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ROLLING CONTACT FATIGUE LIFE FOR ELASTIC LINEAR KINEMATIC-HARDENING PLASTIC MATERIAL

TRWAŁOŚĆ ZMĘCZENIOWA STYKU TOCZNEGO DLA MATERIAŁU SPRĘŻYSTO-PLASTYCZNEGO Z KINEMATYCZNYM WZMOCNIENIEM LINIOWYM

Key-words:

fatigue life, rolling contact, kinematic hardening

Słowa kluczowe:

styk toczny, trwałość zmęczeniowa, wzmocnienie kinematyczne

Summary

To predict the fatigue life of rolling bearings, it is necessary to know the pressure distribution in the contacts of mating elements. The assumption of an appropriate material model in the calculation of the pressure distribution has a significant effect on the value of predicted fatigue life. This paper presents the results of the predicted fatigue life calculations for a radial cylindrical roller bearing, obtained using an algorithm which allows one to perform calculations

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for materials with different stress-strain curves, as well as to take into account the roller chamfers. It has been shown that the assumption of elastic linear kinematic-hardening plastic (ELKP) behaviour of the bearing material, and taking into account the shape of roller chamfers allows one to obtain a fatigue life calculation results, which are consistent with the results of experiments.

INTRODUCTION

The determination of the predicted fatigue life of rolling bearings requires knowledge of the subsurface stress distribution in the contacts of mating elements. Subsurface stress depends mainly on the pressure on the contact surface. The roller bearing pressure distributions differ significantly from the idealised Hertzian contact pressure distribution, among other things, due to the commonly used correction of generators of rolling elements. In such cases, accurate information about a pressure distribution may be achieved using, for example, the finite element method. This method allows one to take into account in the calculation the shape of generators of solids in contact and their finite length. FEM calculations of pressure distributions and corresponding subsurface stress distributions require very high computing power. This is particularly required in cases where it is necessary to examine the phenomena occurring in a single contact and if it is necessary to repeat the calculations for multiple contacts in a complex rolling couple. Determination of the pressure distribution on the contact surface is only one of the stages of a complex iteration process, which is often repeated many times during the numerical solution of the equilibrium equations of one or more rolling couples included in the machine. In such cases, for the determination of pressures and subsurface stresses in the contacts, it is more convenient to use the simplified algorithms.

In the mid-90-ies of the last century, at the Technical University of Lodz, an algorithm that uses a Boussinesq solution for elastic half-space was developed. It was used to analyse the quality and capacity of the rolling contact and fatigue life prediction of rolling couples **[L. 1–5]**. In this algorithm, the contact area was divided into *n* bands, of variable width, perpendicular to the major axis of contact field, and each band was divided into *m* triangles. The pressure acting on each triangle was approximated by the second-degree surface (**Fig. 1**).

Theoretically, as a cylinder of a finite length is pressed against the elastic half-space, the pressure at its ends approaches infinity. In the discrete solution, the more concentrated the discrete division net, the greater are the finite values of the peak pressure. For the calculation of the predicted fatigue life, it is assumed that the contact pressure cannot exceed the shakedown limit. The shakedown limit is the Hertzian pressure p_p, which complies with the relation $p_p / k = 4$ for a line contact. In this ratio, $k = \sigma_o / \sqrt{3}$ is the shear yield strength and σ_o is cyclic tensile yield strength. In the algorithm used in **[L. 1–5]**,

