Numerical, Experimental and Fuzzy Logic Applications for Investigation of Crack Location and Crack Depth Estimation in a Free-Free Aluminum Beam

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Abstract

A beam structure under natural vibration in presence of crack undergoes a sharp change in its dynamic characteristics. In the present study the changes in two important modal vibration parameters like mode shapes and natural frequencies have been extensively studied for crack diagnosis in presence and absence of crack. Numerical and experimental investigations have been carried out using an aluminium Free-Free beam structure with and without crack. The crack presence is indicative of a decrease in local flexibility at crack location and a variation in mode shapes and natural frequencies are noticed. These variations in modal parameters have been used as the tools for crack diagnosis. In the present paper, efforts are made to analyse the presence of a crack using the application of fuzzy logic methodology. Here relative natural frequencies preferably first three are derived from experimental and theoretical investigations are utilised as input data to the fuzzy controller with Gaussian membership functions to obtain crack position and crack depth as output data. The resulted output data from fuzzy logic and the result from corresponding experimental and numerical analysis have been compared.The deviation of result of fuzzy logic from numerical and experimental results have been found to be within a limit of 3% .

Keywords: crack; natural frequency; mode shapes; numerical; fuzzy, etc.

1. Introduction

Maintaining structural integrity is highly essential as cracks are one of the internal damage within the beam structure and its early detection can prevent further degeneration of beam structure. The presence or development of crack in a structure subjected to dynamic vibration is detrimental to the entire system and may lead to decrease in life expectancy.

Crack investigation of a structure based on the changes in its vibration parameters under a dynamic vibration condition have been the area of research since last few decades. In present study, efforts have been made to diagnose the presence of crack in an aluminum beam based on concept of Fuzzy Logic as one of the artificial intelligent techniques and the result outcome is compared with the experimental and theoretical result.

Shen et al. [1] have presented a procedure to evaluate the difference between two corresponding modal characteristics i.e. mode shapes of the beam and its natural frequencies that lead to the identification of crack in a damaged beam. They have simulated the crack by considering an equivalent spring at the crack position. The papers [2, 3] have discussed on related crack characteristics with that of corresponding natural frequencies of the damaged structure. They have compared the result of their methodology with the corresponding result of finite element method to establish the effectiveness of their proposed methodology. Muller et al. [4] have presented a numerical solution that can locate the existence of crack in a rotating shaft–turbo combined rotor assembly. They have established a clear relationship between cracks in the shaft of a turbo-rotor assembly and vibration parameters. The purpose of their study is to show a non-linear effect of the proposed model. The papers [5-7] have introduced a local flexibility at the location of a dynamically vibrating cracked structure and have studied its vibration responses. T Sai et al. [13] have developed a methodology for investigating crack position and crack depth of a vibrating beam structure with an open transverse crack of a stationary shaft without its disengagement from its system assuming a local spring at the crack position. Gounaris et al. [14] have established a suitable methodology for damaged detection in a beam having a transverse cracks which is under a state of dynamic vibration assuming the crack as a transverse open crack. They have presented a co-relation between crack location, crack depth and their corresponding mode difference. The methodology uses Eigen modes of beam structure under vibration mode. Chodros et al. [15] have formulated a model for a Euler-Bernoulli cantilever beam having an edge crack to study the modal characteristics. They have obtained both mode shapes and their corresponding natural frequencies for different crack locations and depths using an Euler-Bernoulli cantilever beam model. They have presented their observation considering variation in crack locations and crack depths. Kisa et al. [16] have established a numerical based procedure for modal analysis of a uniform circular stepped beam with crack. They have demonstrated their combined component mode based synthesis methodology with that of finite element applied to the stepped beam subjected to free vibration. They have established the accuracy by solving some numerical problems. Fabrizio et al. [17] have taken measured value of natural frequency to locate and diagnose the extent of damage in the cracked beam. Duffey et al. [18] have established a novel procedure for crack diagnosis in a vibrating beam structure that exhibits both axial and torsional responses. The proposed methodology can be utilized to investigate the locations of any linear crack in the dynamically vibrating structural beam element using the evaluated modal characteristics. The papers [19, 20] have discussed a crack beam element method for dynamic analysis of cracked vibrating structures. A local flexibility at the crack location site is induced that brings a variation in the dynamic response of a damaged structure which can result in the identification of the crack position and in turn estimation of crack depth. Yang et al. [21] have analyzed the impact of open transverse crack on vibration characteristics of a damaged beam structure. They have presented a suitable numerical method to calculate strain energy possessed by a beam in presence of a crack and computed the equivalent bending stiffness.

