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RELIABILITY MODEL OF MOTORWAY TOLL COLLECTION SYSTEM

MODEL NIEZAWODNOŚCIOWY SYSTEMU POBORU OPŁAT NA AUTOSTRADZIE

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Abstract. *In the paper, a methodology and a concept for establishment of a predictive reliability model of toll collection system at motorway section are presented. Markov processes and methods for projecting exploitation reliability have been used for modelling reliability. Elaboration of the model will allow for defining a projectable availability index of the system and its individual elements. The indices will allow for determining to what extent the system meets specific requirements and for preparing relevant exploitation strategy.*

Keywords*: model, reliability, availability, model, Markov processes, motorway, toll collection system*

Streszczenie: *W referacie przedstawiono metodologię i koncepcję tworzenia predyktywnego modelu niezawodnościowego systemu poboru opłat na autostradzie. W procesie modelowania niezawodności wykorzystano procesy Markowa oraz metody predykcji wykorzystywane do wyznaczania wskaźników niezawodnościowych systemu. Opracowanie modelu umożliwi zdefiniowanie wskaźnika gotowości systemu i jego poszczególnych elementów. Wyznaczone wskaźniki umożliwią określić jaka część systemu spełnia wymagania określone w specyfikacji oraz przygotować odpowiednią strategię eksploatacji systemu. Analiza opracowanego modelu umożliwi określić wstępną gotowość całego systemu potrzebną do wyznaczenia wskaźników niezawodności poszczególnych elementów systemu poboru opłat w następnej części pracy. Celem tworzenia modelu systemu poboru opłat będzie jego analiza i ocena pod względem wymagań niezawodnościowo-eksploatacyjnych określonych przez Generalną Dyrekcję Dróg Krajowych i Autostrad.*

Słowa kluczowe: *gotowość systemu, modelowanie procesu eksploatacji, system poboru opłat, metody predykcji*

1. INTRODUCTION

Majority of European models of Intelligent Transport Systems (ITS) have been prepared basing on guidelines defined under the KAREN project. The project aimed at creation of a program defining the ITS system architecture in the European Union until 2010 [1]. The guidelines defined methodology for creating models of ITS systems, which was mostly based on building of a model taking into consideration data flow between subsystems, modules, and actors using the so-called terminators. The models do not include reliability-exploitation indices allowing for carrying out a reliability and safety analysis of the system. An attempt, which has been presented in this paper, to establish a model aims at development of a research tool allowing for preparation of a work schedule for the toll collection system (SPO) at specific availability and equipment.

2. METHODOLOGY AND ASSUMPTIONS FOR RELIABILITY MODEL OF TOLL COLLECTION SYSTEM AT MOTORWAY

Due to a large complexity of such issue as establishment of the reliability model of Toll Collection System (SPO), the assumptions and methodology for establishment of the model itself are presented below.

Subsequent phases for establishment of the model are as follows:

1. Analysis of individual elements of the Toll Collection System (SPO) from exploitation and reliability point of view,
2. Reliability diagrams of individual elements of the System,
3. Determination of reliability indices values of individual elements of the System using common methods for predicting reliability of technical objects,
4. Determination of a probability function of the System's staying in individual states:
   a) $R_0(t)$ – in state of full serviceability at the moment t,
   b) $Q_{ZB}(t)$ – in state of incomplete serviceability at the moment t,
   c) $Q_B(t)$ – in state of inserviceability at the moment t,
5. Determination of availability indices of the System elements,
6. Reliability diagram of the System,
7. Determination of probability function of the System's staying in individual states,
8. Determination of availability index of SPO system,

Additionally, primary assumptions have been defined when establishing a predictive model for the Toll Collection System:

1. An assumption is made that there are no possibilities for more than one critical element to become damaged, and causing critical damage of the entire object, in a single time span,

2. In case of a critical element damage, the entire object is makes a transition into the repair state,
3. A damage of the object results in its transition into the repair state. The object fully recovers during the repair,
4. Damages intensity value remains constant in time and non-negative: $\lambda \geq 0$.

