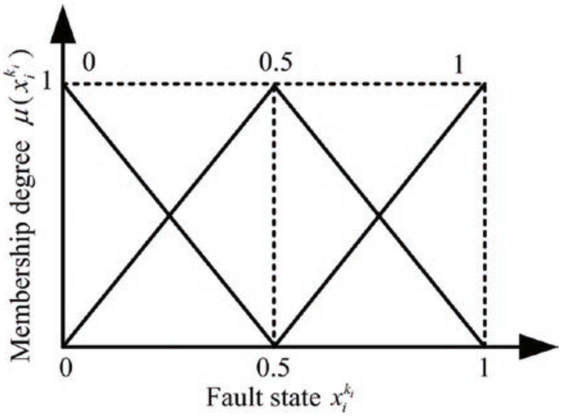


Fig. 4. The membership function of fault state $x_i^{k_i}$ *when r=0*

Table 1. The membership degree of fault states $x_i^{k_i}$ *with two fault states*

Membership degree of $x_i^{k_i}$	0~*r*	*r* ~1-*r*	1-*r* ~1
$\mu_{\tilde{0}}(x_i^{k_i})$	1	$\frac{1-r-x_i^{k_i}}{1-2r}$	0
$\mu_{\tilde{1}}(x_i^{k_i})$	0	$\frac{x_i^{k_i}-r}{1-2r}$	1

Table 2. The membership degree of fault state $x_i^{k_i}$ *with three fault states*

Membership degree of $x_i^{k_i}$	0~*r*	*r*~0.5-*r*	0.5-*r*~0.5+*r*	0.5+*r*~1-*r*	1-*r*~1
$\mu_{\tilde{0}}(x_i^{k_i})$	1	$\frac{0.5-r-x_i^{k_i}}{0.5-2r}$	0	0	0
$\mu_{\widetilde{0.5}}(x_i^{k_i})$	0	$\frac{x_i^{k_i}-r}{0.5-2r}$	1	$\frac{1-r-x_i^{k_i}}{0.5-2r}$	0
$\mu_{\tilde{1}}(x_i^{k_i})$	0	0	0	$\frac{x_i^{k_i}-0.5-r}{0.5-2r}$	1

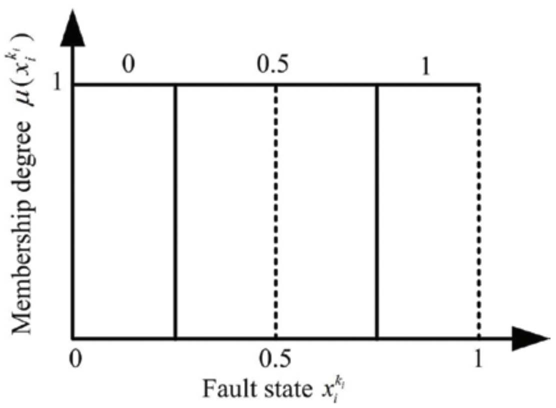


Fig. 5. The membership function of fault state $x_i^{k_i}$ *when r=0.25*

From Table 1 and Table 2, when $x_i^{k_i}\in[0,\ 1]$, $x_i^{k_i}$ all satisfies equation (2) or equation (3). For example, when $x_i^{k_i}\in(r,\ 0.5\text{-}r)$, we substitute $x_i^{k_i}$ in equation (3) and get $\frac{0.5-r-x_i^{k_i}}{0.5-2r}+\frac{x_i^{k_i}-r}{0.5-2r}+0=1$, which verifies the correctness of Table 2. Similarly, Table 1 can be verified.

3.2. The description of conditional probability table

Due to the cognitive limitations on the internal structures, the operational behavior, the constituent element parameters, and the lack of historical data related to the product, the fault logic relationship between the system and its components has a large degree of grey information characteristics in the system. In the process of analysing system reliability, if this relationship is simply represented as the exact value, this will lead to the loss of some important information and the result of system reliability analysis will be deviated. In order to fully exploit system reliability information and clarify the fault logic relationship between the system and its components, interval grey number $\otimes$ defined in interval [0, 1] is used to replace the exact value of conditional probability in traditional Bayesian network. For any grey fuzzy Bayesian network containing *n* nodes with *m* fault states, the conditional probability table can be expressed in Table 3.

In Table 3, each row represents the conditional probability that the child node is in a certain fault state under different combinations of fault states of the parent nodes, for example $P(y=1\,|\,x_1=0,x_2=0,\cdots,x_n=0)=\otimes_{1,1,\cdots,m}$ indicates that the conditional probability that node *y* in the fault state 1 is interval grey number $\otimes_{1,1,\cdots,m}$ when the nodes $x_1, x_2, \ldots, x_n$ are all in the fault state 0, and satisfy the $\otimes_{1,1,\cdots,1}+\cdots+\otimes_{1,1,\cdots,i}+\cdots+\otimes_{1,1,\cdots,m}=1$. In the field of engineering, $\otimes_{1,1,\cdots,m}$ represents the conditional probability of node *y* in completely fault state caused by external factors like human operation errors, environmental factors, and so on.