Ganguli et al. [22] have used fuzzy logic application to a damaged model structure using the concept of decreased stiffness at the damaged site specifically designed for on ground diagnosis of a rotor mounted blade of a helicopter. Behera et al. [23] have discussed about behavior of crack rotor in viscous liquid. Sazonov et al. [24] have established using fuzzy logic application with the concept of finite element methodology for a simple beam structure and have presented an effective methodology for damage diagnosis. Pawar et al. [25] have intensively studied the variation in the natural frequencies in a damaged structure and modeled it for a cantilever beam in presence of crack using finite element methods and also used genetic fuzzy hybrid technique to locate the crack and estimated the crack size effectively. Skarlatos et al. [26] have studied and presented a simple fuzzy logic technique to diagnose the defects developed in railway wheels by vibration measurements under variable train speed using both healthy and defective wheel separately. De et al. [27] have formulated a module that can be applied to detect and diagnose the severity of damage in laboratory models and equipment leading to the minimization of uncertainties in the measurement associated with input and output parameters in a vibrating structures. Kim et al. [28] have demonstrated an online crack identification procedure for existing concrete structures by taking the concept of artificial intelligence technique in fuzzy environment. They have designed their fuzzy inference system using the symptoms of presence of crack and their characteristics. They have presented the result of the application of their proposed system and found the result of the proposed system is matching to the corresponding result of the expert system. Kim et al. [29] have established a computerized assisted programmer using fuzzy set theory that will ensure the symptoms of presence of crack and their characteristics for a reinforced concrete structures. Das and Parhi [30, 31], have studied a Fuzzy logic approach based research paper to locate crack position and to estimate the crack severity by diagnosing the crack depth of a dynamically vibrating beam and have successfully used the first three relative mode shapes and their corresponding relative natural frequencies as data input parameters to the developed Fuzzy inference systems and obtained two relative output parameters as crack position and depth of crack. They have arrived at a conclusion that the output result matched precisely with the experimental values. Sasmal et al. [34] have presented a systematic methodology based on analytical hierarchy approach in a fuzzy surroundings for condition monitoring of constructed bridges to eliminate any imprecision and associated uncertainties in the measurements. Saravanan et al. [35] have studied the dynamic characteristics of machine parts in running condition and monitor the health conditions of inaccessible parts and components effectively. To formulate the rules automatically, they have proposed a model, developed using fuzzy classifier and decision tree. The result of the developed fuzzy inference engine using representative data have been found to be quite encouraging. Chandrashekhar et al. [36] have applied the fuzzy logic approach to avoid any uncertainties in geometry and measurements during the investigation of damage diagnosis. They have shown that the fuzzy logic together with probabilistic analysis can remove the uncertainties in the measurements due to geometry of damaged structures. The paper combines the purviews of probabilistic analysis and theory fuzzy logic to rectify uncertainties associated with structural crack diagnosis. Chandrashekhar et al. [37] have presented a novel concept by exploring a relationship between the changes in material properties and corresponding changes in vibration parameters (frequency) and have presented a fuzzy based model with a novel defuzzifier for crack diagnosis. Beena et al. [38] have used a novel algorithmic based approach on fuzzy cognitive map (FCM), appropriate for structural damage diagnosis. They have used relative changes in frequency deviations as input parameters for the FCM and the result of output parameters are the possible relative crack locations in the damaged structure. The papers [39-51] have presented a discussion about use of various artificial intelligence techniques in engineering problems. In the current paper a systematic AI approach has been used to locate a crack and its depth using natural frequencies as one of the modal characteristics of a beam under vibration.

2. Theoretical Analysis for the determination of beam modal characteristics in the existence of a transverse single crack

A theoretical analysis of a beam with free-free end having a transverse crack subjected to both bending and axial load has been considered in the present study. The stiffness matrix in presence of crack is derived as the inverse of compliance matrix. An equivalent compliance matrix is derived assuming crack node as a cracked element having no mass and length.

Let U_i = additional displacement due to bending load and axial load.

V_t = beam strain energy in presence of crack.

Let U_i = additional or extra displacement due to both axial and bending load.

V_t = strain energy in presence of the crack**.**

$$U_i = \frac{\partial V_t}{\partial P_i} \tag{1}$$

where V_t can be expressed as

$$V_t = \int_0^{a_1} \frac{\partial V_t}{\partial a} da = \int_0^{a_1} J_r(a) da \tag{2}$$

where $J_r = \dfrac{\partial V_t}{\partial a}$ is energy release rate due to strain and a_1 as crack depth.

By the Pari's equation, the additional displacement can be mathematically written as

$$\frac{\partial}{\partial P_i}\left[\int_0^{a_1} J_r(a) da\right] = U_i \tag{3}$$

The components of local flexibility matrix per unit width can be expressed as

$$\alpha_{ij} = \frac{\partial U_i}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j} \int_0^{a_1} J_r(a)\ da \tag{4}$$

The resulting flexibility matrix

[α_{ij}] over the total breadth B for the beam with edge crack can be written as

$$\left[\alpha_{ij}\right] = \frac{\partial U_i}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j} \int_{-B/2}^{+B/2} \int_0^{a_1} J_r(a) dadz \tag{5}$$

As per (Tada. 1973); the expression for release rate of strain energy at crack location is written as

$$J_r = \frac{1-v_1^2}{E}(\mathrm{K}_{11}+\mathrm{K}_{12})^2 \tag{6}$$

$\frac{1}{E} = \frac{1-v_1^2}{E} = \frac{1}{E'}$ (plane strain and plane stress based condition)

