Paper

# Closed-form Distribution and Analysis of a Combined Nakagami-lognormal Shadowing and Unshadowing Fading Channel

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**Abstract—The realistic wireless channels face combined (time shared) Nakagami-lognormal shadowing and unshadowing fading because of time varying nature of radio channel and mobile user. These channels can be modeled as time-shared sum of multipath-shadowing and unshadowing Rician distributions. These fading create severe problems in long distance wireless systems where multipath fading is superimposed on shadowing fading (called multipath-shadowing fading). The multipath effect can be modeled using Rayleigh, Rician, Nakagami-m or Weibull distribution and shadowing effect is modeled using lognormal distribution. In this paper, authors present a new closed-form probability distribution function of a Nakagami-lognormal fading channel. Using this result, the closed-form expression of combined Nakagami-lognormal shadowing and unshadowing fading is presented. The obtained closed-form result facilitates to derive the important performance metrics of a communication system such as amount of fading, outage probability, and average channel capacity in closed-form expressions.**

***Keywords—multipath shadowing fading, outage probability, propagation, wireless communication system.***

## 1. Introduction

The propagation of radio wave through wireless channels is intricate phenomenon characterized by multipath and shadowing effects [1]. This scenario frequently observed with slow moving or stationary user [2]. So the resultant signal at receiver is observed as multipath-shadowing faded signal. Several multipath-shadowing fading models have been presented in [3]–[5]. The precise mathematical expressions and descriptions of this phenomenon are either too complex or unknown for using in real communication system. Based on different experimental observations, lognormal distribution can be used to model shadowing effects [5]–[7] and multipath effect can be captured by using distribution such as Nakagami-m, Rayleigh and Rician distribution [8].

A multipath-shadowing fading scenario are often encountered in a real scenarios. Modeling of composite fading attracts attention due to its role in analyzing the wireless systems such as MIMO and cognitive radio and in the modeling of interferences in cellular phone network. Further, due to mathematical complexity in the closed-form expressions of lognormal based composite models, various approximations to log normal have been suggested. In [9], the Gamma distribution is suggested as an alternative approach to lognormal to model shadowing effect. In [10], a composite Nakagami/N-Gamma distribution is considered and performance metrics such as outage probability and biterror rate for receiver are analyzed. The work in [2] analyses the composite Nakagami-lognormal (NL) fading approximating lognormal shadowing as an Inverse Gaussian (IG) distribution, thus, ensuring the resultant expression in the closed-form. In [11], [12] a multipath-shadowing fading is represented as a series of Nakagami-Mixtures of Gamma distribution. In [13], considering Nakagami-lognormal composite fading, second-order statistics of fading channel has been studied. They expressed probability distribution function in closed form using Gauss-Hermite quadrature function. However, combined NL shadowing and unshadowing fading scenario has got little attention till date in terms of closed-form PDF and is not reported in literature and thus, hampering further analytical derivation of important performance metric such as amount of fading (AOF), average channel capacity, and outage probability.

In the case of mobile user there is the possibility of receiving signal from both line of sight (LOS) and shadowing path. This concept was discussed in [1], [14]. In early 1990s, Lutz *et al.* [15] and Barts and Stutzman [16] found that total fading for land-mobile satellite systems can be viewed as combination of unshadowed fading and a multipath-shadowing fading [1]. Statistical modeling of this realistic scenario has also received a little attention in research community until date.

In this paper, the goal is to obtain the closed-form expression of Nakagami-lognormal (NL) distribution using Holtzman approximation [17] for the expectation of the function of a normal variant. The closed-form expression facilitates to obtain a simple analytical approximation of the probability density function (PDF) of combined NL shadowing and unshadowing (Rician) fading. Further, the proposed closed-form PDF leads to the derivation of the closed-form solution of the performance metrics of communication system such as AOF, outage probability $P_{out}$ and channel capacity C/B using Meijer G function for both multipath-shadowing fading as well as combined shadowing and unshadowing fading channel.