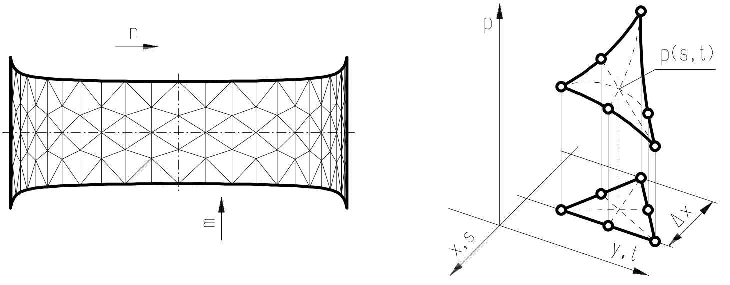

Fig. 1. Contact field discrete division for the contact stress determination
Rys. 1. Podział pola styku na elementy dyskretne przy wyznaczaniu rozkładu nacisków

the shakedown limit value was set to p_p = 4.5 GPa, according to Yhland **[L. 6]**. Despite this assumption, the predicted fatigue life for cylindrical roller bearings with rollers and raceways with rectilinear generators have taken unrealistically low values, that are not confirmed by experiments **[L. 7]**. The results far more accurately reflect the reality when they are obtained by taking into account the shape of roller chamfers **[L. 5]**. Nevertheless, the predicted fatigue life of roller bearings, with rollers with rectilinear generators, was different than fatigue life measured during experiments.

The value of the shakedown limit p_p given by Yhland applies to elastic-perfectly plastic materials. Meanwhile, experimental studies which have been carried out by Hahn, Bhargava et al **[L. 8]** have shown that bearing steels have properties typical for elastic linear kinematic-hardening plastic (ELKP) material. In this case, the relationship between strain and stress can be presented in the form of a three-parameter, bilinear representation. The parameters describing the ELKP model and the hysteresis loop (**Fig. 2**) are the Young's modulus *E*, the plastic modulus *M* and the kinematic yield strength σ_k.

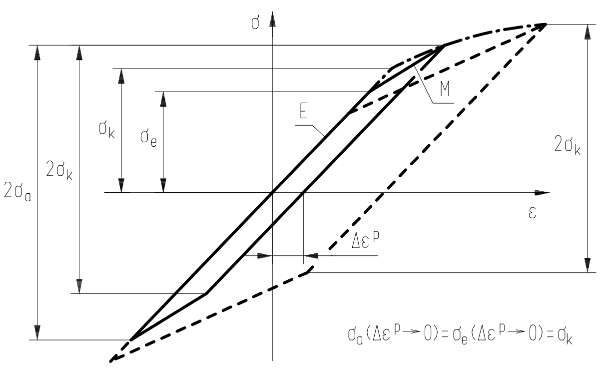

Fig. 2. The cyclic stress-strain hysteresis loops for elastic linear kinematic-hardening plastic material
Rys. 2. Pętle histerezy dla materiału sprężysto-plastycznego z kinematycznym wzmocnieniem liniowym

The subject of study by the authors of **[L. 8]** were specimens of AISI 52100 steel (equivalent of bearing steel ŁH15) with a hardness of HRC 62. Specimens were subjected to cyclic torsion. The results allowed the authors to determine the parameters necessary to plot the hysteresis loop typical for ELKP materials. The studies have shown that the shape of hysteresis loops is typical for the kinematic hardening, as is indicated by approximately the same value of kinematic yield strength σ_k. The plastic modulus M, which for an ideal ELKP material should be constant and independent of the plastic strain $\Delta\varepsilon^p$, in fact, decreases with increasing strain. The authors also noted that the dependence of stress amplitude σ_a on strain ε is nonlinear for a range of plastic deformation ($\sigma > \sigma_k$). A small increase in stress is accompanied by a large increase in strain (dash dot line in **Fig. 2**).

Based on the results presented in **[L. 8]**, for the purpose of determining the distribution of pressure, a three-parameter, bilinear representation was applied in this study. The model was described by Young's modulus E = 208 GPa, the plastic modulus M = 140 GPa and the kinematic yield strength, which according to [**L. 8**] for rolling bearings is σ_k = 880 MPa. The last value corresponds to the shakedown pressure p_k = 2040 MPa. At the same time the previous assumption, that the contact pressure cannot exceed the shakedown limit for the elastic-perfectly plastic material (p_p = 4.5 GPa), was assumed valid. With these assumptions, the stress-strain curve has the form shown in **Fig. 3** (solid line). The curve describing the dependence of contact pressure on strain, plotted in **Fig. 3**

