Analysis of the Effectiveness of Determining the Horizontal Curvature of a Track Axis Using a Moving Chord

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Summary
The paper addresses the issue of determining the horizontal curvature of a railway track, noting that it is most often done indirectly – on the basis of measured sags from a chord stretched along the track. Further use of this method would not be justified if there were a direct method for determining the curvature. Therefore, the assumptions of the method for determining the horizontal curvature from "Archives of Civil Engineering", iss. 4/2020, are presented. This method is based on changes in the slope angles of the moving chord in the Cartesian coordinate system. Two important details are examined: the influence of the length of the chord on the obtained values of curvature and the possibility of determining the location of border points between particular geometrical elements. The analysed variants resulted from the type of transition curves used. It has been found that the length of the chord does not play a significant role in determining the curvature and does not limit the application of this method. At the same time, attention is drawn to the precision of determining the nature of the curvature and its compliance with the theoretical course on transition curves. The analysis shows that, in the moving chord method, it is possible to determine the location of the border points between the individual geometrical elements, but the required chord length must be adapted to the type of transition curve.

Keywords: railway, horizontal curvature, moving chord, analysis methodology

1. Introduction

Defining and evaluating the shape of the track axis in the horizontal plane determines the achievable speed of trains. This operation involves determining the basic geometry of the route: location and length of straight sections, location of circular arcs with determination of their radius and length and location of transition curves with determination of their type and length.

The rules for conducting appropriate measurements are similar in various railway managements [3, 5–7, 22–24]. The methods used have a very long tradition and, although they are subject to various innovations, they are highly labour-intensive and require significant financial resources. A radical improvement in the existing situation should be ensured by the method of mobile satellite measurements developed in Poland for over 10 years [13–18, 27–30]. Measurements result in a set of figures which, after appropriate processing, form a set of coordinates in the relevant Cartesian system; in Poland it is the National Spatial Reference Frame PL 2000 [26]. The goal of the ongoing BRIK research project [4, 33] is to obtain a deployment solution.

Now that the problem of determining route coordinates has been solved, the key issue is to identify the geometrical elements present. The method used for this purpose is based on the so-called curvature plot [9, 21], which is the most commonly used tool for assigning track points to sections with a defined geometry. From a modelling standpoint, this plot is the same as the horizontal sag plot and the cant plot, if any.

In engineering practice, identification of the nature of horizontal curvature occurring in a given track geometry is most often performed in an indirect way – on the basis of sags measured from a chord, stretched along the track. This paper presents the possibilities of using a horizontal sag plot to estimate track curvature values, then the assumptions of a new method for determining horizontal curvature described in [12] are presented. It addresses two important specific issues: the effect of chord length on the curvature values obtained and the possibility of determining the position of boundary points between geometrical elements.

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2. Possibilities of using horizontal sagitta plots

The value of the horizontal sag f is determined for a given point along the track length, located at a distance of l from the starting point. It is therefore a function of $f(l)$, which means that there is an analogy to both the definition of curvature $k(l)$ and the way the longitudinal profile of a railway is presented.

Figure 1 shows the idea of measuring horizontal sagittas in a railway track. The horizontal sagitta f at point A, located at a distance l from the starting point of a given track section, is the distance of that point from the centre of the chord CD (i.e. point B), with the ends of the chord (i.e. points C and D) aligned so that point A projects perpendicularly onto the chord CD at point B. For a circular arc, the curvature can be determined from the measured value of the sagitta using a formula in which λ is half the length of the chord:

$$k = \frac{2f}{\lambda^2 + f^2}, \tag{1}$$

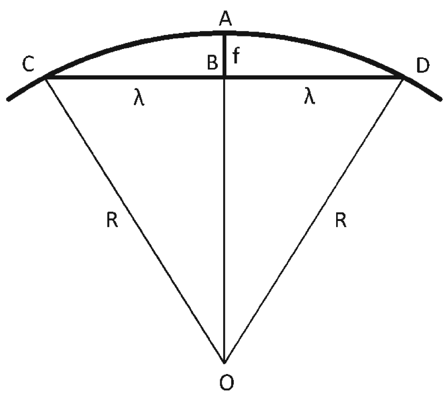

Fig. 1. Explanation of the principle for measuring horizontal sagittas [author's study]

On a circular arc, the curvature is the reciprocal of the radius, which can be written as follows:

$$R = \frac{\lambda^2 + f^2}{2f} \tag{2}$$

In practice, the value f^2 – compared to the value λ^2 – is very small, so a simplified formula is applied for general use:

$$f = \frac{\lambda^2}{2R} \tag{3}$$

however, this simplification, with current computing capabilities, no longer has any justification.