3. DESCRIPTION AND ANALYSIS OF TOLL COLLECTION SYSTEM AT MOTORWAY

Toll collection for using motorways forces implementation of constantly modernised Systems that would make access or exit to or from a toll payable section difficult to the least possible extent. They are the most complex Systems, as they must be applying a few different processes in real time simultaneously and for all vehicles of their users.

Due to a diversity of users and other exploitation factors, many solutions of the Toll Collection Systems are applicable. The main division refers to toll payable when in motion, or when a vehicle stops. This is of a crucial impact on maintenance of traffic flow at the beginnings and the ends of sections. As it is not too complicated with a stopped vehicle, nevertheless at full speed very short times are required for communication between a gate and a vehicle. Information flow and range are very developed. Certain communication sections are additionally self-controlled during the process, in order to eliminate possible mistakes while toll collection.

Modern Toll Collection Systems being used at motorways in Poland are the Systems based on manual toll collection for transit. Applying this type of systems is conditioned by a low intensity of vehicle traffic on toll payable roads, which has been documented inter alia in periodical research on traffic intensity conducted by General Directorate for National Roads and Motorways (GDDKiA) in 2005.

Analysis of this type of Systems allows for separating basic elements that are part of the toll collection process:

- toll collection area (PPO) with single PPO bays,
- PPO single exit bay (exit island with a kiosk),
- PPO single access-exit bay (access-exit island with a ticket automaton and a kiosk),
- PPO single access bay (access island with a ticket automaton).

The self-service gate diagram has been illustrated on Fig. 1 together with denotations of all the elements. The PPO, presented on Fig. 2, has 3 exit islands (with kiosks), 2 access islands (with ticket automata) and one centre exit-access island (with a kiosk and ticket automaton).

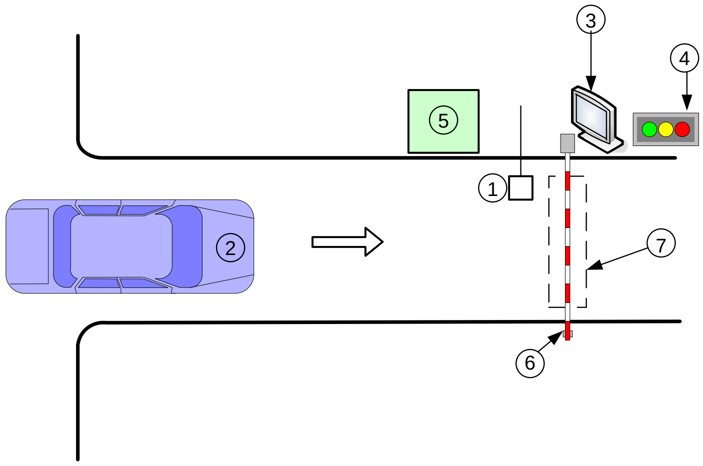

Fig. 1. Diagram of Self-Service Gate with the Following Elements: 1 – Ticket Automaton, 2 – Vehicle, 3 – Information Board, 4 – Signalling Device, 5 – Toll Collection Kiosk, 6 – Automatic Access Barrier, 7 – Transit Loop