Factors affecting stress intensity represented as K_{11}, K_{12} for mode I (crack opening) subjected under the load P_1 and P_2 defined as axial and bending load respectively. Mathematically the Factors affecting stress intensity be expressed in the form as below;

$$K_{ij} = \sigma_i \sqrt{\pi a}\left(F_i\left(\frac{a}{W}\right)\right) \tag{7}$$

where σ_i is stress at the cross location of crack due to axial and bending load, so,

$$K_{11} = \frac{P_1}{BW}\sqrt{\pi a}\left(F_1\left(\frac{a}{W}\right)\right),\ K_{12} = \frac{6P_2}{BW^2}\sqrt{\pi a}\left(F_2\left(\frac{a}{W}\right)\right)$$

where terms F_1 and F_2 can be expressed as

$$F_1\left(\frac{a}{W}\right) = \left\{\frac{2W}{\pi a}\tan\left(\frac{\pi a}{2W}\right)\right\}^{0.5}\left\{\frac{0.751+2.02a/W+0.369(1-\sin(\pi a/2W))^3}{\cos(o.5\pi a/W)}\right\} \tag{8}$$

$$F_2\left(\frac{a}{W}\right) = \left\{\frac{2W}{\pi a}\tan\left(\frac{0.5\pi a}{W}\right)\right\}^{0.5}\left\{\frac{0.922+0.191(1-\sin(\pi a/2W))^4}{\cos(0.5\pi a/W)}\right\} \tag{9}$$

Using equation (6) for strain energy release rate and putting it in equation (4), the flexibility matrix can be mathematically written as:

$$\alpha_{ij} = \frac{\partial^2}{\partial P_i \partial P_j}\frac{B}{E'}\int_0^{a_1}\left(K_{11}+K_{12}\right)^2 da \tag{10}$$

Taking a/W=φ, then $da/W = \mathrm{d}\varphi$

We obtain $\mathrm{W\,d}\varphi = \mathrm{da}$ and then φ=0; when $a = 0$

$a = a_1,\ \varphi = a_1/W = \varphi_1$

From the relation above, the equation (10) becomes,

$$\alpha_{ij} = \frac{BW}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int_0^{\varphi_1}\left(K_{11}+K_{12}\right)^2 d\varphi \tag{11}$$

From the equation (10), calculating $\alpha_{11}, \alpha_{12}(=\alpha_{21})$ and α_{22} we get

$$\alpha_{11} = \frac{BW}{E'} \int_0^{\varphi_1} \frac{\pi a}{B^2 W^2} 2(\mathrm{F}_1(\varphi_1))^2 d\varphi == \frac{2\pi}{BE'} \int_0^{\varphi_1} (\varphi_1)(\mathrm{F}_1(\varphi_1))^2 d(\varphi_1) \tag{12}$$

$$\alpha_{12} = \alpha_{21} = \frac{12\pi}{E'BW} \int_0^{\varphi_1} (\varphi_1)\, \mathrm{F}_1(\varphi_1)\, \mathrm{F}_2(\varphi_1) d(\varphi_1) \tag{13}$$

$$\alpha_{22} = \frac{72\pi}{E'BW^2} \int_0^{\varphi_1} \mathrm{F}_2(\varphi_1)\, \mathrm{F}_2(\varphi_1)\ d(\varphi_1) \tag{14}$$

The inversion of compliance matrix gives rise to a local stiffness matrix expressed below as $K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}^{-1}$.

Figure 1 Represents a free-free beam with amplitudes in longitudinal vibration for the sections as U_1, U_2 and vibrational amplitudes under the application of bending at the sections as Y_1, Y_2 respectively. In following Figure 1, L_1 and L represent crack distance from one end of free-free beam and length of the respectively.

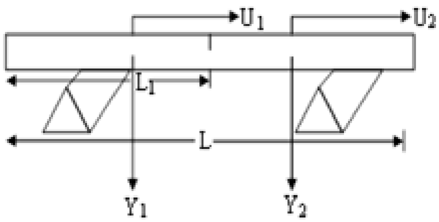

Figure 1. Free-free beam

normal function defined for of the system

$$\bar{U}_1 = \mathrm{C}_1\cos(\bar{\mathrm{K}}_u \bar{\mathrm{x}}) + \mathrm{C}_2\sin(\bar{\mathrm{K}}_u \bar{\mathrm{x}}) \tag{15a}$$

$$\bar{U}_2 = \mathrm{C}_3\cos(\bar{\mathrm{K}}_u \bar{\mathrm{x}}) + \mathrm{C}_4\sin(\bar{\mathrm{K}}_u \bar{\mathrm{x}}) \tag{15b}$$

$$\bar{Y}_1(\bar{\mathrm{x}}) = \mathrm{C}_5\cosh(\bar{\mathrm{K}}_y \bar{\mathrm{x}}) + \mathrm{C}_6\sinh(\bar{\mathrm{K}}_y \bar{\mathrm{x}}) + \mathrm{C}_7\cos(\bar{\mathrm{K}}_y \bar{\mathrm{x}}) + \mathrm{C}_8\sin(\bar{\mathrm{K}}_y \bar{\mathrm{x}}) \tag{15c}$$