On transition curves, a direct transition from the measured sagitta to curvature is unauthorised, but it is still common to simplify by assuming that the curve segment associated with the chord is a circular arc and relations (1) ÷ (3) can be used for it.

The sagitta plot method is still very popular on railways. The sagitta plot is very similar to a curvature plot, so some people consider this method as a way to determine the track curvature. From a formal point of view, this is obviously unjustified. Sagitta values are in millimetres, while the unit of curvature is rad/m. In addition, account should be taken of the fact that the $f(l)$ sagitta plot shows the values of the horizontal sagittas measured, but does not specify the directions in which those sags are measured. The reference line is related to the chord directions, which are constantly changing.

It should be noted that the measurement of sags (horizontal and vertical) has for many years been the basis of diagnostic methods related to the assessment of the geometry of railway tracks [19–20, 25, 31–32]. It was used in measuring wagons and is now used by measuring draisines. The terms "alignment plot" and "longitudinal level plot" occurring in these measurements actually mean the sag plots measured in the corresponding railway track. A similar situation is also found in commercial railway design support programs, including [1, 2, 8]. The generated "curvature plots" were used as a starting point for calculating the geometrical parameters of curvilinear track systems in the horizontal plane. However, it seems that a self-interpretation of the concept of curvature has been applied, probably using a sag plot.

For the measurement of sags, appropriate measuring equipment should be available (in this respect manufacturers provide a wide assortment) and refer to the structural element that physically exists. Therefore, the described measurement cannot be made for the track axis, since the sagitta should be measured for the selected track, and both ends of the chord must be on that track. This is a typical situation for gauges used in track diagnostics. However, if the ends of the chord were located on the track axis, then it would be possible to determine the track sagitta. This situation occurs when the track axis coordinates are available (designed or measured), while the chord is the moving rigid base of the measuring wagon, determined by the centre plate axes of its bogies (in the case of a bogie wagon) or the crosspoints of the longitudinal axis of the wagon with the axes of wheel sets (in the case of a two-axle wagon) [33].

From the plot $f(l)$, the location of straight sections and sections located on circular arcs and transition curves can be determined. The characteristic parameters, i.e. the beginnings and ends of these sections, are parameters to the length l. On straight sections, this is not a problem, but it is no longer the case on curved sections. The railway route is determined by the coordinates of the points which are defined in the Cartesian coordinate system (this is due to the requirements for determining the track axis). The transition from a linear system of variable *l* to a plane coordinate system of *x*, *y* (in the local coordinate system) or *Y*, *X* (in the PL-2000 system) is difficult and creates problems in interpreting the geometry. Therefore, a way of identifying the geometrical shape of the track in a rectangular coordinate system should be sought. This would, of course, undermine the point of continuing to use the sagitta plot method for this purpose if there was a method for directly determining curvature.

3. New method for determining track curvature

The measure of the curvature of a route is the ratio of the angle by which the direction of a vehicle's longitudinal axis changes after passing a certain curve to the length of that curve. According to the definition, the curvature of curve *K* at point *M* is called the limit towards which the ratio of the acute angle $\Delta\Theta$, contained between the tangents to curve *K* at points *M* and M_1, to the curve length Δl, when point M_1 follows curve *K* to point *M*, moves.

$$k = \lim_{\Delta l \to \infty} \left| \frac{\Delta\Theta}{\Delta l} \right| \tag{4}$$

A practical way to determine curvature (for small values Δl) is to use a simplified formula:

$$k(l) \cong \frac{\Delta\Theta}{\Delta l} \tag{5}$$

From the definition of curvature, it is necessary to manipulate the angles of the tangent to the geometry. In the case of an analytical record of the curve, this is not a problem at all, but in a real railway track, usually deformed as a result of operation, it is very difficult to determine the position of tangent lines. The situation is completely different for stretched chords, the position of which is always unambiguously determined.

The idea was put forward that, when determining the track curvature, one should not operate with tangents but with corresponding chords. It was assumed that for the small track sections under consideration, they are parallel to each other, while the points of tangency project perpendicularly to the centre of the given chord. This is reflected in the proposed method for changing the angles of the measuring chord (referred to as the moving chord method), which is presented in [12]. Figure 2 shows the block diagram of curvature determination in the proposed method.