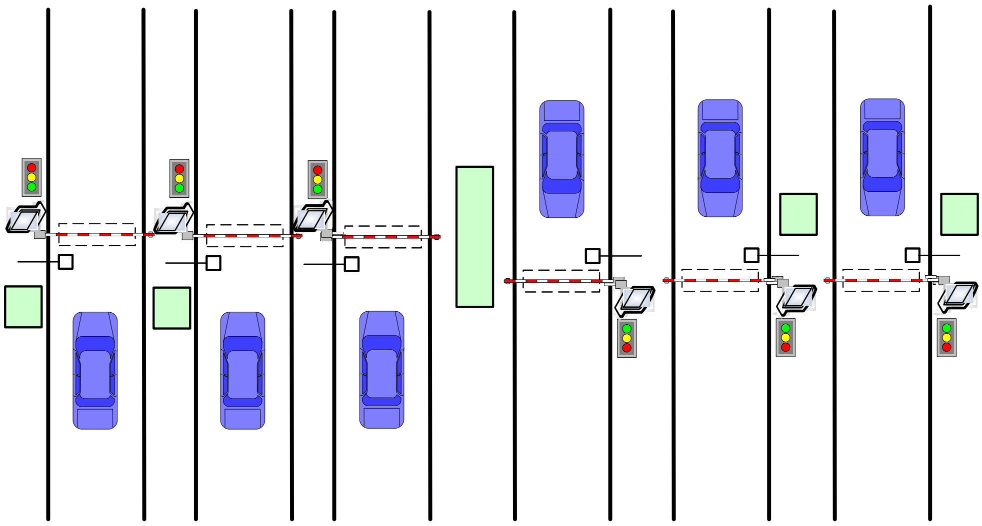

Fig. 2. Diagram of Toll Collection Area

4. RELIABILITY ANALYSIS OF TOLL COLLECTION SYSTEM

Based on operation of a single self-service gate bay, a statement can be made that it has a series reliability structure. A damage of any of the critical elements (damage of the elements leads to critical damage of the object[1]) participating in the toll

[1]The object critical damage stands for critical damages of a machine and/or wrong actions taken by an operator and which can result in failures and accidents, with possible undesired effects in a form of

collection process effects in transition of the bay from its full serviceability state $R_0(t)$ to the inserviceability state $Q_B(t)$. The following critical elements can be distinguished at the exit bay:

1) Ticket automaton;
2) Automatic exit barrier from the island lane at vehicle access;
3) Transit loop.

An assumption has been made that the object, consisting of n = 3 independent critical elements, has a series reliability structure. At the moment of damage to any of the elements there is a loss of the object's serviceability and its transition to the inserviceability state. The damaged element is being repaired. Repair of the critical element recovers its full serviceability.

The following states have been adopted:

k – state of the object's inserviceability, caused by a damage of element with number k,
k = 1, 2, …, n,
k = n+1= 4 – state of the object's serviceability.

Referring to the hereto considered bay, it has been assumed that n = 3.
Fig. 3 illustrates relations occurring in the hereto considered bay in its exploitation aspect.

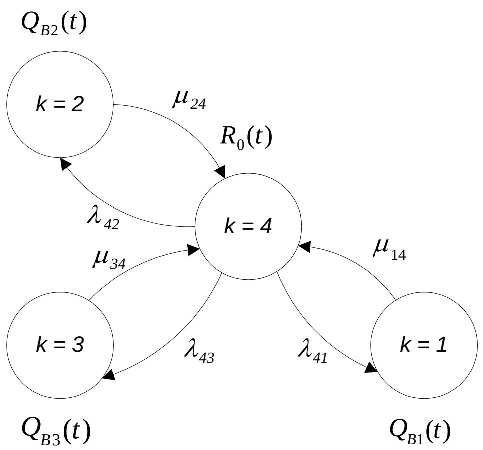

Fig. 3. Relations Occurring in a Single Exit Toll Collection Bay

Denotations on Fig. 3 are as follows:

k – state of the object's inserviceability, caused by a damage of element with number k = 1,…,3,

human injuries, degradation of natural environment and property loss, as well as financial losses and also they can cause various types of civil and legal consequences for decision-makers and operators.

k = n + 1 = 4 – state of the object's serviceability,
$R_0(t)$ – probability function of the bay staying in state of full serviceability,
$Q_{Bk}(t)$ – probability function of the bay staying in state of inserviceability referring to element with number k = 1,…,3,
λ_{4k} – intensity of damages of the bay element with number k = 1,…,3,
μ_{k4} – intensity of repairs of the bay element with number k = 1,…,3,

Transitions intensity matrix corresponding to a graph on Fig. 3 is illustrated as follows:

$$\Lambda = \begin{bmatrix} -\mu_{14} & 0 & 0 & \mu_1 \\ 0 & -\mu_{24} & 0 & \mu_2 \\ 0 & 0 & -\mu_{34} & \mu_3 \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & \lambda \end{bmatrix} \tag{1}$$