3.3. System reliability characteristic quantities

3.3.1. Fault state of leaf node

In the grey fuzzy Bayesian network, we assume the root node variable is x_i (*i*=1, 2, …, *n*), the intermediate node variable is y_j (*j*=1,2,…, *m*), and the leaf node variable is *T*. According to the bucket elimination, if the current fault states of nodes x_i are x_1', x_2', …, x_n', the grey fuzzy possibility of the leaf node *T* in the fault state T_q is:

$$P_{\otimes}(T=T_q)=\sum_{\substack{x_1',\cdots,x_n'\\ y_1,\cdots,y_m}} P_{\otimes}(x_1',\cdots,x_n';y_1,\cdots,y_m;T=T_q)$$

$$=\sum_{\lambda(T)} P_{\otimes}(T=T_q\,|\,\lambda(T))\sum_{\lambda(y_1)} P_{\otimes}(y_1\,|\,\lambda(y_1))\times\cdots\times\sum_{\lambda(y_m)} P_{\otimes}(y_m\,|\,\lambda(y_m))\mu_{\tilde{x}_1^{k_1}}(x_1')\times\cdots\times\mu_{\tilde{x}_n^{k_n}}(x_n') \tag{4}$$

In equation (4), $P_{\otimes}(T=T_q)$ is the grey fuzzy possibility of the leaf node *T* in the fault state T_q; $\lambda(T)$ is the parent nodes set of leaf node *T*; $\lambda(y_i)$ is the parent nodes set of intermediate node y_j; $\mu_{\tilde{x}_i^{k_i}}(x_i')$ is the membership degree of the current fault state x_i' corresponding to the fuzzy set.

3.3.2. Grey fuzzy state importance measures

The state importance measure $I_{T_q}^{De}(x_i)$ indicates the possibility which separately causes the system leaf node *T* to be the fault state T_q when the root node x_i is in the fault state x_i'. It reflects the influence degree of the root node x_i in the fault state x_i' to the leaf node *T* in the

Table 3. Conditional probability table of grey fuzzy Bayesian network

x_1	x_2	...	x_n	$P(y=0 \mid x_1,x_2,\cdots,x_n)$	...	$P(y=i \mid x_1,x_2,\cdots,x_n)$	...	$P(y=1 \mid x_1,x_2,\cdots,x_n)$
0	0	...	0	$\otimes_{1,1,\cdots,1}$	...	$\otimes_{1,1,\cdots,i}$	...	$\otimes_{1,1,\cdots,m}$
⋮	⋮		⋮	⋮		⋮		⋮
i	i		i	$\otimes_{i,i,\cdots,1}$	⋮	$\otimes_{i,i,\cdots,i}$	⋮	$\otimes_{i,i,\cdots,m}$
⋮	⋮		⋮	⋮		⋮		⋮
1	1		1	$\otimes_{m,m,\cdots,1}$	...	$\otimes_{m,m,\cdots,i}$	...	$\otimes_{m,m,\cdots,m}$

fault state T_q. The grey fuzzy state importance measure of the root node x_i can be defined as:

$$I_{T_q}^{De}(x_i) = \max\{[P_\otimes(T=T_q \mid x_i = x_i') - P_\otimes(T=T_q \mid x_i = 0)], 0\} \quad (5)$$

In equation (5), $P_\otimes(T=T_q \mid x_i = x_i')$ indicates the grey fuzzy possibility of the leaf node T in the fault state T_q when the root node x_i is in the fault state x_i'; $P_\otimes(T=T_q \mid x_i = 0)$ indicates the grey fuzzy possibility of the leaf node T in the fault state T_q when the root node x_i is in the fault state 0.

3.4. The algorithm for solving system reliability characteristic quantities

When fault state of the component is described by the membership function with the fuzzy support radius variable r, and the conditional probability table containing the interval grey number $\otimes_i$ is used to describe the uncertain fault logic relationship between the system and its components, the parameter planning model of the system reliability characteristic quantities can be constructed, as shown in equation (6). The system reliability characteristic quantities can be obtained by the parameter planning model:

$$\max(\min) f(\otimes_1,\otimes_2,\cdots,\otimes_n)$$
$$s.t.\begin{cases} a_1 \le \otimes_1 \le b_1 \\ a_2 \le \otimes_2 \le b_2 \\ \vdots \\ a_n \le \otimes_n \le b_n \end{cases} \quad (6)$$

The essence of the above parameter planning model is to solve the problem of the extreme value of the function mapping by a series of interval grey numbers in a certain interval, which can be obtained by commercial optimization software, such as Matlab, Isight, and so on. The objective function in the parameter planning model is obtained by above Tables and equations. For the comparison of the reliability characteristic quantities between the nodes, each size of quantity can be determined in the light of the interval value size comparison rule proposed by Nakahara et al.[20].