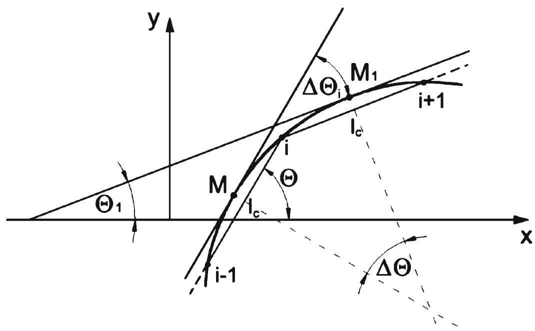

Fig. 2. Block diagram of the curvature determination by the moving chord method [author's study]

The curvature at point *i* is determined from the following formula:

$$k_i = \frac{\Delta\Theta_i}{l_c} \tag{6}$$

in which l_c is the length of the chord and the angle $\Delta\Theta_i$ results from the difference in the angles of the chords that meet at the point *i*, i.e:

$$\Delta\Theta_i = \Theta_{i \div (i+1)} - \Theta_{(i-1) \div i} \tag{7}$$

The application of the described procedure requires knowledge of the coordinates of a given curve in the Cartesian system (written analytically or in a discrete manner), because the values of the angles $\Theta_{(i-1)\div 1}$ result from the slope coefficients of the lines describing both chords.

One paper [12] presents the verification of the proposed method for curvature determination on an explicitly defined elementary track geometry, consisting of a circular arc and two symmetrically aligned transition curves (of the same type and length), calculated according to the principles of the analytical design method [10]. Several geometry cases were considered for different train speeds, while also varying the types of transition curves used and the route turning angles. The obtained curvature plots were fully consistent with the plots constituting the basis of the corresponding geometrical solution. This was true for circular arc sections as well as transition curve sections.

It is also pointed out that the track curvature can be determined by the proposed method, both with respect to the abscissa axis in the Cartesian coordinate system and to the length parameter in the linear system. However, one basic condition must be met – it is necessary to know the coordinates of the points of a given route section in the Cartesian system. Most often, these will be values determined from measurements.

It was also pointed out that the proposed method offers great potential for application. The practical aspect of the presented considerations may be revealed when the geometrical characteristics of the track axis determined by measurements are not known and the primary objective becomes the determination of these characteristics. In this situation, the described method perfectly fits the assumptions of mobile satellite measurements. These measurements determine the coordinates of the track axis in a rectangular coordinate system, in very large numbers and in a very short time.

In [12], a gauge chord of the assumed length $l_c = 5$ m was used and the curvature $k(x)$ was determined. Two important specific issues are addressed: the impact of the chord length on the curvature values obtained, and the possibility of determining the position of boundary points between geometrical elements. In contrast to that work [12], this paper operates with the curvature $k(l)$.

4. Effect of chord length on curvature values

The effect of moving chord length was considered for the elementary geometry used in [12], determined according to the principles of the analytical design method [10]. It was assumed that the route turning angle $\alpha = \pi/4$ rad and train speed $V = 120$ km/h (which results in the radius of circular arc $R = 800$ m with cant on the curve $h = 85$ mm). The analysed variants resulted from the type of transition curves used. The lengths of these curves varied and were driven by the need to maintain limit values of the relevant kinematic parameters. The following types of transition curves were adopted:

- a 105-m-long clothoid (variant 1), described by parametric equations:

$$x(l) = l - \frac{1}{40R^2l_k^2}l^5 + \frac{1}{3456R^4l_k^4}l^9 - \frac{1}{599040R^6l_k^6}l^{13}, \quad (8)$$

$$y(l) = \frac{1}{6Rl_k}l^3 - \frac{1}{336R^3l_k^3}l^7 + \frac{1}{42240R^5l_k^5}l^{11}; \quad (9)$$

- a 150-long Bloss transition curve (variant 2), described by parametric equations:

$$x(l) = l - \frac{1}{14R^2l_k^4}l^7 + \frac{1}{16R^2l_k^5}l^8 - \frac{1}{72R^2l_k^6}l^9 + \frac{1}{312R^4l_k^8}l^{13} - \\ + \frac{1}{168R^4l_k^9}l^{14} + \frac{1}{240R^4l_k^{10}}l^{15} - \frac{1}{768R^4l_k^{11}}l^{16}, \quad (10)$$

$$y(l) = \frac{1}{4Rl_k^2}l^4 - \frac{1}{10Rl_k^3}l^5 - \frac{1}{60R^3l_k^6}l^{10} + \frac{1}{44R^3l_k^7}l^{11} - \\ + \frac{1}{96R^3l_k^8}l^{12} + \frac{1}{624R^3l_k^9}l^{13} + \frac{1}{1920R^5l_k^{10}}l^{16}; \quad (11)$$