Intensity of transitions:

$$\lambda = \sum_{i=1}^{3} \lambda_{4i}$$
$$\mu = \sum_{i=1}^{3} \mu_i \tag{2}$$

The Chapman-Kolmogorov equation system has form of:

$$R'_0(t) = P'_{n+1}(t) = -P_4(t)\sum_{j=1}^{3}\lambda_i + \sum_{j=1}^{3} P_j(t)\,\mu_j$$
$$Q'_{Bk}(t) = P'_i(t) = P_4(t)\lambda_i - P_i(t)\,\mu_i \tag{3}$$

For initial conditions:

$$P_4(t) + \sum_{j=1}^{4} P_j(t) = 1;$$
$$P_4(0) = 1;$$
$$P_i(0) = 0;$$
$$i = 1,\ldots,3 \tag{4}$$

where:

$R_0(t) = P_4(t)$ - probability of the bay staying in state of full serviceability,
$Q_{Bk}(t) = P_i(t)$ - probability of the bay staying in state of inserviceability.

Then, stationary availability of the object amounts to:

$$K_g = P_4 = \frac{1}{1 + \sum_{j=1}^{3} \frac{\lambda_{4j}}{\mu_{j4}}} \qquad j = 1,2 \ldots 3$$
$$P_i = \frac{\lambda}{\mu \left(1 + \sum_{j=1}^{3} \frac{\lambda_{4j}}{\mu_{j4}} \right)} \qquad i = 1,2 \ldots 3 \tag{5}$$

Making a similar analysis of the remaining types of toll collection bays, which can be found at toll collection area illustrated on Fig. 2, the analogical stationary availabilities of objects are presented in the following form.

For a single access toll collection bay, we have as follows:

$$K_g = P_5 = \frac{1}{1 + \sum_{j=1}^{4} \frac{\lambda_{5j}}{\mu_{j5}}} \qquad j = 1,2,\ldots 4$$
$$P_i = \frac{\lambda}{\mu \left(1 + \sum_{j=1}^{4} \frac{\lambda_{5j}}{\mu_{j5}} \right)} \qquad i = 1,2,\ldots 4 \tag{6}$$

For a single access-exit toll collection bay, the stationary availability of the object is as follows:

$$K_g = P_5 = \frac{1}{1 + \sum_{j=1}^{4} \frac{\lambda_{5j}}{\mu_{j5}} + \frac{\lambda_{ZB}}{\mu_{ZB}}} \qquad j = 1,2 \ldots 4$$
$$P_i = \frac{\lambda}{\mu \left(1 + \sum_{j=1}^{4} \frac{\lambda_{5j}}{\mu_{j5}} + \frac{\lambda_{ZB}}{\mu_{ZB}} \right)} \qquad i = 1,2 \ldots 4 \tag{7}$$

Analysing the toll collection process, taking place at the Toll Collection Area (PPO) consisting of bays, a statement can be made that the entire system has a k out of n type of threshold reliability structure. The system is serviceable whenever there is at least k out of n of its elements (toll collection bays). The System has been found fully operational when there is a possibility for servicing a vehicle both accessing and exiting the Toll Collection Area. Each Toll Collection Area is equipped with access-exit bay with a kiosk and ticket automaton. The bay may service vehicles moving both ways. On this basis, a statement can be made that the system has a special case of threshold structure namely structure 1 out of n, that is, a parallel structure. The Fig. 4 below presents relations occurring at the Toll Collection Area (PPO) consisting of 6 toll collection bays 6.

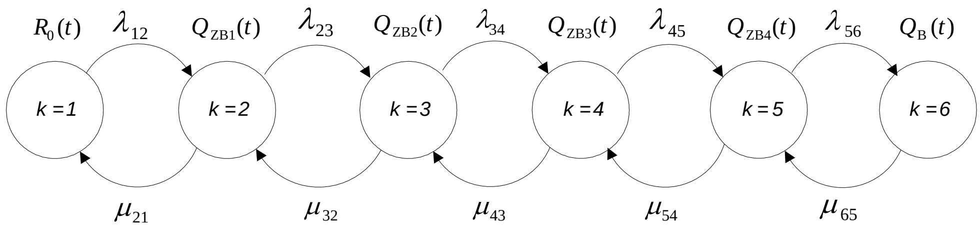

Fig. 4. Relations Occurring at the Toll Collection Area (PPO)