The remainder of the paper is organized as follows. In Section 2 the closed-form expression of NL distribution is obtained and using it, the distribution of combined fading is expressed. AOF for the composite fading channels is derived followed by combined fading using Kummer confluent hyper geometric function in Section 3. In Section 4, performance measure such as $P_{out}$ of the communication system is analyzed. The derivation of average channel capacity for both the composite fading as well as combined fading is derived in Section 5. This is followed by results and discussion in Section 6. Finally conclusions are given in Section 7.

# 2. The PDF of Combined NL Shadowing and Unshadowing Distribution

## 2.1. PDF of NL Shadowing

In this section, a closed-form of Nakagami-lognormal composite fading is derived where Nakagami-m represents multipath effect and lognormal model capture the effect of shadowing. PDF of signal to noise ratio (SNR) of NL shadowing can be obtained by averaging the conditional PDF of NL distribution over lognormal fading. Conditional Nakagami-m distribution is given as [1], [2]:

$$p\left(\frac{\gamma}{w}\right) = \frac{m^m \gamma^{m-1}}{w^m \Gamma(m)} \mathrm{e}^{-\frac{m\gamma}{w}}; \qquad \gamma \geq 0, \tag{1}$$

where $\Gamma(\cdot)$ is Gamma function and $m$ is the Nakagami-m fading parameter. The parameter $\gamma$ is the average SNR at the receiver. Here, $w$ is slowly varying power and modeled using lognormal distribution:

$$p(w) = \frac{1}{\sigma w \sqrt{2\pi}} \mathrm{e}^{-\frac{(\log_e w - \mu)^2}{2\sigma^2}}; \qquad w \geq 0, \tag{2}$$

where the parameters $\mu$ and $\sigma$ are mean and standard deviation, respectively of random variable (RV) $\log_e w$. They can be expressed in decibels by $\sigma_{dB} = \xi\sigma$ and $\mu_{dB} = \xi\mu$, where $\xi = 10/\ln 10$ [6]. Averaging the PDF of Eq. (1) w.r.t. Eq. (2), we have PDF of composite NL fading:

$$p(\gamma) = \int_0^\infty p\left(\frac{\gamma}{w}\right) p(w) \mathrm{d}w. \tag{3}$$

Substituting the PDFs from Eqs. (1) and (2), into Eq. (3), we have

$$p(\gamma) = \int_0^\infty \left\{ \frac{m^m \gamma^{m-1}}{w^m \Gamma(m)} exp\left(-\frac{m\gamma}{w}\right) \right\} \times \times \left\{ \frac{1}{\sigma w \sqrt{2\pi}} \mathrm{e}^{-\frac{(\log_e w - \mu)^2}{2\sigma^2}} \right\} \mathrm{d}w. \tag{4}$$

It is difficult to calculate the result in closed-form. In this work, the approach proposed by Holtzman [17], [18]. Taking $\log_e w = x$ in Eq. (2):

$$p(\gamma) = \int_0^\infty \psi(\gamma; x) \left\{ \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}} \right\} \mathrm{d}x. \tag{5}$$

Then, finally PDF of NL fading is:

$$p(\gamma) = \tfrac{2}{3}\psi(\gamma;\mu) + \tfrac{1}{6}\psi(\gamma;\mu + \sigma\sqrt{3}) + + \tfrac{1}{6}\psi(\gamma;\mu - \sigma\sqrt{3}), \tag{6}$$

where

$$\psi(\gamma; x) = \frac{m^m \gamma^{m-1}}{\mathrm{e}^{xm}\Gamma(m)} \mathrm{e}^{-\frac{m\gamma}{\mathrm{e}^x}}. \tag{7}$$

Using Eqs. (6) and (7), CDF of NL fading is derived as:

$$P(\gamma) = \sum_{i=1}^{3} d_i \Gamma(\gamma, m), \tag{8}$$

where, $\Gamma(\cdot,\cdot)$ is incomplete Gamma function and $d_i (i = 1,2,3)$ are $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{6}$.