4. Numerical examples

The two sets of numerical examples are conducted in this section. The first set is a validation experiment based on the example presented by Chen et al.[6]. The second set is an example of satellite propulsion system, which is exemplified to show the advantages of the proposed method in terms of coping with complex uncertainty multi-state systems.

4.1. Set of experiments #1: validation example

To verify the effectiveness of the proposed method, the hydraulic suspension system in the large hydraulic truck is presented in this section. The detailed results and related discussions are as follows.

The large hydraulic truck is a special vehicles with electro-hydraulic driving, steering and lifting. It possesses the characteristics of heavy load handling, manoeuvrability, high stability, and so on, which is widely used in high-speed railway construction, shipbuilding, highway bridges, petrochemical, military and other fields. The hydraulic suspension system is the control system of driving and steering, which plays an important role in the large hydraulic truck.

Take the Bayesian network model of the hydraulic suspension system in reference [6] as an example, according to the presented method in our study, the membership function with fuzzy support variables is established and substitute into the Bayesian network model. It is assumed that the fuzzy support variable $r = 0.1$ and the values in the conditional probability table are all the exact values in the grey fuzzy Bayesian network. In this situation, the model parameters that we constructed are the same as those in reference [6].

4.1.1. Fault states of leaf node for the hydraulic suspension system

With Table 2, equation (4) and equation (6), the grey fuzzy possibility of leaf node T in different fault states is obtained, as shown in Table 4.

The analysis results in Table 4 show that the maximum and minimum values of grey fuzzy possibility of leaf node T in different fault states are the same. Therefore, the analysis results are the same as the previous methods in reference [6], and the correctness and feasibility of the proposed method can be verified in the reliability analysis of leaf node.

Table 4. Grey fuzzy possibility of leaf node T in different fault states

Leaf node	Fault state	Interval value
T	0	[0.082, 0.082]
	0.5	[0.111, 0.111]
	1	[0.807, 0.807]

4.1.2. Grey fuzzy state importance measures for the hydraulic suspension system

With Table 2, and equation (4) to equation (6), state importance measures of root nodes are obtained in the grey fuzzy Bayesian network, as shown in Table 5.

Table 5 shows that the results of the grey fuzzy state importance measures of root nodes are the same as the results in reference [6], and the correctness and feasibility of the proposed method can be verified in the analysis of state importance measures.

Table 5. Grey fuzzy state importance measures of root nodes

x_i	Grey fuzzy state importance measures	
	$I_{0.5}^{De}(x_i)$	$I_1^{De}(x_i)$
x_1	[0.096, 0.096]	[0.000, 0.000]
x_2	[0.000, 0.000]	[0.000, 0.000]
x_3	[0.000, 0.000]	[0.013, 0.013]
x_4	[0.000, 0.000]	[0.031, 0.031]
x_5	[0.000, 0.000]	[0.041, 0.041]
x_6	[0.000, 0.000]	[0.011, 0.011]
x_7	[0.000, 0.000]	[0.029, 0.029]
x_8	[0.000, 0.000]	[0.073, 0.073]
x_9	[0.000, 0.000]	[0.071, 0.071]

From the discussion in this section, it is conclude that in the process of modeling and method validation, setting the fuzzy support radius variable as a fixed value (r = 0.1) and setting the value of the conditional probability table as all the exact values is a special form of the proposed method based on the grey fuzzy Bayesian network, which does not affect the accuracy of verification results. In Section 4.2, we apply the proposed method to the satellite propulsion system to illustrate the advantages of this method.

4.2. Set of experiments #2: Satellite propulsion system

The satellite propulsion system is the power system that implements functions such as satellite aberration, attitude control, orbit reposition, and so on. Its performance directly affects the control accuracy and longevity of the satellite. According to statistics, due to the adverse environment in outer space, the fault possibility of the satellite propulsion system is relatively higher, which is of great significance for the reliability study.

4.2.1. Satellite propulsion system modeling

The structure of monopropellant propulsion system is small and compacted, which is the most commonly used propulsion system in the field of low and medium orbit satellites. It mainly includes tank (TK), feeding valve (FDV), filter (F), self-locking valve (SLV), pressure sensor (PS) and thruster (TH), as shown in Fig.6. The tank provides propellant for thrusters, and the amount of propellant flowing out is determined by the number of thrusters currently in operation. The feeding valve is adding and discharging the pressurizing gas and propellant of the storage tank. The filter is used to filter impurities from the propellant to prevent blockage of the piping system. The self-locking valve is used to control the opening and closing of pipelines and ensure the one-way flow of propellant. The pressure sensor measures the current pressure of the propellant in the pipeline in real time and sends the measured value to the ground receiving equipment. The thruster is the core component of the propulsion system, providing propulsion for the system.