Denotations on Fig. 4 are as follows:

k – individual states of the entire object that is the Toll collection area, k = 1,…,6,

R_0(t) – probability function of the Toll Collection Area staying in state of full serviceability,

Q_B(t) – probability function of the Toll Collection Area staying in state of inserviceability,

Q_{ZBk}(t) – probability function of the Toll Collection Area staying in state of partial serviceability with respect to bay k = 1,…,4, ,

λ_{nk} – intensities of damages of individual bays with number k = 2,..,6, n = k - 1

μ_{kn} – intensities of repairs of individual bays with number k = 2,…,6, n = k – 1

Transition probabilities in graph on Fig. 4 can be arranged in transition matrix:

$$M = \begin{bmatrix} 0 & \lambda_{12} & 0 & 0 & 0 & 0 \\ \mu_{21} & 0 & \lambda_{23} & 0 & 0 & 0 \\ 0 & \mu_{32} & 0 & \lambda_{34} & 0 & 0 \\ 0 & 0 & \mu_{43} & 0 & \lambda_{45} & 0 \\ 0 & 0 & 0 & \mu_{54} & 0 & \lambda_{56} \\ 0 & 0 & 0 & 0 & \mu_{65} & 0 \end{bmatrix} \tag{8}$$

As it is noticeable, Fig. 4 illustrates the process that is a special case of Markov chain. The case allows for transitions only between neighbouring states and it is called the birth-death process. Static balance diagrams are the convenient way for describing the birth-death process. This way of description can be used to make the following system of equations:

$$
\begin{aligned}
&\lambda_{12}R_0(t)^{(n)} = \mu_{21}Q_{ZB1}(t)^{(n)} \\
&\lambda_{12}R_0(t)^{(n)} + \mu_{32}Q_{ZB2}(t)^{(n)} = (\lambda_{23} + \mu_{21})Q_{ZB1}(t)^{(n)} \\
&\lambda_{23}Q_{ZB1}(t)^{(n)} + \mu_{43}Q_{ZB3}(t)^{(n)} = (\lambda_{34} + \mu_{32})Q_{ZB2}(t)^{(n)} \\
&\lambda_{34}Q_{ZB2}(t)^{(n)} + \mu_{54}Q_{ZB4}(t)^{(n)} = (\lambda_{45} + \mu_{43})Q_{ZB3}(t)^{(n)} \\
&\lambda_{56}Q_{ZB4}(t)^{(n)} = \mu_{65}Q_{B}(t)^{(n)}
\end{aligned} \tag{9}
$$

for initial conditions:

$$\sum_{j=6} p_{ij} = 1 \tag{10}$$

$$p_1^{(1)} = 1, \;\; p_2^{(1)} = p_3^{(1)} = p_4^{(1)} = p_5^{(1)} = p_6^{(1)} = 0\,,$$

where:

p_{ij} – probability of the Toll Collection System’s transition from state i to state j;

$p_i^{(1)}$ – initial probability of finding the Toll Collection System in state i;

$p_i^{(n)}$ – probability of finding the Toll Collection System at the moment n in state i;

Values of damages intensity λ will be determined using the standard for reliability assessment of electronic systems MIL-HDBK-217, and NSWC standard for predicting reliability of mechanical elements. Values of repairs intensity μ will be estimated basing on MIL-HDBK-472 standard and by using 2, 5A and 5B procedures as well as MIL-STD-1388 further on in this work.

The reliability and safety indices that have been determined will be helpful when establishing a predictive model of the Toll Collection System, allowing for preparation of the System’s work schedule at defined availability and agreed equipment.

5. CONCLUSION

The hereto performed reliability analysis of the model allows for a preliminary description of availability of the real System and for determining reliability indices in the next phase, using methods for predicting reliability of technical objects. The purpose for establishing a model of the System will be an analysis and assessment of the System, allowing for further development of its work schedule at minimum specified reliability-exploitation parameters, prepared by the General Directorate for National Roads and Motorways (GDDKiA).

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