## 2.2. PDF of the Combined NL Shadowing and Unshadowing Fading

Considering the case of mobile user there is a possibility of receiving signal from both LOS and shadowing path. So combined PDF of instantaneous SNR $\gamma$ is [14]

$$p(\gamma) = (1-A) \times \text{PDF of Rician fading} + + A \times \text{PDF of NL composite shadowing fading}, \tag{9}$$

where $A$ is shadowing time-share factor and $0 \leq A \leq 1$. When $A = 1$, only NL shadowing exists and for $A = 0$, only unshadowing (Rician) exists. For $0 < A < 1$, a mobile user moves between unshadowing and composite shadowing fading.
Rice distribution of $\gamma$ (SNR per symbol) is given as [14]

$$p_{rice}(\gamma) = \frac{(1+K)\mathrm{e}^{-K}}{\overline{\gamma}} \mathrm{e}^{-\frac{(1+K)\gamma}{\overline{\gamma}}} I_o\left(2\sqrt{\frac{K(1+K)\gamma}{\overline{\gamma}}}\right), \tag{10}$$

where $I_0(\cdot)$ is modified Bessel function of first kind with zero order, $\overline{\gamma}$ is average SNR per symbol and $K$ is Rician factor [1]. Rice distribution is often used to model the propagation channel consisting of LOS and some multipath components. Using $p(\gamma)$ from Eqs. (6), (7), (9), and (10), PDF of combined NL and unshadowing fading is given as:

$$p(\gamma) = (1-A) \times \frac{(1+K)\mathrm{e}^{-K}}{\overline{\gamma}} \mathrm{e}^{-\frac{(1+K)\gamma}{\overline{\gamma}}} \times I_o\left(2\sqrt{\frac{K(1+K)\gamma}{\overline{\gamma}}}\right) + A\sum_{i=1}^{3} d_i \psi(\gamma; x_i), \tag{11}$$

where $x_i (i = 1,2,3)$ are $\mu, \mu + \sigma\sqrt{3}, \mu - \sigma\sqrt{3}$, $\psi(\gamma; x_i)$ is defined in Eq. (7) and $d_i (i = 1,2,3)$ are $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{6}$.

# 3. Amount of Fading

Amount of fading is a measure of severity of fading of the channel. In this section, the AOF of the NL fading is computed. The amount of fading is defined as [1]:

$$AOF = \frac{E[\gamma^2]}{(E[\gamma])^2} - 1. \tag{12}$$

Considering the closed-form expressions given in Eqs. (6) and (7), $k$-th moment of $\gamma$ is given as:

$$E[\gamma^k] = \int_0^\infty \gamma^k \frac{m^m \gamma^{m-1}}{\overline{\gamma}^m \Gamma(m)} \mathrm{e}^{-\frac{m\gamma}{\overline{\gamma}}} \mathrm{d}\gamma, \tag{13}$$

where $\overline{\gamma} = \mathrm{e}^x$.

Substituting $\frac{m\gamma}{\overline{\gamma}} = R$ in Eq. (13) and after some simple calculations:

$$E[\gamma^k] = \frac{\Gamma(m+k)}{\Gamma(m)} \left(\frac{\mathrm{e}^x}{m}\right)^k. \tag{14}$$

Thus, considering all the three terms of Eq. (6):

$$E[\gamma^k] = \sum_{i=1}^{3} \frac{d_i \Gamma(m+k)}{\Gamma(m)} \left(\frac{e^{x_i}}{m}\right)^k. \tag{15}$$

From Eq. (12), AOF for NL fading is given as:

$$\mathrm{AOF} = \left[\ dfrac\Gamma(m+2)\Gamma(m)(\Gamma)^2(m+1)m\right] \times \times \left[\frac{\left\{\frac{2}{3}\mathrm{e}^{2\mu} + \frac{1}{6}\mathrm{e}^{2\mu+2\sigma\sqrt{3}} + \frac{1}{6}\mathrm{e}^{2\mu-2\sigma\sqrt{3}}\right\}}{\left\{\frac{2}{3}\mathrm{e}^{\mu} + \frac{1}{6}\mathrm{e}^{\mu+\sigma\sqrt{3}} + \frac{1}{6}\mathrm{e}^{\mu-\sigma\sqrt{3}}\right\}^2}\right] - 1, \tag{16}$$