The monopropellant propulsion system adopts redundant structure, where the TH2 is backup branch for the TH1 branch, and if there is a normal operation, the system will work properly. If both branches fail, the system is in the fault state. Because of the uncertain fault logic relationship between the system and its components, when any branch is in fault and another branch is in semi-fault, the system may be in fault, semi-fault, or work properly. Based on Fig.6, Bayesian network of monopropellant propulsion system is constructed, as shown in Fig.7. Node y_1 of series subsystem represents the fault state of the TH1 branch that is formed by connecting PS1, SLV1, F1 and TH1 in series. Similarly, node y_2 of series subsystem represents the fault state of the TH2 branch that is formed by connecting PS2, SLV2, F2 and TH2 in series. And node y_3 represents the fault state of a subsystem that is formed by connecting PS3, FDV1, FDV2 and TK in series. Leaf node T represents reliability of the entire monopropellant propulsion system which is made up of a parallel system y (formed by connecting y_1 and y_2 in parallel) and a series subsystem y_3 in series.

According to analysing the system fault modes and fault mechanisms, the components possessing the three states are the filter and

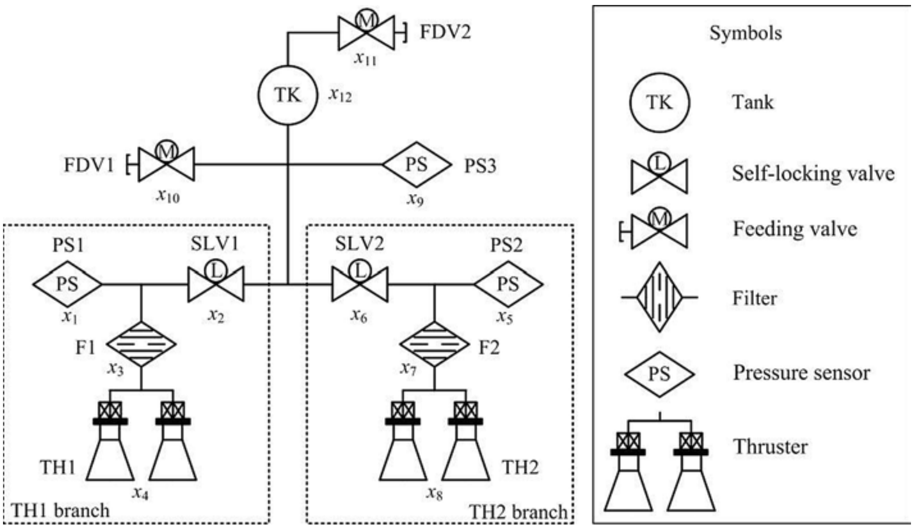


Fig.6. The structure of monopropellant propulsion system

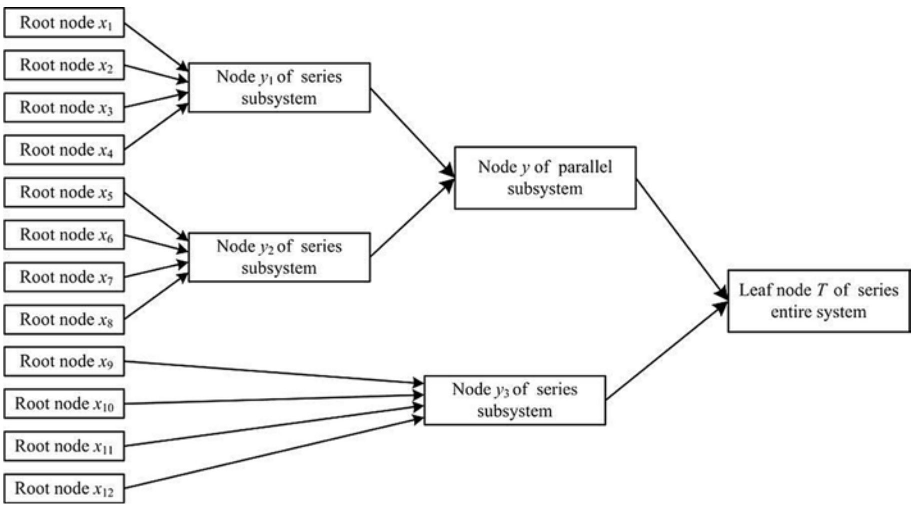


Fig.7. Bayesian network of monopropellant propulsion system

Table 6. Conditional probability table of node y_1

x_1	x_2	x_3	x_4	$P(y_1=\|x_1\sim x_4)$		
				0	0.5	1
0	0	0	0	1	0	0
0	0	0	0.5	$\otimes_1$ [0.18,0.32]	$\otimes_2$ [0.38,0.53]	0.25
0	0	0	1	0	0	1
0	0	0.5	0	0.25	0.6	0.15
0	0	0.5	0.5	0.19	$\otimes_3$ [0.42, 0.58]	$\otimes_4$ [0.27, 0.38]
0	0	0.5	1	0	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	1	0	0	1