- a 135-m-long curve proposed in [11] (variant 3), described by parametric equations:

$$x(l) = l - \frac{1}{40R^2l_k^2}l^5 - \frac{1}{36R^2l_k^3}l^6 + \frac{5}{504R^2l_k^4}l^7 + \frac{1}{96R^2l_k^5}l^8 + \\ + \left(\frac{1}{3456R^4l_k^4} - \frac{1}{288R^2l_k^6}\right)l^9 + + \frac{1}{1440R^4l_k^5}l^{10} + \\ + \frac{1}{6336R^4l_k^6}l^{11} - \frac{19}{31104R^4l_k^7}l^{12}, \quad (12)$$

$$y(l) = \frac{1}{6Rl_k}l^3 + \frac{1}{12Rl_k^2}l^4 - \frac{1}{20Rl_k^3}l^5 - \frac{1}{336R^3l_k^3}l^7 - \\ + \frac{1}{192R^3l_k^4}l^8 + \frac{1}{2592R^3l_k^5}l^9 + \frac{69}{19440R^3l_k^6}l^{10} + \\ + \left(\frac{1}{42240R^5l_k^5} - \frac{1}{6336R^3l_k^7}\right)l^{11} \\ + \left(\frac{1}{13824R^5l_k^6} - \frac{1}{1152R^3l_k^8}\right)l^{12}. \quad (13)$$

Figures 3, 4, and 5 show plots of the ordinates of the curvature $k(l)$ determined in the considered variants for the assumed moving chord lengths $l_c = 5$ m, 10 m, and 20 m.

As can be seen, the curvature plots overlap in each variant. This means that in the range $l_c = 5 \div 20$ m, the chord length does not play a significant role in determining the curvature and does not create a limitation when using the presented method.

At the same time, attention is drawn to the precision of determining the nature of the curvature and compliance with the theoretical course on the transition curves. The two clothoid curves in Figure 3 have a linear curvature, the Bloss curves in Figure 4 have an S curve, and the new transition curves in Figure 5 have a smoothed end curve.

Relatively small inconsistencies in the ordinates of the curvature in Figures 3, 4 and 5 occur only in the areas of transition from straight sections to transition curves and from transition curves to the circular arc. This remark is in no way relevant to the Bloss curve. In particular, some small perturbations in the transition sections from the transition curve to the circular arc are locally evident in the transition curve from [11]

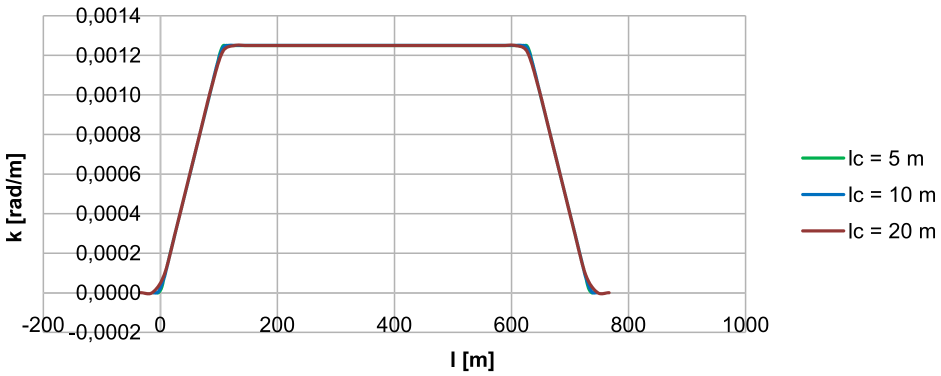

Fig. 3. Plot of the curvature ordinates $k(l)$ in variant I for the assumed lengths of the moving chord l_c = 5 m, 10 m and 20 m ($\alpha = \pi/4$ rad, R = 800 m, clothoid l_k = 105 m) [author's study]

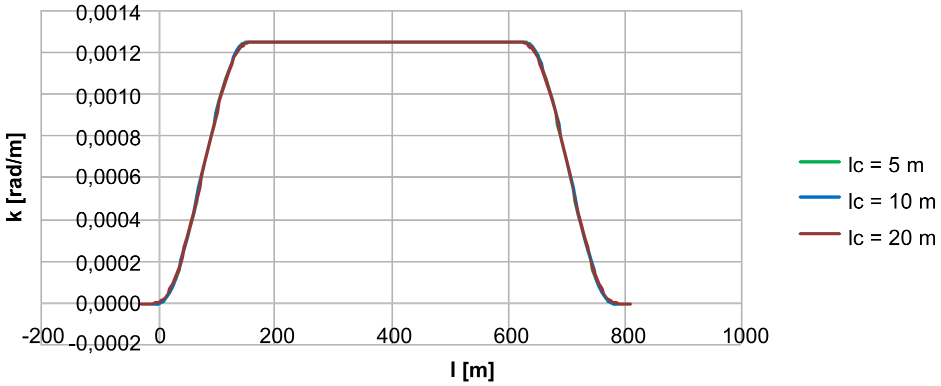

Fig. 4. Plot of the curvature ordinates $k(l)$ in variant II for the assumed lengths of the moving chord l_c = 5 m, 10 m and 20 m ($\alpha = \pi/4$ rad, R = 800 m, Bloss transition curve l_k = 150 m) [author's study]