$k$-th moment of output SNR of combined fading is:

$$E[\gamma^k] = (1-A)\int_0^\infty \gamma^k p_{rice}(\gamma) d\gamma + A\int_0^\infty \gamma^k p_{comp}(\gamma)\mathrm{d}\gamma. \tag{17}$$

The $k$-th moment of Rice distribution is given as [1]:

$$E[\gamma^k] = \frac{\Gamma(1+k)}{(1+k)^k} {}_1F_1(-k,1;-K)\overline{\gamma}^k, \tag{18}$$

where ${}_1F_1(\cdot,\cdot;\cdot)$ is Kummer confluent hyper geometric function.

Thus, the $k$-th moment of combined NL and unshadowing fading is obtained by substituting Eqs. (15) and (18) in Eq. (17):

$$E[\gamma^k] = (1-A)\frac{\Gamma(1+k)}{(1+k)^k} {}_1F_1(-k,1;-K)\overline{\gamma}^k + + \sum_{i=1}^{3} \frac{d_i\Gamma(m+k)}{\Gamma(m)} \left(\frac{\mathrm{e}^{x_i}}{m}\right)^k. \tag{19}$$

Amount of fading can be obtained using Eqs. (19) and (12).

# 4. Outage Probability

The outage probability $P_{out}$ is one of the standard performance criterion of communication systems operating over fading channels and it is defined as [1]:

$$P_{out}(\gamma_{th}) = \int_0^{\gamma_{th}} p(\gamma)\mathrm{d}\gamma. \tag{20}$$

Substitution of Eq. (10) into Eq. (20) yields $P_{out}$ of Rician fading channel and is expressed as:

$$P_{out}(\gamma_{th})_{rice} = \int_0^{\gamma_{th}} \frac{(1+K)\mathrm{e}^{-K}}{\overline{\gamma}} \mathrm{e}^{-\frac{(1+K)\gamma}{\overline{\gamma}}} \times \times I_o\left(2\sqrt{\frac{K(1+K)\gamma}{\overline{\gamma}}}\right)\mathrm{d}\gamma. \tag{21}$$

To solve this, $I_0(z)$ is replaced in (21) using [19] $a = \frac{1+K}{\overline{\gamma}}$, $P_{out}(\gamma_{th})_{rice}$ is given as:

$$P_{out}(\gamma_{th})_{rice} = \sum_{n=0}^{\infty} \frac{a^{n+1}e^{-K}K^n}{n!^2} \int_0^{\gamma_{th}} \gamma^n \mathrm{e}^{-a\gamma}\mathrm{d}\gamma. \tag{22}$$

Using Appendix 1, Eq. (22) can be computed as:

$$P_{out}(\gamma_{th})_{rice} = \sum_{n=0}^{\infty} \frac{a^{n+1}e^{-K}K^n}{n!^2} \times \times\left[(-a)^{n-1}(-1)^n\Gamma[1+n,a\gamma_{th}] - (-a)^{n-1}(-1)^n\Gamma[1+n,0]\right]. \tag{23}$$

Considering the incomplete Gamma function and its relation with Kummer's confluent hypergeometric function from [20], the Eq. (23) can also be expressed as:

$$P_{out}(\gamma_{th})_{rice} = \sum_{n=0}^{\infty} \frac{a^{n+1}e^{-K}K^n}{n!^2} a^{-n-1}(1+n)^{-1} \times \times (a\gamma_{th})^{n+1} \times {}_1F_1(n+1, 2+n; -a\gamma_{th}). \tag{24}$$

$P_{out}$ of NL fading can be obtained by substituting Eq. (6) into Eq. (20), which yields:

$$P_{out}(\gamma_{th})_{NL} = \sum_{i=1}^{3} d_i \Gamma(\gamma_{th}, m). \tag{25}$$