Table 7. Conditional probability table of node y_2

x_5	x_6	x_7	x_8	$P(y_2=\|x_5\sim x_8)$		
				0	0.5	1
0	0	0	0	1	0	0
0	0	0	0.5	$\otimes_5$ [0.18,0.32]	$\otimes_6$ [0.38,0.53]	0.25
0	0	0	1	0	0	1
0	0	0.5	0	0.25	0.6	0.15
0	0	0.5	0.5	0.19	$\otimes_7$ [0.42, 0.58]	$\otimes_8$ [0.27, 0.38]
0	0	0.5	1	0	0	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	1	1	1	0	0	1

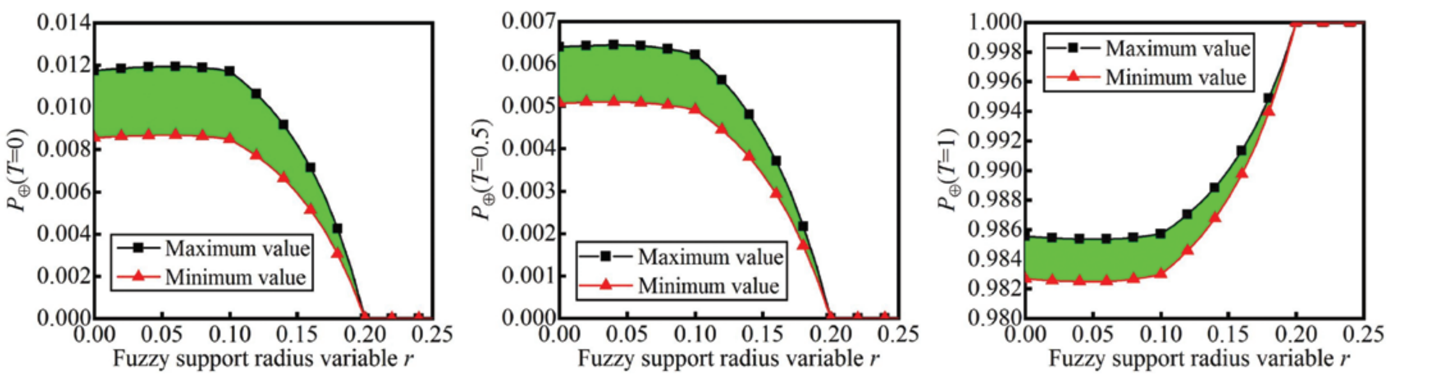
Table 8. Conditional probability table of leaf node T

y_1	y_2	y_3	$P(T=\|y_1\sim y_3)$		
			0	0.5	1
0	0	0	1	0	0
0	0	1	0	0	1
0	0.5	0	1	0	0
0	0.5	1	0	0	1
0	1	0	1	0	0
0	1	1	0	0	1
0.5	0	0	1	0	0
0.5	0	1	0	0	1
0.5	0.5	0	0.26	$\otimes_9$ [0.35, 0.48]	$\otimes_{10}$ [0.25, 0.36]
0.5	0.5	1	0	0	1
0.5	1	0	$\otimes_{11}$ [0.15, 0.28]	0.42	$\otimes_{12}$ [0.32, 0.45]
0.5	1	1	0	0	1
1	0	0	1	0	0
1	0	1	0	0	1
1	0.5	0	$\otimes_{13}$ [0.15, 0.28]	0.42	$\otimes_{14}$ [0.32, 0.45]
1	0.5	1	0	0	1
1	1	0	0	0	1
1	1	1	0	0	1

the thrusters. The multistate of the filter is reflected in a variety of fault modes, namely normal, poor filtering effect and blockage. When the filtering effect is not good, the branch in which the filter is normal, semi-fault and fault has certain possibility. The multistate of the thrusters is shown in the number of the fault thruster. That is, $x_i^0 = 0$, $x_i^{0.5} = 0.5$, $x_i^1 = 1$, i = 3, 4, 7, 8. And other components are deemed to be fault-free and fault, that is, $x_i^0 = 0$, $x_i^1 = 1$, i = 1, 2, 5, 6, 9, 10, 11, 12. According to historical data, engineering experience and expert knowledge [11, 32], with interval grey number ⊗ [19], the conditional probability table is constructed, as shown in Table 6 to Table 8. Each row in Table 6 to Table 8 represents the conditional probability of child node fault under different combinations of fault states of parent nodes.