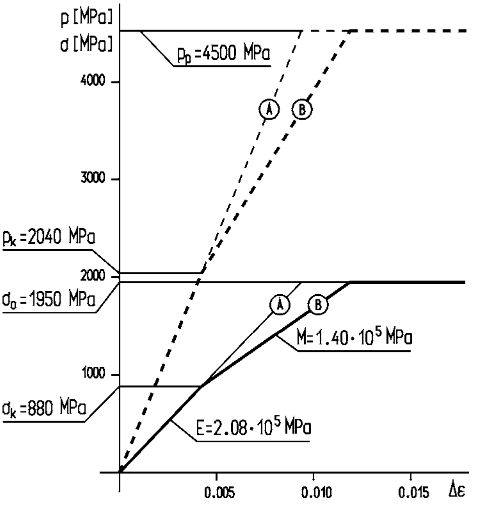

Fig. 3. Stress-strain and contact pressure-strain curves for elastic-perfectly plastic material (A) and for ELKP material (B)

Rys. 3. Charakterystyki opisujące naprężenie oraz naciski kontaktowe w funkcji odkształcenia dla materiału sprężysto-idealnie plastycznego (A) i materiału sprężysto-plastycznego z kinematycznym wzmocnieniem liniowym (B) dashed lines has a similar shape. It should be noted that the shape of the two curves is more appropriate for non-linear elastic kinematic-hardening plastic materials [**L. 9**]; although, their initial shape is typical for the ELKP materials.

This paper presents a comparison of calculation results of rolling contact fatigue life obtained for the two material models used in the determination of pressure distribution for the elastic linear kinematic-hardening plastic material (curve marked in **Fig. 3** with the letter B) and for the elastic-perfectly plastic material (curve marked with the letter A).

OBJECT OF RESEARCH

Comparisons of the effects of the assumed material model on calculation results of predicted fatigue life were made using the example of radial-cylindrical roller bearings NJ 312. Calculations of the predicted fatigue life L_{10} were carried out for radial bearing load $F_r = 38500$ N, with the radial clearance in the bearing $g = 0.07$ mm, and the following cases of roller generators profile (**Fig. 4**):

a) Rollers with rectilinear generators;
b) Rollers with chord-arch (ZB) correction: $R_c = 1320$ mm, $2x_c = 10$ mm, the maximum correction co-ordinate $h_c = 0.018$ mm; and,
c) Rollers with modified logarithmic correction [**L. 10**]: the relative correction co-ordinate $h_m / h_L = 3$, the exponent of logarithmic correction equation $\varepsilon_K = 3$, the maximum correction co-ordinate $h_m = 0.036$ mm.

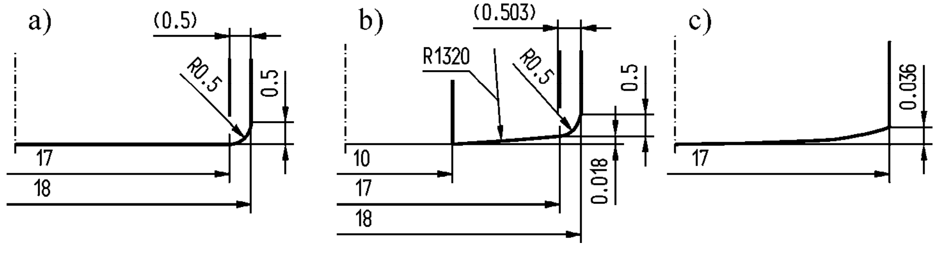

Fig. 4. Analysed roller-generator profiles
Rys. 4. Analizowane profile tworzących wałeczków

The calculation of the fatigue life was carried out taking into account roller chamfer faces (cases a) and b)), and without taking them into consideration (cases a), b) and c)). In the computation, a discrete division mesh with the following parameters: n from the range 20-40, $m = 7$ was used. The computations were carried out using the methodology described in [**L. 4**], using computer programs ROLL1, ROLL2, ROLL3 and ROLL4. ROLL4 was used to determine pressure and subsurface stress distributions, and allows one to perform computations for different material models. The methodology takes into account the effect of radial clearance in the bearing on the number of rollers, which participate in the transfer of radial load. In contrast to the method described in [**L. 11, 12**], the methodology assumes always an odd number of loaded rollers.