Using the results of Eqs. (23) and (25), $P_{out}$ of combined NL and unshadowing channel can be written in closed form as:

$$P_{out}(\gamma_{th}) = \sum_{n=0}^{\infty} \frac{(1-A)a^{n+1}e^{-K}K^n}{n!^2} \times \times\left[(-a)^{n-1}(-1)^n\Gamma[1+n,a\gamma_{th}] - (-a)^{n-1}(-1)^n \times \times \Gamma[1+n,0]\right] + \sum_{i=1}^{3} d_i\Gamma(\gamma_{th}, m). \tag{26}$$

# 5. Average Channel Capacity

The average channel capacity for fading channel is a significant performance metric as it gives an estimation of the information rate that the channel can support with small probability of error. Channel capacity is defined as [21]:

$$\frac{C}{B} = \int_0^\infty \log_2(1+\gamma)p(\gamma)\mathrm{d}\gamma. \tag{27}$$

Using Eq. (10) into Eq. (27), the average channel capacity of Rician fading channel is given as:

$$\frac{C}{B}_{rice} = \int_0^\infty \log_2(1+\gamma) \frac{(1+K)\mathrm{e}^{-K}}{\overline{\gamma}} \mathrm{e}^{-\frac{(1+K)\gamma}{\overline{\gamma}}} \times \times I_o\left(2\sqrt{\frac{K(1+K)\gamma}{\overline{\gamma}}}\right)\mathrm{d}\gamma. \tag{28}$$

After substituting the modified Bessel function from [19] into Eq. (28) and noting the Meijer G functions from Appendix 1, an average channel capacity for Rician fading may be written as:

$$\frac{C}{B}_{rice} = \sum_{n=0}^{\infty} \frac{a^{n+1} \mathrm{e}^{-K} K^n}{\ln(2) n!^2} \int_0^{\infty} \gamma^n \mathrm{G}_{2,2}^{1,2}\left[\gamma \middle| \begin{matrix} 1,1 \\ 1,0 \end{matrix}\right] \times \\ \times \mathrm{G}_{0,1}^{1,0}\left[a\gamma \middle| 0\right] \mathrm{d}\gamma, \quad (29)$$

where $a = \frac{1+K}{\bar{\gamma}}$, replacing $n$ by $n-1$ in Eq. (29) and using Appendix 1 and [21], the average channel capacity of Rician fading channel is expressed as:

$$\frac{C}{B}_{rice} = \sum_{n=-1}^{\infty} \frac{a^n \mathrm{e}^{-K} K^{n-1}}{\ln(2)(n-1)!^2} \mathrm{G}_{2,3}^{3,1}\left[a \middle| \begin{matrix} -n, 1-n \\ 0, -n, -n \end{matrix}\right]. \quad (30)$$

After substituting Eq. (6) in Eq. (27), the channel capacity for the NL shadowing fading is given as:

$$\frac{C}{B}_{NL} = \int_0^{\infty} \log_2(1+\gamma) \left\{ \left(\frac{2}{3} \psi(\gamma;\mu)\right) + \\ + \frac{1}{6} \psi(\gamma; \mu + \sigma\sqrt{3}) + \frac{1}{6} \psi(\gamma; \mu - \sigma\sqrt{3}) \right\} \mathrm{d}\gamma. \quad (31)$$

From Eq. (7):

$$\frac{C}{B}_{NL} = \sum_{i=1}^{3} \frac{{b_i}^m d_i}{ln(2)\Gamma(m)} \int_0^{\infty} \gamma^{m-1} \ln(1+\gamma) \mathrm{e}^{-\gamma b_i} \mathrm{d}\gamma, \quad (32)$$

where $b_i = m \cdot \mathrm{e}^{-x_i}$.