$$\bar{Y}_2(\bar{\mathrm{x}}) = \mathrm{C}_9\cosh(\bar{\mathrm{K}}_y \bar{\mathrm{x}}) + \mathrm{C}_{10}\sinh(\bar{\mathrm{K}}_y \bar{\mathrm{x}}) + \mathrm{C}_{11}\cos(\bar{\mathrm{K}}_y \bar{\mathrm{x}}) + \mathrm{C}_{12}\sin(\bar{\mathrm{K}}_y \bar{\mathrm{x}}) \tag{15d}$$

where,

$$\frac{U}{\bar{U}} = \frac{X}{\bar{\mathrm{X}}} = \frac{Y}{\bar{\mathrm{Y}}} = L, \frac{L_1}{L} = \gamma$$

$$\frac{L\omega}{C_u} = \bar{K}_u, \left(\frac{L^2\omega}{C_y}\right)^{1/2} = \bar{K}_y \;, \left(\frac{E}{\rho}\right)^{1/2} = C_u \;, \left(\frac{IE}{\mu}\right)^{1/2} = C_y \;, \mu = \rho A$$

$$C_i \left(i = 1{,}12 \right)$$

are constants and its values can be calculated using following boundary conditions.

$$\bar{U}_1'(0) = 0;\; \bar{Y}_1''(0) = 0; \bar{Y}_1'''(0) = 0; \bar{U}_2'(1) = 0;\; \bar{Y}_2''(1) = 0;\; \bar{Y}_2'''(1) = 0;$$

At the cracked location:

$$\overline{U}_1(\gamma) = \bar{U}_2'(\gamma);\; \overline{Y}_1(\gamma) = \overline{Y}_2(\gamma);\; \bar{Y}_1''(\gamma) = \bar{Y}_2''(\gamma);\; \bar{Y}_1'''(\gamma) = \bar{Y}_2'''(\gamma)$$
$$\bar{U}_2'(\gamma) = \bar{U}_3'(\gamma);\; \bar{Y}_2(\gamma) = \bar{Y}_3(\gamma);\; \bar{Y}_2''(\gamma) = \bar{Y}_3''(\gamma);\; \bar{Y}_2'''(\gamma) = \bar{Y}_3'''(\gamma)$$

Also at the section of the crack i.e. at distance L_1, due to absence of axial displacement on both left side and right side of the location of the crack, we have:

$$AE\,\frac{dU_1(L_1)}{dX} = K_{11}\,U_2(L_1) + \left(K_{12}\,\frac{dY_2(L_1)}{dX} - K_{12}\,\frac{dY_1(L_1)}{dX}\right) - K_{11}U_1(L_1)$$

Multiplying, the term $\dfrac{AE}{K_{11}K_{12}L}$ in the expression above, we get

$$M_1\, M_2\overline{U}\,'(\gamma) = M_2\overline{U}_2(\gamma) + M_1\bar{Y}_2'(\gamma) - M_1\,\bar{Y}_1'(\gamma)$$

Similarly, at crack location due to slope discontinuity at both sides of the crack

$$EI\,\frac{d^2Y_1(L_1)}{dX^2} = (K_{21}\,U_2(L_1) - K_{21}U_1(L_1)) + \left(K_{22}\,\frac{dY_2(L_1)}{dX} - K_{22}\,\frac{dY_1(L_1)}{dX}\right)$$

Multiplying, the term $\dfrac{EI}{L^2K_{22}K_{21}}$ on both sides of the expression above , we get

$$M_3\, M_4\overline{Y_1}\,''(\gamma) = M_3\overline{U}_2(\gamma) - M_3\overline{U}_1(\gamma) + M_4\bar{Y}_2'(\gamma) - M_4\,\bar{Y}_1'(\gamma)$$

where,

$$M_1 = \frac{AE}{LK_{11}}, M_2 = \frac{AE}{K_{12}}, M_3 = \frac{EI}{LK_{22}}, M_4 = \frac{EI}{L^2K_{21}}$$

The boundary conditions as defined above and normal functions along with equation (15) result in the system characteristic equation as

$$\left|\psi\right| = 0 \tag{16}$$

where ψ is a 12x12 matrix.

This characteristic equation in determinant form expressed above is a function of relative crack position (γ), local stiffness matrix (K) , natural circular frequency (ω) and relative crack depth (φ).

Figure 2 represents the pictorial view of a free-free end beam geometry with dimensions (a_1) as crack depth, (L_1) as crack location and (BxW) as cross-sectional area which is under the influence of axial force (P_1) and a bending moment (P_2) that subject the beam to a coupling effect.