The basic parameters of the bearing that was the object of the analysis are presented in **Table 1**.

Table 1.Parameters of the examined roller bearing NJ 312 [L. 13]
Tabela 1. Parametry badanego łożyska walcowego NJ 312 **[L. 13]**

Bearing bore diameter	d **= 60 mm**
Bearing outside diameter	D **= 130 mm**
Bearing width	B **= 31 mm**
Diameter of the inner ring raceway	d_{bi} = 77 mm
Roller diameter	D_w = 18 mm
Roller length	L_w = 18 mm
Roller chamfer	r_c **= 0.5 mm**
Number of rollers in the bearing	Z_w **= 12**

Data used in the fatigue life calculations are the same as in the experiment conducted by Waligóra [**L. 7**]. This makes it possible to compare the results of calculations with experimental results.

RESULTS OF FATIGUE LIFE CALCULATIONS

Fig. 5 shows the results of the calculations of the predicted fatigue life of the test bearings obtained for the two material models and for different mesh densities used in the computations, including various types of corrections.

As shown in **Figure 5**, the assumption in the calculation of the pressure distribution curve of the material, corresponding to the elastic linear kinematic-hardening plastic material, results in obtaining a greater bearing fatigue life than for elastic-perfectly plastic material. The difference is smallest in the case of bearings with a logarithmic correction of roller generators. The maximum correction co-ordinate for logarithmic correction is so large that, even with a significant load length of the contact field, it is usually less than the length of the roller generator, which makes contact at the ends without pressure peaks. This correction type also ensures the highest fatigue life of the bearing.

The greater difference between the results of fatigue life calculations for A and B material models can be seen in the case of the chord-arch (ZB) correction. The ZB correction, although it best approximates the logarithmic shape of roller generators, does not prevent the occurrence of pressure peaks. Pressure values determined in accordance with the material model B are smaller than for the material model A (**Fig. 6**). This results in a greater predicted fatigue life.

The biggest difference occurs for the roller bearings without correction. For rectilinear generators, the pressure distributions are always characterised by the presence of pressure peaks at the ends of the contact, regardless of whether or not the computations of pressure distributions include roller chamfers. However, in this second case peak pressure values are so large that the calculated fatigue life becomes unrealistically small compared to the life of the bearing in which the ZB correction was applied.