Using Meijer G function from Appendix 1 in Eq. (32):

$$\frac{C}{B}_{NL} = \sum_{i=1}^{3} \frac{{b_i}^m d_i}{\ln(2)\Gamma(m)} \int_0^{\infty} \gamma^{m-1} \mathrm{G}_{2,2}^{1,2}\left[\gamma \middle| \begin{matrix} 1,1 \\ 1,0 \end{matrix}\right] \mathrm{G}_{0,1}^{1,0}\left[b_i \gamma \middle| 0\right] \mathrm{d}\gamma. \quad (33)$$

Using Appendix 1, average channel capacity of NL fading is expressed as:

$$\frac{C}{B}_{NL} = \sum_{i=1}^{3} \frac{{b_i}^m d_i}{ln(2)\Gamma(m)} \mathrm{G}_{2,3}^{3,1}\left[b_i \middle| \begin{matrix} -m+1, -m \\ 0, -m+1, -m+1 \end{matrix}\right]. \quad (34)$$

From Eqs. (30) and (34), average channel capacity of combined NL and unshadowing channel is given in closed form as:

$$\frac{C}{B}_{rice} = \sum_{n=-1}^{\infty} \frac{(1-A) a^n e^{-K} K^{n-1}}{\ln(2)(n-1)!^2} \mathrm{G}_{2,3}^{3,1}\left[a \middle| \begin{matrix} -n, 1-n \\ 0, -n, -n \end{matrix}\right] + \\ + \sum_{i=1}^{3} \frac{A {b_i}^m d_i}{\ln(2)\Gamma(m)} \mathrm{G}_{2,3}^{3,1}\left[b_i \middle| \begin{matrix} -m+1, -m \\ 0, -m+1, -m+1 \end{matrix}\right]. \quad (35)$$

## 6. Results and Discussion

In Figs. 1, 2, and 3 exact plot using Eq. (4) and proposed closed-form solution using Eq. (11) are plotted. The proposed closed-form solution perfectly matches the exact solution, which confirms accuracy of proposed closed-form solution. In Figs. 1 and 2, values of $A$ and $K$ are fixed (0.25 and 11.9 dB, respectively) and $m$ takes values 0.5, 2, and 4. In Fig. 3, $m$ takes value 2 whereas values of $K$ and $A$ as per Table 1. Values of $\mu$ and $\sigma$ are taken as $-3.914$ and 0.806, respectively for heavy shadowing and $-0.115$ and 0.161, respectively for average shadowing [2]. Different shadowing cases given in [22] have been considered such as urban, sub-urban and highway scenarios. The corresponding values of $A$ (parameter showing occurrence of shadowing) and Rice factor $K$, for these scenarios are presented in Table 1. It is observed from the PDF of the combined NL shadowing and unshadow-

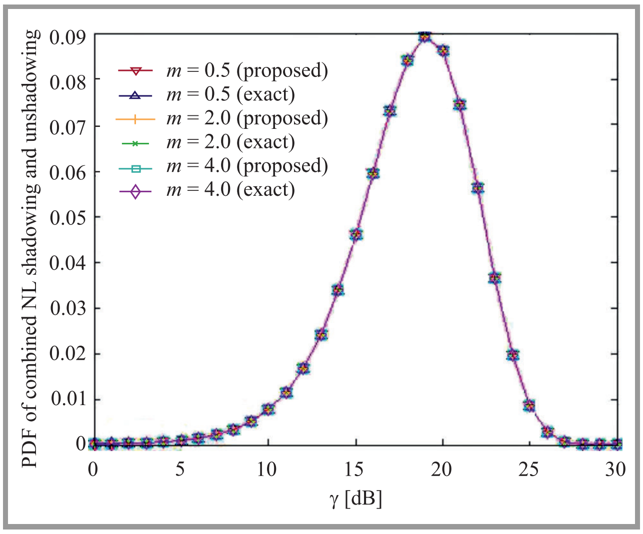


***Fig. 1.*** PDF of combined NL shadowing and unshadowing for heavy shadowing ($\mu = -3.914$, $\sigma = 0.806$, $A = 0.25$, $K =$ 11.9 dB.
(See color pictures online at www.nit.eu/publications/journal-jtit)