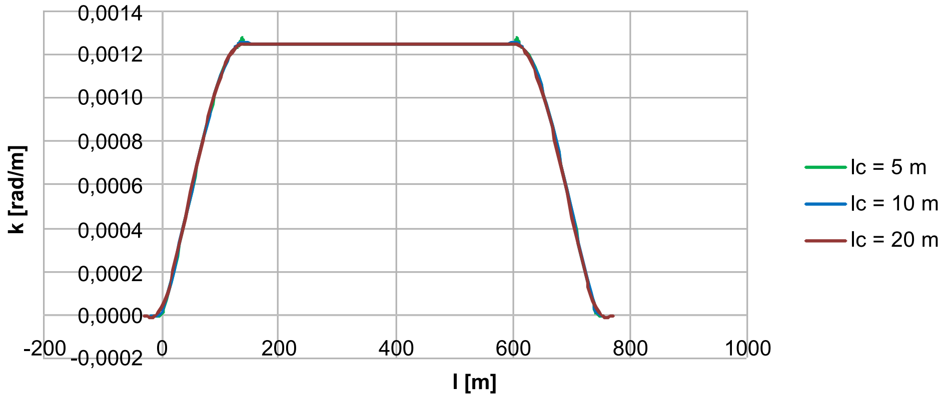

Fig. 5. Plot of the curvature ordinates $k(l)$ in variant III for the assumed lengths of the moving chord l_c = 5 m, 10 m and 20 m ($\alpha = \pi/4$ rad, R = 800 m, new curve l_k = 135 m) [author's study]

(Figure 5). As it turns out, this disturbance can be reduced by extending the measurement chord; however, this does not seem advisable due to the loss of the ability to find a clear boundary between the transition curve and the circular arc, which is analysed in the next section of the paper.

5. Determination of the location of boundary points between geometrical elements

As shown, determining the curvature along the track length is not a special problem. What remains to be clarified is the location of the connection points between geometrical elements – straight sections with transition curves and transition curves with circular arcs. This may prove to be particularly important from a practical standpoint. Figure 6 presents the curvature ordinate plots $k(l)$ in the transition section from the circular arc to the transition curve in the form of a clothoid (variant 1) for the assumed moving chord lengths l_c = 5 m, 10 m and 20 m. Figure 7 shows analogous plots prepared for the connection of the transition curve with a straight track section. These figures also include a plot of theoretical curvature.

It is clear from Figures 6 and 7 that a long measuring chord (l_c = 20 m) cannot be used to achieve the intended purpose. This is also true for the other options considered. A chord of the length l_c = 5 m is not suitable either, so an even shorter chord, i.e. l_c = 2 m, should be used. Figures 8 and 9 show the corresponding curvature plots *k(l)*. The desired value of the connection point abscissa in Figure 8 is l = 628.3185 m, while in Figure 9 it is l = 733.3185 m. As can be seen, the chord with l_c = 2 m is close to obtaining these values.

When using the Bloss curve, the use of a chord of the length l_c = 2 m is not necessary. The curvature plot of $k(l)$ obtained with the chord l_c = 5 m basically coincides with the theoretical plot, as seen in Figures 10 and 11. The desired abscissa of the connection points (l = 628.3185 m in Figure 10 and l = 778.3185 m in Figure 11) can be determined directly.