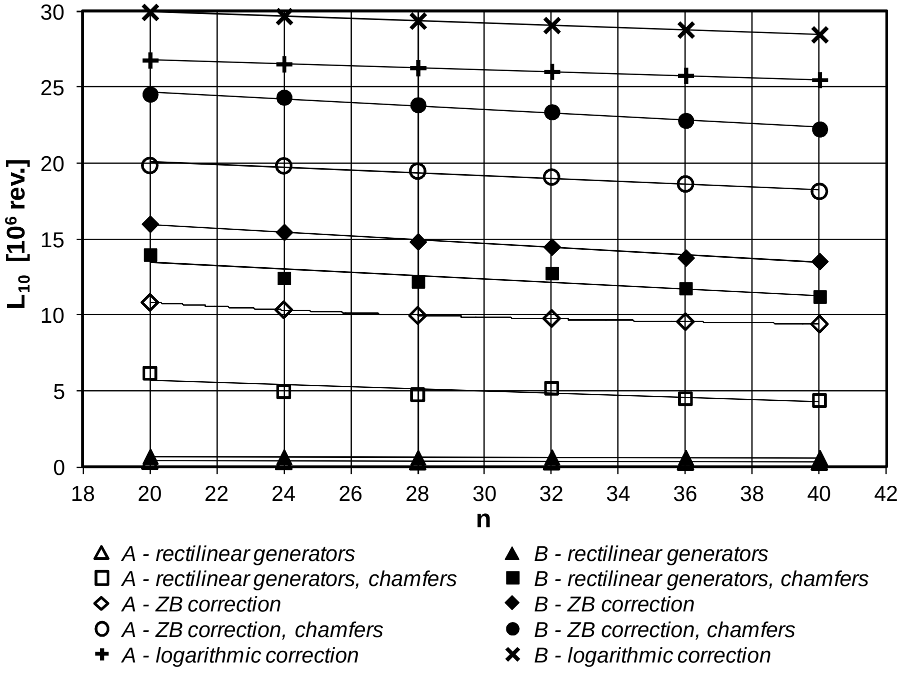

Fig. 5. The fatigue life of NJ 312 bearing for elastic-perfectly plastic material (A) and for ELKP material (B)

Rys. 5. Trwałość zmęczeniowa łożyska NJ 312 dla materiału sprężysto-idealnie plastycznego (A) i materiału sprężysto-plastycznego z kinematycznym wzmocnieniem liniowym (B)

Fatigue test results described in [**L. 7**] indicate that the bearing with chord-arch correction of roller generators reaches the fatigue life approximately two times greater than the bearing with rollers with rectilinear generators (**Fig. 7**). A similar relationship between fatigue life for bearings with ZB correction and bearings without correction can be obtained, if the pressure distribution computations are established material parameters appropriate for the ELKP material. In the case of elastic-perfectly plastic material, the predicted fatigue life of roller bearings without correction is more than 3 times smaller than the bearing life with ZB correction.

In all analysed cases, the predicted fatigue life decreases slightly with an increase in the mesh density. It can be concluded that the effect of mesh density on the results of fatigue-life calculations is negligibly small.

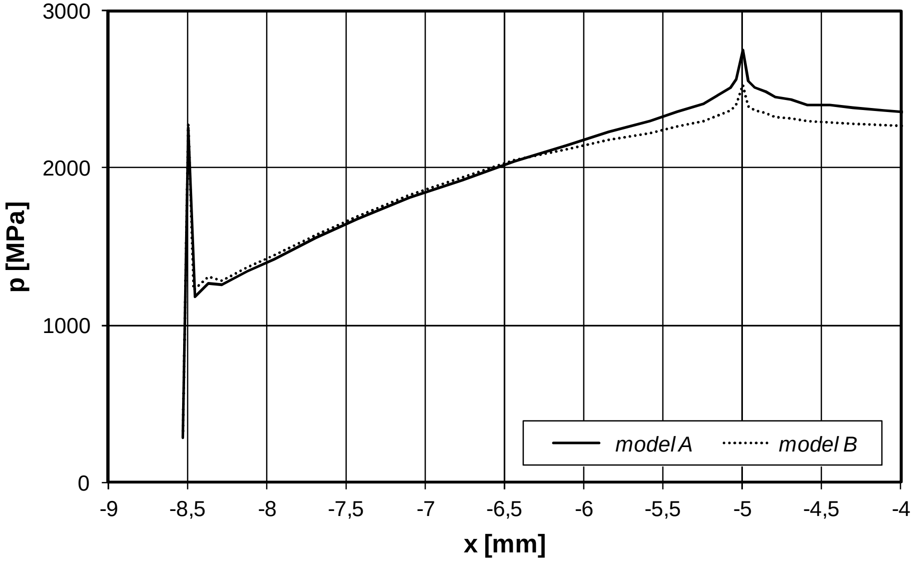

Fig. 6. Pressure distributions in the most heavily loaded roller-inner ring contact, *n* = 40, roller load Q_r = 16061 N
Rys. 6. Rozkłady nacisków w styku najbardziej obciążonego wałeczka z bieżnią pierścienia wewnętrznego, *n* = 40, obciążenie wałeczka Q_r = 16061 N