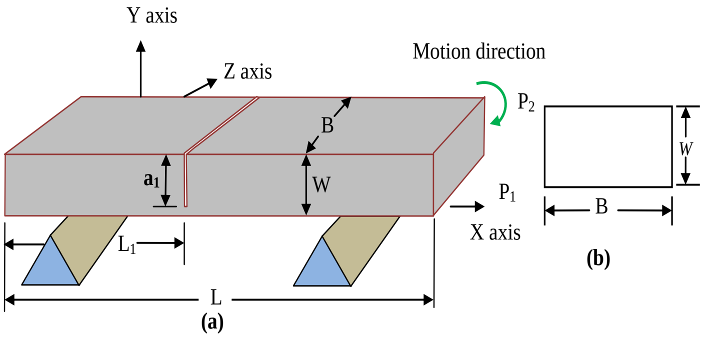

Figure 2. Beam Geometry (a) free-free beam (b) beam crossectional view

3. Numerical calculations for natural frequencies considering parameters like relative crack depths and its locations to establish its validation with that of corresponding experimental result

Tables 1-5 present a numerical analysis to calculate first three natural frequencies(relative) by varying crack depths (relative) at different crack locations(relative) to establish a database for validating the results with that of corresponding experimental results.

Table 1. Relative natural frequencies for fixed crack location = 0.15 at various crack depths

Free-Free			
Relative crack depth	**crack location(relative) = 0.15**		
	Relative 1st natural frequency	**Relative 2nd natural frequency**	**Relative 3rd natural frequency**
0.05	0.96427	0.97409	0.95994
0.15	0.96204	0.97248	0.95744
0.25	0.95982	0.97087	0.95495
0.35	0.95759	0.96925	0.95245
0.45	0.95537	0.96764	0.94996
0.55	0.95314	0.96603	0.94746

Table 2. Relative natural frequencies for fixed crack location = 0.25 at various crack depths

Free-Free			
Relative crack depth	**crack location(relative) = 0.25**		
	Relative 1st natural frequency	**Relative 2nd natural frequency**	**Relative 3rd natural frequency**
0.05	0.98238	0.93647	0.93146
0.15	0.98128	0.93251	0.92720
0.25	0.98018	0.92856	0.92293
0.35	0.97909	0.92460	0.91866
0.45	0.97799	0.92065	0.91439
0.55	0.97689	0.91669	0.91012

Table 3. Relative natural frequencies for fixed crack location = 0.35 at various crack depths

Free-Free			
Crack depth (relative)	**crack location(relative) = 0.35**		
	Relative 1st natural frequency	**Relative 2nd natural frequency**	**Relative 3rd natural frequency**
0.05	0.96076	0.93628	0.97909
0.15	0.95795	0.93232	0.97779
0.25	0.95549	0.92835	0.97649
0.35	0.95330	0.92438	0.97518
0.45	0.95056	0.92041	0.97388
0.55	0.94809	0.91644	0.97258

Table 4. Relative natural frequencies for fixed crack location = 0.45 at various crack depths

Free-Free beam			
Relative crack depth	**crack location(relative) = 0.45**		
	Relative 1st natural frequency)	**Relative 2nd natural frequency)**	**Relative 3rd natural frequency**
0.05	0.93660	0.95338	0.93502
0.15	0.93265	0.95048	0.93097
0.25	0.92869	0.94758	0.92692
0.35	0.92475	0.94468	0.92287
0.45	0.92080	0.94177	0.91883
0.55	0.91685	0.93887	0.91478

Table 5. Relative natural frequencies for fixed crack location = 0.55 at various crack depths

Free-Free beam			
Crack depth (relative)	**crack location(relative) = 0.55**		
	Relative first natural frequency	**Relative second natural frequency**	**Relative third natural frequency**
0.05	0.93642	0.95151	0.93596
0.15	0.92959	0.94849	0.93197
0.25	0.92546	0.94547	0.92798
0.35	0.92133	0.94245	0.92399
0.45	0.91720	0.93943	0.92000
0.55	0.91308	0.93641	0.91601

4. Experimental procedure

Initially an aluminum Free-Free beam specimen of length 100cm, width 50mm and thickness 8 mm is prepared for the purpose of carrying out experiment. Before the experiment is carried out, a transverse crack is created using a wire cut electrode discharge machine. Following the same procedure, a no of specimen is made with cracks at different relative crack locations from fix end (0.15, 0.25, 0.35, 0.45, 0.55) with different relative crack depths (0.15, 0.25, 0.35, 0.45, 0.55) in each specimen. The specimen under test is a free-free beam. The specimen was put to test to study its vibration characteristics. A vibration testing machine (pulse lite 3560-L machine) was used as a part of the experimental procedure to study the variations in mode shapes and natural frequency of a vibrating beam with and without a crack. Specific experiments were conducted step by step with varying locations and crack depths in a systematic manner. As a part of experimental procedure, the specimen was placed in a test rig fig. (3) with proper end conditions and tests were conducted systematically considering different parameters like crack locations, and crack depths. The initial excitation was given on the middle span of the beam using a specialized hammer. To register the 1st, 2nd and 3rd modes of vibration, a piezoelectric accelerometer was placed along the length of beam. The signals representing natural frequency and mode shapes were captured in a vibration analyzer consisting of frequency response spectrum, printer and a desk top computer with dual channel analyzer.

Figure 3. Experimental setup of a free-free aluminum beam

5. Graphical presentation of relative mode shapes of experimental and numerical result

In the following section a graphical presentation of 1st, 2nd and 3rd mode shapes resulted from numerical and experimental investigations have been presented systematically to understand the influence of locations and depths of crack on its mode shapes in a vibrating beam structures. The significant variations in the modal parameter like mode shapes will assist in locating the crack and estimating the crack depth.