4.2.2. Fault states of leaf node T for satellite propulsion system

If the current fault state of root nodes are $x'_1 = 0.3$, $x'_2 = 0.4$, $x'_3 = 0.2$, $x'_4 = 0.6$, $x'_5 = 0.3$, $x'_6 = 0.7$, $x'_7 = 0.1$, $x'_8 = 0.7$, $x'_9 = 0.3$, $x'_{10} = 0.4$, $x'_{11} = 0.2$, $x'_{12} = 0.8$. The membership degree of fault state of x_i can be calculated from Table 1 and Table 2, as shown in Table 9 and Table 10.



(a) Leaf node T=0 (b) Leaf node T=0.5 (c) Leaf node T=1

Fig. 8. The grey fuzzy possibility of leaf node T

Table 9. The membership degree of fault state of the root nodes with two fault states

Fault state of x_i	Fuzzy support radius variable r	Membership degree	
		0	1
$x'_1 = 0.3$	$0 \le r \le 0.3$	$\frac{0.7-r}{1-2r}$	$\frac{0.3-r}{1-2r}$
	$0.3 < r \le 0.5$	1	0
$x'_2 = 0.4$	$0 \le r \le 0.4$	$\frac{0.6-r}{1-2r}$	$\frac{0.4-r}{1-2r}$
	$0.4 < r \le 0.5$	1	0
$x'_5 = 0.3$	$0 \le r \le 0.3$	$\frac{0.7-r}{1-2r}$	$\frac{0.3-r}{1-2r}$
	$0.3 < r \le 0.5$	1	0
$x'_6 = 0.7$	$0 \le r \le 0.3$	$\frac{0.3-r}{1-2r}$	$\frac{0.7-r}{1-2r}$
	$0.3 < r \le 0.5$	0	1
$x'_9 = 0.3$	$0 \le r \le 0.3$	$\frac{0.7-r}{1-2r}$	$\frac{0.3-r}{1-2r}$
	$0.3 < r \le 0.5$	1	0
$x'_{10} = 0.4$	$0 \le r \le 0.4$	$\frac{0.6-r}{1-2r}$	$\frac{0.4-r}{1-2r}$
	$0.4 < r \le 0.5$	1	0
$x'_{11} = 0.2$	$0 \le r \le 0.2$	$\frac{0.8-r}{1-2r}$	$\frac{0.2-r}{1-2r}$
	$0.2 < r \le 0.5$	1	0
$x'_{12} = 0.8$	$0 \le r \le 0.2$	$\frac{0.2-r}{1-2r}$	$\frac{0.8-r}{1-2r}$
	$0.2 < r \le 0.5$	0	1

Table 10. The membership degree of fault state of the root nodes with three fault states

Fault state of x_i	Fuzzy support radius variable r	Membership degree		
		0	0.5	1
x'_3=0.2	$0 \le r \le 0.2$	$\frac{0.3-r}{0.5-2r}$	$\frac{0.2-r}{0.5-2r}$	0
	$0.2 < r \le 0.25$	1	0	0
x'_4=0.6	$0 \le r \le 0.1$	0	$\frac{0.4-r}{0.5-2r}$	$\frac{0.1-r}{0.5-2r}$
	$0.1 < r \le 0.25$	0	1	0
x'_7=0.1	$0 \le r \le 0.1$	$\frac{0.4-r}{0.5-2r}$	$\frac{0.1-r}{0.5-2r}$	0
	$0.1 < r \le 0.25$	1	0	0
x'_8=0.7	$0 \le r \le 0.2$	0	$\frac{0.3-r}{0.5-2r}$	$\frac{0.2-r}{0.5-2r}$
	$0.2 < r \le 0.25$	0	1	0

Table 11. Grey fuzzy possibility of leaf node T in fault state 0

Fault state	Fuzzy support radius variable r	Interval value	Difference between maximum and minimum
T=0	0.05	[0.00869,0.01193]	0.00324
	0.10	[0.00849,0.01170]	0.00321
	0.15	[0.00515,0.00715]	0.00200
	0.20	[0.00000,0.00000]	0.00000

Table 12. Grey fuzzy possibility of leaf node T in different fault states

Leaf node	Fault state	Interval value
T	0	[0.007, 0.010]
	0.5	[0.004, 0.005]
	1	[0.985, 0.987]

The objective function in the parameter programming model can be obtained from Table 9, Table 10 and equation (4). And the grey fuzzy possibility of the leaf node T in different fault states can be obtained by solved the parametric programming model with Matlab. The extreme value of the grey fuzzy possibility of the leaf node T in different fault states varies with the fuzzy support radius variable r, as shown in Fig. 8.