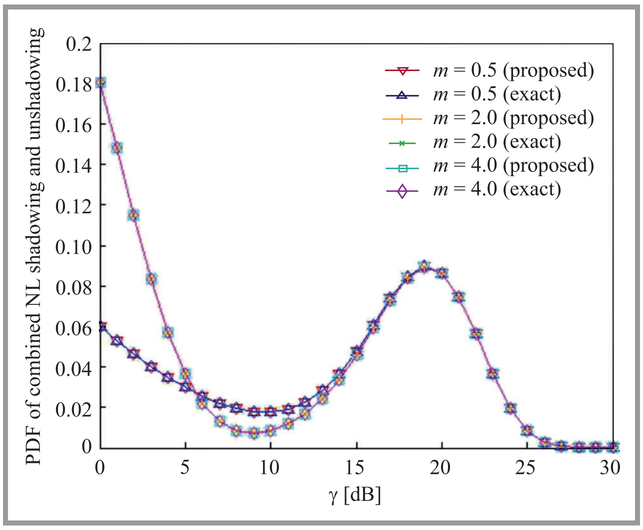


***Fig. 2.*** PDF of combined NL shadowing and unshadowing for average shadowing ($\mu = -0.115$, $\sigma = 0.0.161$, $A = 0.25$, $K =$ 11.9 dB.

ing fading that for urban scenario, the PDF is closer to NL fading and as the user moves from urban to highway, the combined PDF gets closer to Rician distribution. Thus, for highway scenario, the fading is dominated by Rician distribution. Hence, the PDF approaches the perfect Rician curve.

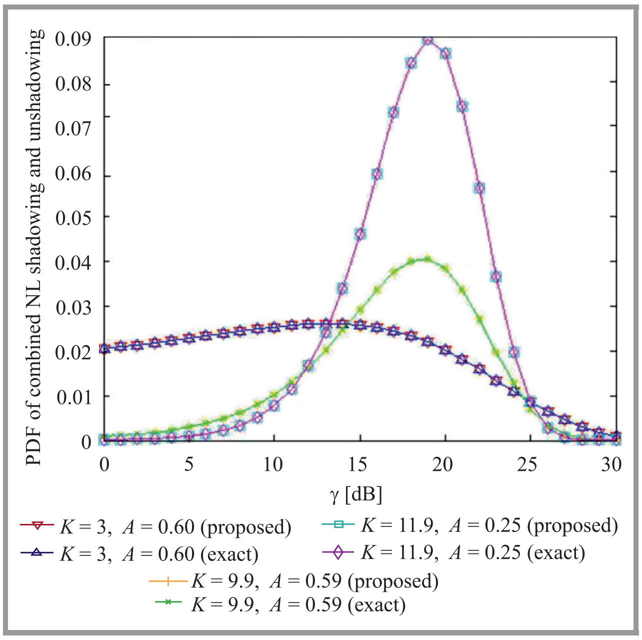


***Fig. 3.*** PDF of combined NL shadowing and unshadowing for urban, suburban and highway ($\mu = -3.914$, $\sigma = 0.806$, $m = 2$).

In Fig. 4, AOF has been plotted for different time-share factor $A$. This figure gives details about variation in the total amount of fading with variation in probabilistic change in the fading conditions for a combined fading scenario. Initially, with $A = 0$, only Rician condition dominates and hence AOF remains very low. With increase in $A$, fading is dominated by multipath shadowing condition and hence

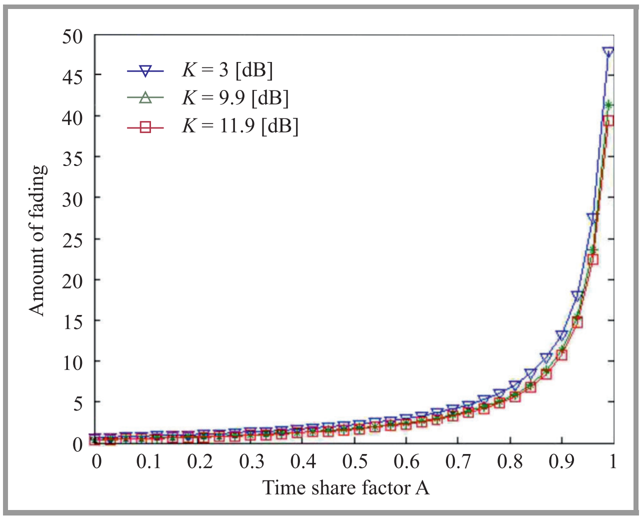


***Fig. 4.*** Amount of fading for combined NL shadowing and unshadowing ($\mu = -3.914$, $\sigma = 0.806$, $m = 0.5, \overline{\gamma} = 10$ dB.