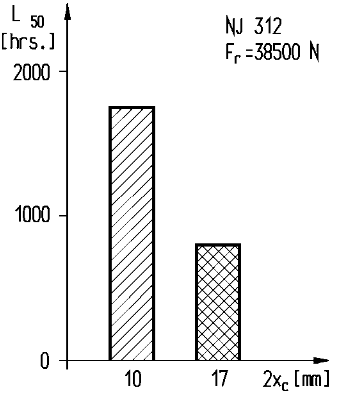

Fig. 7. L_{50} fatigue life of cylindrical roller bearing NJ 312 [L. 7]
Rys. 7. Trwałości zmęczeniowa L_{50} łożyska walcowego NJ 312 według **[L. 7]**

CONCLUSIONS

When calculating the predicted fatigue life of rolling bearings, it is preferable to assume that the bearing elements are made of the elastic linear kinematic-hardening plastic material. This allows one to achieve results consistent with the experimental results. Such an assumption is particularly important for the calculation of the fatigue life of rolling elements with rectilinear generators, where the application of elastic-perfectly plastic material model results in obtaining undervalued, predicted fatigue life.

In the case of rectilinear generators of rolling elements or rolling elements in which the correction of small maximum correction co-ordinate is used which does not fully eliminate pressure peaks, accurate fatigue life calculation results can be achieved provided that the roller chamfers are included. The omission of chamfers results in a unrealistically low values of fatigue life, which are not supported by experimental studies. In the case of the logarithmic correction of roller generators, the shape of roller chamfers can be omitted in the calculation of the pressure distribution, since the surface of the chamfers is not in contact with the surface of the bearing race.

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Streszczenie

Do prognozowania trwałości zmęczeniowej łożysk tocznych konieczna jest znajomość rozkładów nacisków w stykach współpracujących elementów. Przyjęcie właściwego modelu materiałowego podczas obliczeń rozkładów nacisków ma istotny wpływ na wartość prognozowanej trwałości. W artykule przedstawiono wyniki obliczeń prognozowanej trwałości zmęczeniowej promieniowego łożyska walcowego, otrzymane za pomocą algorytmu pozwalającego na wykonanie obliczeń dla różnych krzywych materiałowych, a także na uwzględnienie sfazowań czół wałeczków. Obliczenia trwałości zmęczeniowej przeprowadzono dla wałeczków o tworzących prostoliniowych oraz wałeczków, dla których zastosowano dwa rodzaje korekcji tworzących: korekcję cięciwowo-łukową (ZB) i korekcję logarytmiczną modyfikowaną. Rozpatrzono dwa modele materiałowe: sprężysto-idealnie plastyczny model materiału oraz model sprężysto-plastyczny z kinematycznym wzmocnieniem liniowym. Wyniki obliczeń prognozowanej trwałości zmęczeniowej dla obydwu modeli materiałowych i różnych rodzajów korekcji przedstawiono w funkcji parametru gęstości siatki przedziałów dyskretnych przyjętej podczas obliczeń rozkładów nacisków w stykach wałeczków z bieżniami łożyska. Prognozowaną trwałość łożyska walcowego z wałeczkami bez korekcji i z korekcją ZB porównano z wynikami badań eksperymentalnych przeprowadzonych przez Waligórę. Wykazano, że zastosowanie modelu sprężysto-plastycznego z kinematycznym wzmocnieniem liniowym przy jednoczesnym uwzględnieniu kształtu sfazowań wałeczków umożliwia otrzymanie wyników obliczeń trwałości zmęczeniowej, które są zgodne z wynikami doświadczeń. Przyjęcie takiego założenia ma szczególne znaczenie w przypadku obliczeń trwałości zmęczeniowej elementów tocznych o prostoliniowych tworzących.