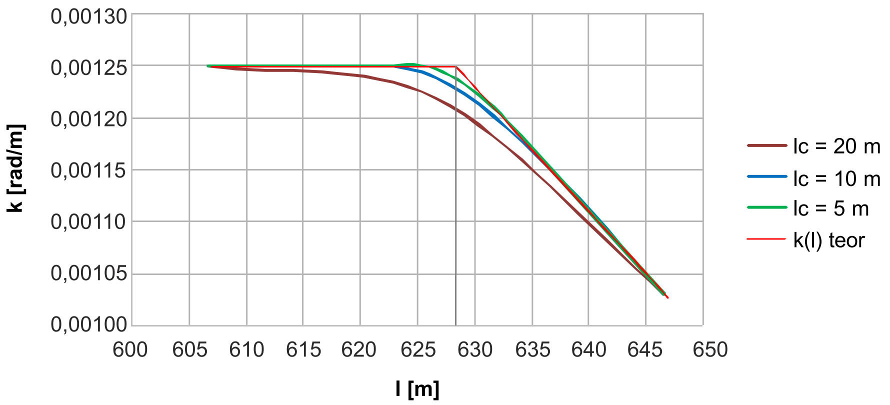

Fig. 6. Plot of the curvature ordinates $k(l)$ theor in the section of transition from circular arc to transition curve in the form of a clothoid (variant I) for the assumed lengths of the moving chord l_c = 5 m, 10 m and 20 m ($\alpha = \pi/4$ rad, R = 800 m, clothoid l_k = 105 m) [author's study]

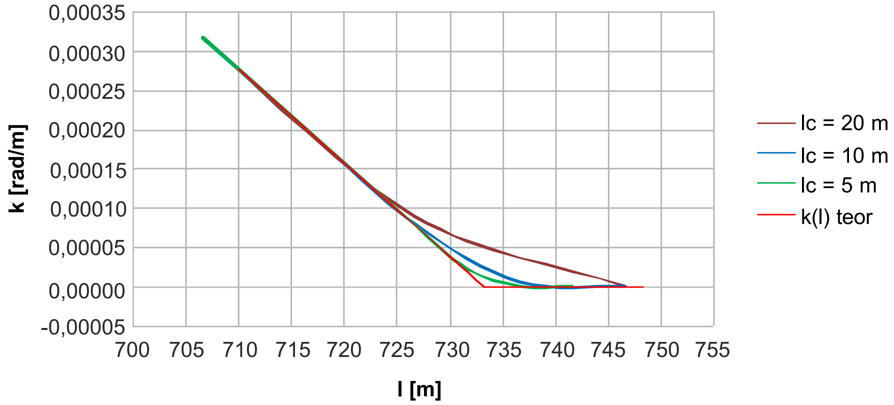

Fig. 7. Plot of the curvature ordinates $k(l)$ theor in the section of transition from transition curve in the form of a clothoid to a straight section (variant I) for the assumed lengths of the moving chord l_c = 5 m, 10 m and 20 m ($\alpha = \pi/4$ rad, R = 800 m, clothoid l_k = 105 m) [author's study]

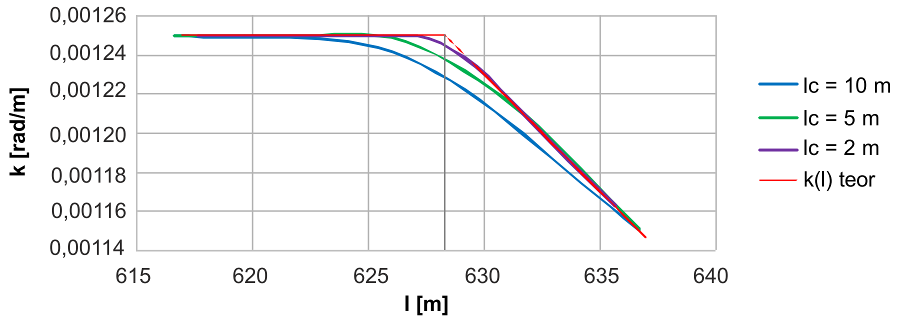

Fig. 8. Plot of the curvature ordinates $k(l)$ theor in the section of transition from circular arc to transition curve in the form of a clothoid (variant I) for the assumed lengths of the moving chord $l_c = 2$ m, 5 m and 10 m ($\alpha = \pi/4$ rad, $R = 800$ m, clothoid $l_k = 105$ m) [author's study]

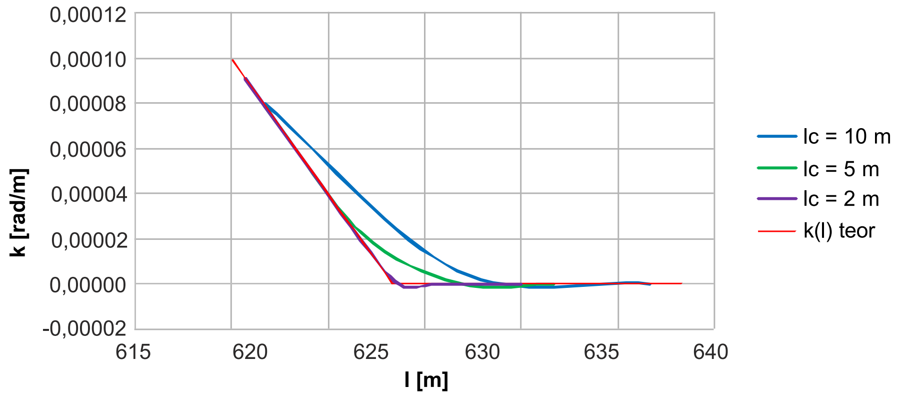

Fig. 9. Plot of the curvature ordinates $k(l)$ theor in the section of transition from transition curve in the form of clothoid to a straight section (variant I) for the assumed lengths of the moving chord $l_c = 2$ m, 5 m and 10 m ($\alpha = \pi/4$ rad, $R = 800$ m, clothoid $l_k = 105$ m) [author's study]