(1) From the perspective of system reliability, under the condition of the current fault state of the components, due to the uncertainty caused by the complex structures, the limited test samples, and the insufficient reliability data of the modern systems, the grey fuzzy possibility of leaf node T may be at any point on two curves and in the green area between two curves. Obviously, the results of system reliability analysis are quite different due to uncertainty.

(2) When $0 \le r < 0.2$, the uncertainty of the root node decreases with the increase of the variable r, meanwhile, the difference between the maximum and minimum of the grey fuzzy possibility of fault state of the leaf node T decreases. Take the grey fuzzy possibility of the leaf node $T = 0$ for an example, as shown in Table 11.

(3) When $0.2 < r \le 0.25$, it is calculated that the membership degree of the fault states of the nodes y_1, y_2 change with the value of the variable r, and the membership degree of the fault states of the nodes y_3 is 0. And it is calculated from Table 8 that the grey fuzzy possibility of the leaf node T in different fault states is a straight line which is independent of the value of the variable r.

(4) For generality, the midpoint of the fuzzy support radius variable r is selected, namely $r = 0.125$, to analyse the reliability of the system. Besides, we can also choose the value of the variable r based on expert knowledge. And the membership functions of the fault state of root nodes are trapezoidal membership functions, and grey fuzzy possibility of leaf node T in different fault states is obtained, as shown in Table 12.

From Table 12, according to the comparison rule of interval values size from the reference [20], $P_{\otimes}(T=1) > P_{\otimes}(T=0) > P_{\otimes}(T=0.5)$ is obtained. Under the current fault state of the components, the fault possibility and the fault-free possibility of the satellite propulsion system are higher than the semi-fault possibility, and the fault possibility of system is biggest.

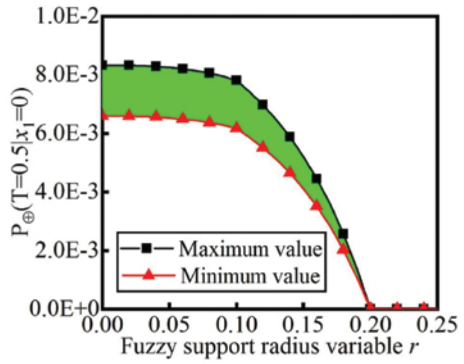


Fig. 9. $P_{\otimes}(T = 0.5 \mid x_1 = 0)$ *changes with variable r*

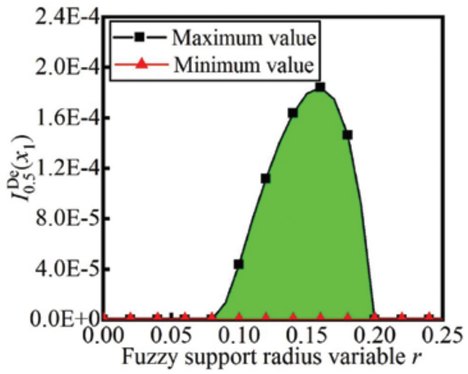


Fig. 10. $I_{0.5}^{De}(x_1)$ *changes with variable r*

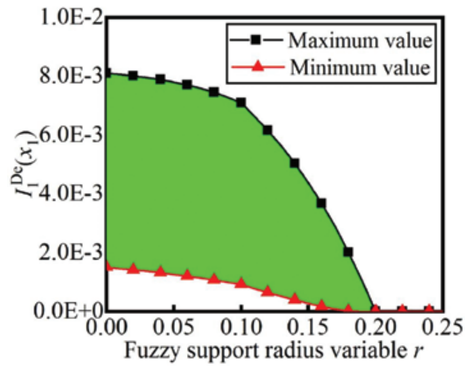


Fig. 11. $I_1^{De}(x_1)$ *changes with variable r*

4.2.3. Grey fuzzy state importance measures for satellite propulsion system

According to the calculation and analysis in Table 9, Table 10, and equation (4) to equation (6), when the fault state of root node x_1 is 0, grey fuzzy possibility of leaf node T in the fault state 0.5 varies with the fuzzy support radius variable r, as shown in Fig.9. The maximum and minimum values of the grey fuzzy state importance of the root node x_1 with leaf node T in the fault state 0.5 varies with the fuzzy support radius variable r, which can be obtained by the equation (5), as shown in Fig.10. Similarly, the curves of the grey fuzzy state importance of the root node x_1 with leaf node T in the fault state 1 can be obtained, as shown in Fig.11. Due to space limitations, the grey fuzzy state importance measures of other root nodes are not listed one by one.