The following figures (Figures 4-12) represent a graphical presentation of 1st, 2nd and 3rd modes of transverse vibration for validating numerical and experimental results at various relative crack locations and depths in presence of a transverse edged crack. A significant variation in relative amplitudes are observed beyond the crack location for any given relative crack location and depth.

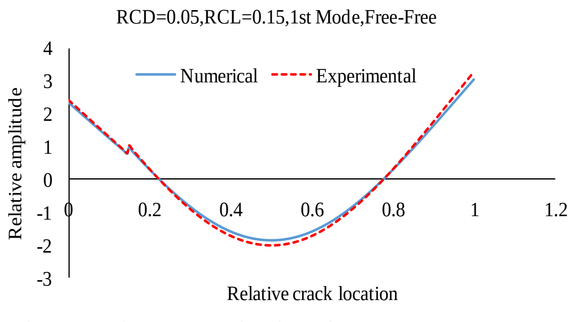

Figure 4. View of 1st mode of vibration for free-free beam with RCD = 0.05, RCL = 0.15

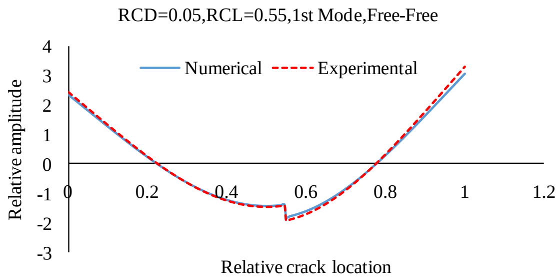

Figure 5. View of 1st mode of vibration for free-free beam with RCD = 0.05, RCL = 0.55

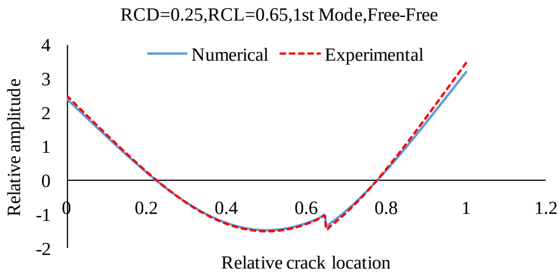

Figure 6. View of 1st mode of vibration for free-free beam with RCD = 0.25, RCL = 0.65

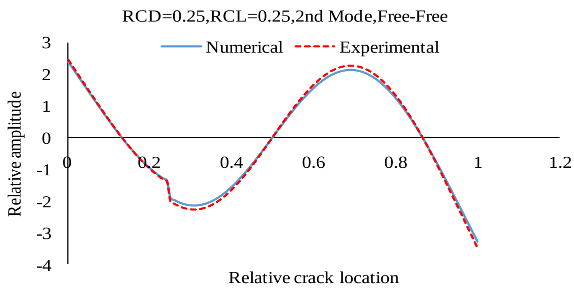

Figure 7. View of 2nd mode of vibration for free-free beam with RCD = 0.25, RCL = 0.25

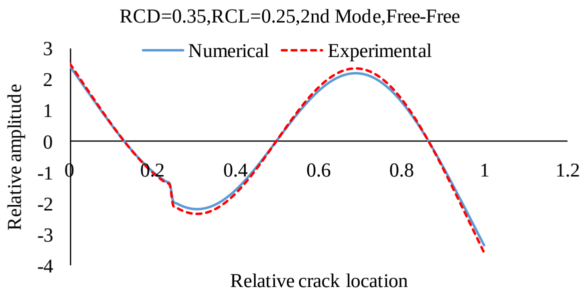

Figure 8. View of 2nd mode of vibration for free-free beam with RCD = 0.35, RCL = 0.25

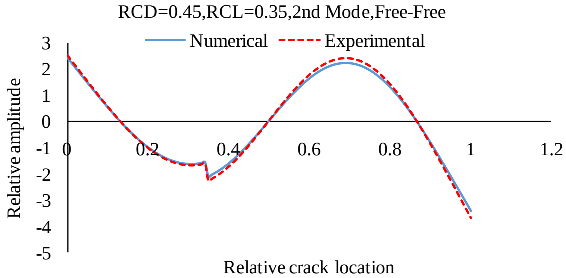

Figure 9. View of 2[nd] mode of vibration for free-free beam with RCD = 0.45, RCL = 0.35

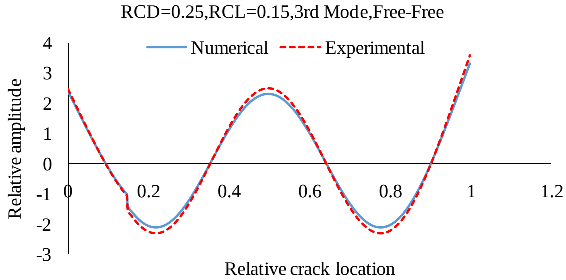

Figure 10. View of 3[rd] mode of vibration for free-free beam with RCD = 0.25, RCL = 0.15