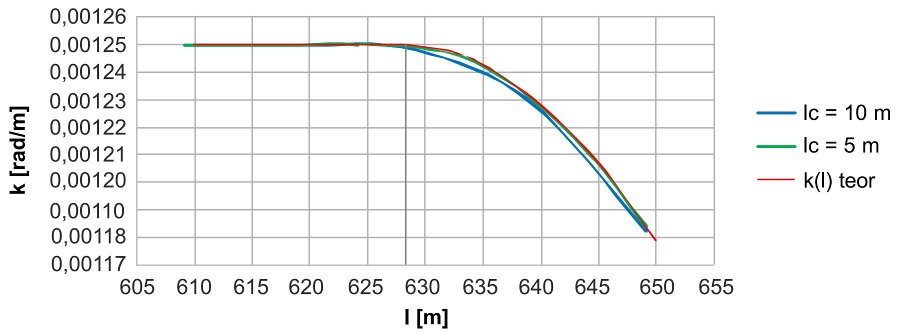

Fig. 10. Plot of the curvature ordinates $k(l)$ theor in the section of transition from circular arc to Bloss transition curve (variant II) for the assumed lengths of the moving chord $l_c = 5$ m and 10 m ($\alpha = \pi/4$ rad, $R = 800$ m, $l_k = 150$ m) [author's study]

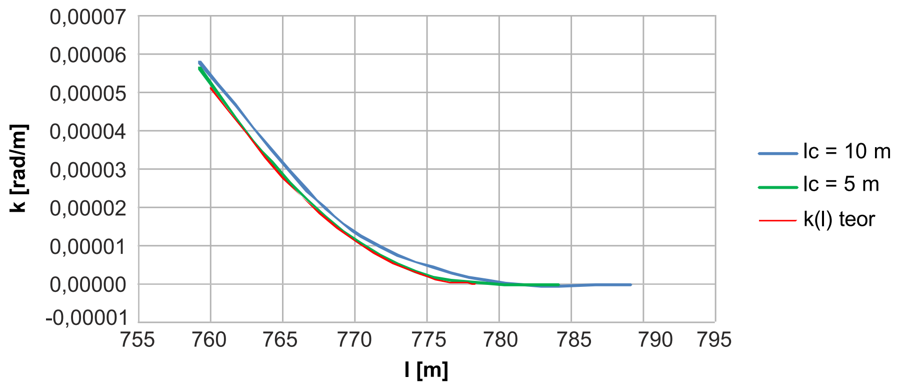

Fig. 11. Plot of the curvature ordinates $k(l)$ theor in the section of transition from Bloss transition curve to a straight line (variant II) for the assumed lengths of the moving chord $l_c = 5$ m and 10 m ($\alpha = \pi/4$ rad, R = 800 m, $l_k = 150$ m) [author's study]

For the new transition curve (variant III), the situation in the transition section from circular arc to a curve looks quite different than in Figures 8 and 10. It is shown in Figure 12.

As can be seen, in the analysed section there is a clear disturbance in the curvature plot – the ordinates $k(l)$ for $l_c = 5$ m and 2 m are overestimated, but, thanks to this, the determination of the abscissa of the connection point of the circular arc with the transition curve (which in this case is $l = 605.8185$ m) becomes very precise. The situation at the connection point of the transition curve with the straight line (Fig. 13) is very similar to that in variant I. The desired value of the abscissa is $l = 740.8185$ m.

The similarity of the curvature ordinate plots $k(l)$ in the transition section from the transition curve to the straight curve in variants I and III is completely understandable – it explains the offset present in the theoretical curvature plot for the clothoid and the new curve.

The analysis shows that, in the moving chord method, it is possible to determine the position of boundary points between geometrical elements, but for the transition curve in the form of a clothoid and a new curve, a chord of the length $l_c = 2$ m should be used, while for the Bloss curve it can be a chord of the length $l_c = 5$ m. Some inaccuracies are to be expected at both ends of the clothoid (at the connections with the straight sections and the circular arc) and in the initial section of the new transition curve (at the connections with the straight sections). Determining the position of the boundary points between the circular arc and the Bloss curve, as well as between the circular arc and the new transition curve, should not pose any problem.