From Fig.10 and Fig.11:

(1) Analysing component from the state importance measures, under the condition of the current fault state of the components, due to the uncertainty caused by the complex structures, the limited test samples, and the insufficient reliability data of the modern systems, $I_{T_q}^{De}(x_i)$ may be at any point on two curves and in the green area between two curves. Obviously, the results of the state importance measures are greatly influenced by the uncertainty.

(2) The state importance measures of root nodes are affected by current fault state of the components and the value of the variable r. When the current fault state of component or variable r is different, the interval values of the state importance measures of root nodes are different, and the weak links of the system are also different. When $r = 0.125$, the interval values of grey fuzzy state importance measures of root nodes x_i with leaf node T in the fault states 0.5 and 1 are obtained, as shown in Table 13.

Table 13. Grey fuzzy state importance measures of root nodes

x_i	Grey fuzzy state importance measures	
	$I_{0.5}^{De}(x_i)$	$I_1^{De}(x_i)$
x_1	[0.000,0.000]	[0.001,0.006]
x_2	[0.000,0.000]	[0.003,0.009]
x_3	[0.000,0.001]	[0.000,0.003]
x_4	[0.002,0.003]	[0.005,0.008]
x_5	[0.000,0.001]	[0.000,0.003]
x_6	[0.000,0.000]	[0.004,0.010]
x_7	[0.000,0.001]	[0.000,0.002]
x_8	[0.000,0.002]	[0.002,0.006]
x_9	[0.000,0.000]	[0.001,0.007]
x_{10}	[0.000,0.000]	[0.005,0.011]
x_{11}	[0.000,0.001]	[0.000,0.004]
x_{12}	[0.000,0.000]	[0.111,0.137]

According to the state importance measures of root nodes, the weak links of the system can be identified. And the reliability of the system can be improved effectively by improving the reliability of the weak nodes.

From Table 13, based on the comparison rules of interval size from the reference [20], the grey fuzzy state importance measures of root nodes such as x_3, x_5, x_7, x_8 with leaf node T in the fault state 0.5 is weaker, and x_4 is the weakest link for the fault state of the system. And the order of the grey fuzzy state importance measures of root nodes with leaf node T in the fault state 1 is: $I_1^{De}(x_{12}) > I_1^{De}(x_{10}) > I_1^{De}(x_6) > I_1^{De}(x_4) > I_1^{De}(x_2) > I_1^{De}(x_9) > I_1^{De}(x_8) > I_1^{De}(x_1) > I_1^{De}(x_{11}) > I_1^{De}(x_3)\,(I_1^{De}(x_5)) > I_1^{De}(x_7)$, obviously, x_{12} is the weakest link for the fault state of the system.

From the discussion in Section 4, it can be concluded that the proposed method can characterize and quantify the fuzzy uncertainty of the fault state of the system with its components and the uncertainty of the logical relationship between the system and its components in an actual system. Besides, utilizing unique bidirectional reasoning ability of Bayesian network, reliability characteristic quantities of system such as reliability of leaf node and state importance measures of root nodes can be effectively analyzed. And some subjective information, such as expert knowledge, may be required for our work. The results are more consistent with the actual engineering situation.

5. Conclusions

A new complex system reliability analysis method based on non-deterministic membership functions and interval characteristic quantities is proposed by introducing fuzzy mathematics and grey system theory to Bayesian network. The trapezoidal fuzzy number (TrFN) with fuzzy support radius variable r is constructed to describe the fuzzy uncertainty of the fault state of the system and its components. The conditional probability table with interval grey numbers is constructed to effectively express the uncertain fault logic relationship between the system and its components. Moreover, a parameter planning model of the system reliability characteristic quantities is constructed, and the system reliability characteristic quantities are expressed as the form of interval values.

Two sets of numerical experiments are carried out and they show the validity and advantages of the proposed method. The obtained results are expressed in the form of interval values, which can better represent reliability characteristic quantities under uncertain conditions caused by the complex structures, the limited test samples, the insufficient reliability data, and so on. It also shows that the proposed method is a powerful reliability analysis method for complex uncertainty multi-state system.

The highlights of the innovations of this article are as follows: (i) the uncertainty, which is caused by human subjective factors on the selection of the membership function and out of strict boundary among states during the fault evolution process, can be quantified by the trapezoidal membership function with fuzzy support radius variable; (ii) the uncertain fault logical relationship between the system and its components can be expressed by the conditional probability table with interval grey numbers, instead of simply setting it to a determined one; (iii) the proposed method can make full use of historical data, engineering experience and expert knowledge, and can effectively analyse the reliability of the system without requiring accurate values in engineering applications.

However, during the reliability analysis, there may be some problems need to be solved, such as common cause failure, multiple failure modes, chain failure, and so on. How to analyse the reliability of complex uncertainty multi-state system with these problems will be studied in our further works.

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