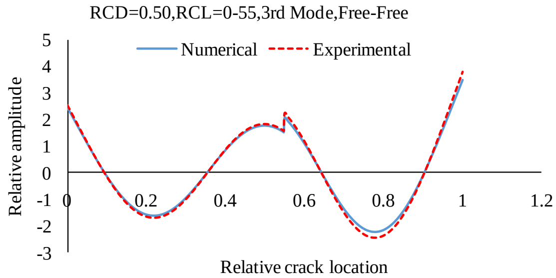

Figure 11. View of 3[rd] mode of vibration for free-free beam with RCD = 0.50, RCL = 0.55

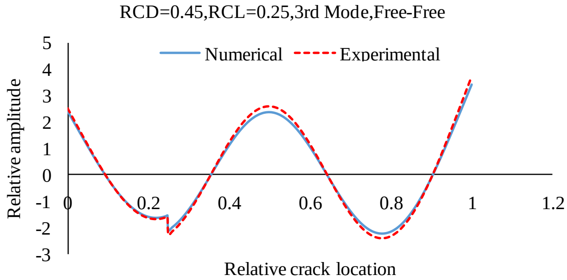

Figure 12. View of 3[rd] mode of vibration for free-free beam with RCD = 0.45, RCL = 0.25

6. Application of fuzzy Logic for crack diagnosis

Fuzzy logic refers to a computing based approach that considers quantity of truths without assigning numerical for true or false i.e. (1,0). This approach uses specific functions for its linguistic variables. Fuzzy logic has wide area of applications ranging from control theory to artificial intelligence. Traditional computing makes use of precise data with certainty but soft computing can use imprecise data and can compute to generate precise output. Fuzzy logic employs words rather than numbers for defining certain mapping rules.

In this present chapter, an algorithm for crack detection using the concept of fuzzy logic has been formulated. This formulation uses three no of input modal data i.e. three initial relative natural frequencies to obtain two output parameters i.e. first two relative crack positions and first two relative crack intensity or depth. The fuzzy system has been designed using the modal characteristic vibration data of a cracked beam under transverse vibration with different end conditions. The current chapter presents a brief account of a working of a fuzzy logic system which is categorized under fuzzy inference system. Figs 13(a), 13(b) and 13(c) represent trapezoidal, Gaussian and triangular membership functions which use first three input data as relative natural frequencies and give rise to two output data as relative crack location and depth. Fig 14 represents a fuzzy inference system. Table 6 represents the comparison of output of different membership functions with that of Experimental results in an attempt to search for a particular membership functions that has close output results as with that of experimental results. The result using Gaussian membership function controller are closely matching with the experimental results. Table 7 shows the comparative result of Fuzzy logic, numerical and experimental investigations considering five different test samples.

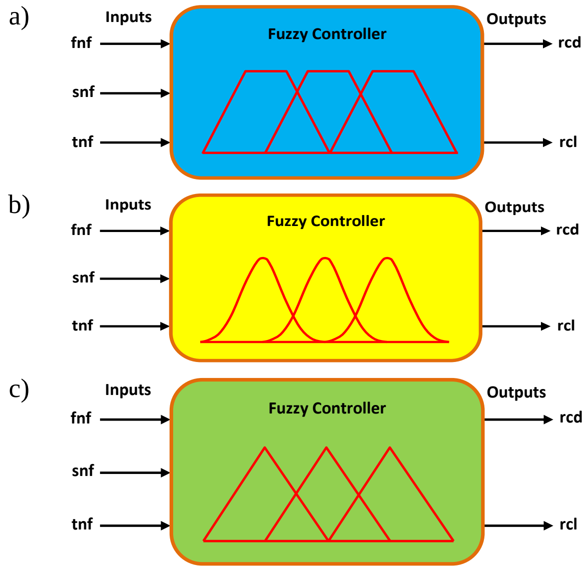

Figure 13. Fuzzy controller: a) Trapezoidal, b) Gaussian, c) Triangular

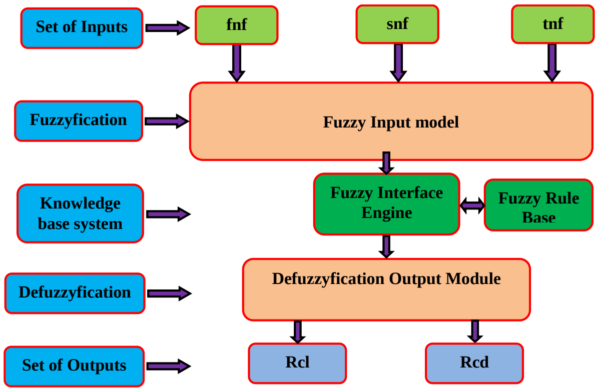

Figure 14. Flow diagram of a Fuzzy inference system

Table 6. The comparative results from theoretical and various Fuzzy Controller Analysis for free-free beam