6. Conclusions

If there was a method to directly determine the track curvature, it would undermine the point of con-

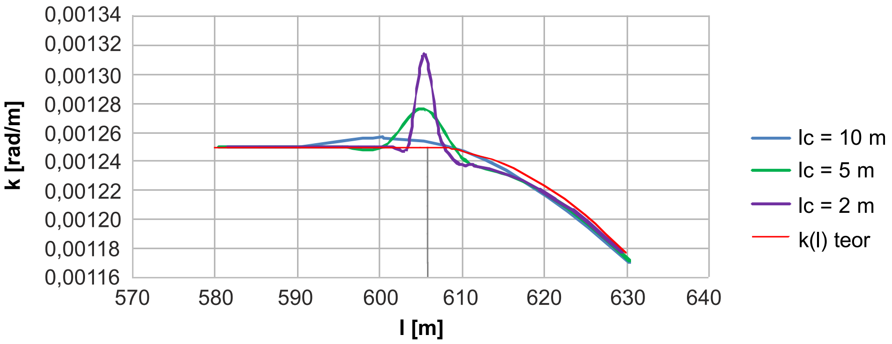

Fig. 12. Plot of the curvature ordinates $k(l)$ theor in the section of transition from circular arc to a transition curve curve (variant III) for the assumed lengths of the moving chord $l_c = 2$ m, 5 m and 10 m ($\alpha = \pi/4$ rad, $R = 800$ m, $l_k = 135$ m) [author's study]

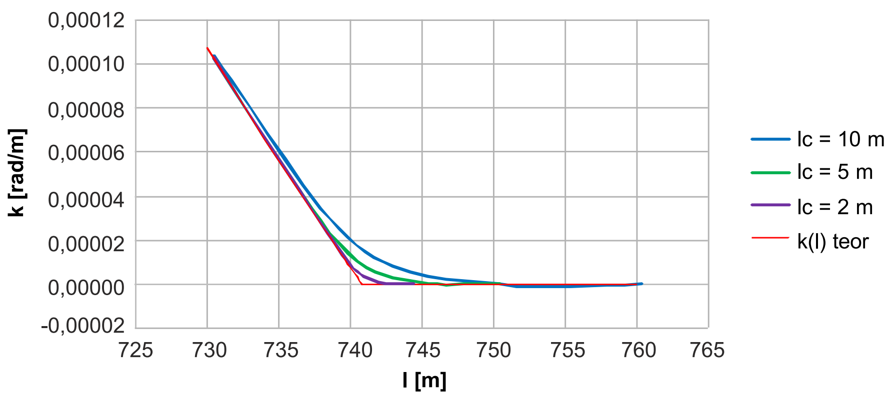

Fig. 13. Plot of the curvature ordinates $k(l)$ theor in the section of transition from the new transition curve to a straight line (variant III) for the assumed lengths of the moving chord $l_c = 2$ m, 5 m and 10 m ($\alpha = \pi/4$ rad, $R = 800$ m, $l_k = 135$ m) [author's study]

tinuing to use sagitta plots for this purpose. One paper [12] presents the assumptions of such a method for determining the horizontal curvature, the basis of which are the changes in the angles of the moving chord in the Cartesian coordinate system. The verification of the proposed method was conducted on an explicitly defined elementary track geometry, consisting of a circular curve and two symmetrically aligned transition curves (of the same type and length), calculated according to the principles of the analytical design method [10]. High application options created by the described method were indicated. This may be useful when the geometrical characteristics of the track axis determined by measurements are not known and the primary objective becomes the determination of these characteristics.

Two important specific issues are addressed: the impact of the chord length on the curvature values obtained, and the possibility of determining the position of boundary points between geometrical elements. The analysed variants resulted from the type of transition curves used. It was found that chord lengths in the range of 5 to 20 m do not play a significant role in determining curvature and do not create limitations to the use of the described method. At the same time, attention is drawn to the precision of determining the nature of the curvature and its compliance with the theoretical course on the transition curves.

The analysis shows that, in the moving chord method, it is possible to determine the position of the boundary points between the geometrical elements, but the required chord length must be adapted to the type of transition curve. For example, for a transition curve in the form of a clothoid (with linear curvature) a chord of the length l_c = 2 m should be used locally, while for a Bloss curve (i.e. smooth curve) it may be a chord of the length lc = 5 m. In addition, for a clothoid, some inaccuracies at both ends (i.e. at the connections with straight sections and circular arc) are to be expected.

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