Relative 1st natural frequency "rfnf"	Relative 2nd natural frequency "rsnf"	Relative 3rd natural frequency "rtnf"	Experimental		Fuzzy Triangular Controller		Fuzzy Trapezoidal Controller		Fuzzy Gaussian Controller	
			Relative crack depth	Relative crack location	Relative crack depth	Relative crack location	Relative crack depth	Relative crack location	Relative crack depth	Relative crack location
0.98187	0.93305	0.92793	0.14718	0.24607	0.14429	0.24338	0.14441	0.24356	0.14467	0.24377
0.95818	0.93311	0.97832	0.14529	0.34796	0.14234	0.34357	0.14249	0.34381	0.14286	0.34394
0.92913	0.94803	0.92738	0.24706	0.44901	0.24427	0.44549	0.24442	0.44553	0.24467	0.44579
0.92278	0.94391	0.92481	0.34391	0.54311	0.34219	0.54220	0.34227	0.54234	0.34248	0.54251
0.91344	0.93667	0.91666	0.54833	0.54858	0.54344	0.54488	0.54359	0.54499	0.54386	0.54528

Table 7. The percentage deviation of fuzzy result from experimental result in a free-free beam

Relative 1st natural frequency "rfnf""	Relative 2nd natural frequency "rsnf"	Relative 3rd natural frequency "rsnf"	Fuzzy model		Numerical model		Experimental model		% deviation of fuzzy output from experimental result	
			Relative crack depth	Relative crack location	Relative crack depth	Relative crack location	Relative crack depth	Relative crack location	Relative crack depth	Relative crack location
0.98187	0.93305	0.92793	0.14467	0.24377	0.14701	0.24603	0.14718	0.24607	1.73	0.94
0.95818	0.93311	0.97832	0.14286	0.34394	0.14507	0.34789	0.14529	0.34796	1.70	1.16
0.92913	0.94803	0.92738	0.24467	0.44579	0.24687	0.44889	0.24706	0.44901	0.97	0.72
0.92278	0.94391	0.92481	0.34248	0.54251	0.34379	0.54288	0.34391	0.54311	0.41	0.11
0.91344	0.93667	0.91666	0.54386	0.54528	0.54811	0.54840	0.54833	0.54858	0.82	0.60

Figure 15(a)-(b) present a graph of the relative crack depths and locations against no. of test samples for Fuzzy logic, numerical and experimental results.

a) b)

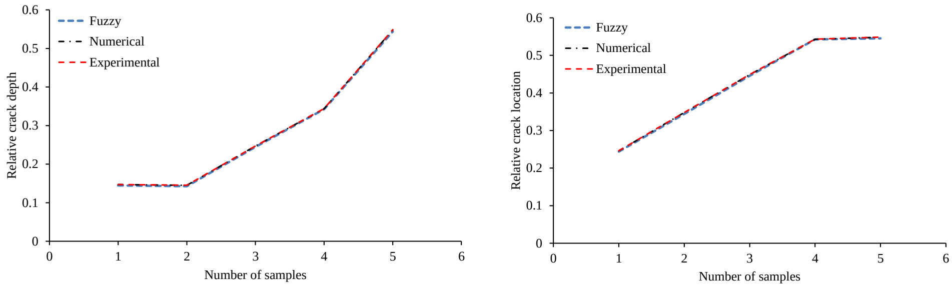

Figure 15. a) Crack depth; b) Crack location of different samples for experimental, numerical and fuzzy-logic technique

7. Conclusion

The objective of the present paper aims at establishing an effective technique to diagnose cracks in the damaged vibrating beam structure under a complex loading pattern. So a systematic study has been conducted to ensure changes on modal characteristics of vibrating beam. As an initial step numerical result has been validated with that of corresponding experimental results with and without crack. It may be established here that a local flexibility is induced at the crack location leading to a change in the structural integrity sensitive indicators like relative mode shapes and relative natural frequency.

It may be noted that the depth of crack and position of the crack influence both mode shapes and natural frequencies in a damaged beam structure under dynamic vibration. In present research efforts have been made to locate the position of crack and its severity by adopting fuzzy logic as one of the artificial intelligence technique. In such techniques relative natural frequencies obtained from numerical data are used as inputs to a reverse inference engine controller for securing crack locations and estimating crack depths. Further to state here that the relative natural frequency drops with higher crack depths irrespective of crack locations.

A noticeable variation in the relative natural frequencies and mode shapes have been found following a change in the crack locations and depths. Changes become prominent in the relative natural frequencies with higher crack depths. The presence of crack brings a noticeable variation in its relative mode shapes.

The whole study aims at developing an effective crack diagnostic tool for correct prediction of crack depths and its locations in a damaged beam like structure. The influence of modal characteristics like relative natural frequencies and mode shapes in a vibrating beam in crack presence have been studied successfully using the proposed artificial intelligence technique.

It may be noted that the Gaussian fuzzy controller gives more accurate result as compared to trapezoidal and triangular fuzzy controllers as evident from the Table 6.

The deviation in the result of output parameters i.e. crack depths and crack locations calculated experimentally and numerically have been compared with that of corresponding Fuzzy logic and the computed deviation percentage does not exceed beyond 3% percent as evident from the Table 7.

Application

1) The model designed using fuzzy logic technique can be employed for crack diagnosis in fatigue crack, turbo machinery, ship and plane structures, robots etc.

2) Since this is a non-destructive technique for crack diagnosis, so the present study will be quite useful for crack diagnosis and on line condition monitoring of structural members.

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