Table 1
Parameters $A$ and $K$ for various scenarios

| Environment | $A$ | $K$ [dB] |
|---|---|---|
| Urban | 0.60 | 3 |
| Suburban | 0.59 | 9.9 |
| Highway | 0.25 | 11.9 |

AOF increases with increase in $A$. One can also observe that AOF slightly decreases with increase in Rice factor $K$ due to obvious reason.

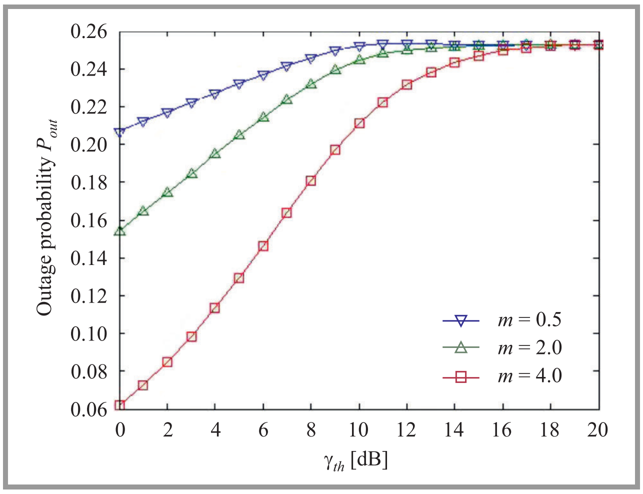


***Fig. 5.*** Outage probability for heavy shadowing($\mu = -3.914$, $\sigma = 0.806$, $\overline{\gamma} = 10$ dB, $A = 0.25$ and $K = 11.9$ dB.

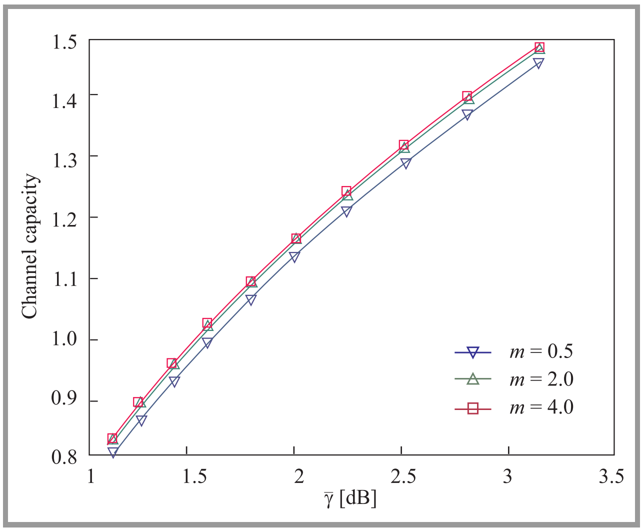


***Fig. 6.*** Average channel capacity for combined NL shadowing and unshadowing for different values of $m$ parameter ($\mu = -0.115$, $\sigma = 0.161$, $A = 0.25$, $K = 11.9$ dB.

$P_{out}$ of combined fading is illustrated in Fig. 5 for different threshold level $\gamma_{th}$. As expected $P_{out}$ increases with increase in threshold level $\gamma_{th}$. Increasing $m$ indicates the less fluctuation of the multipath effect of composite fading, so $P_{out}$ is expected to decrease with increase in $m$ parameter and it is observed from the Fig. 5.

Figure 6 gives the numerical results for the average channel capacity vs average SNR $\overline{\gamma}$ for combined fading for different $m$ parameter. As is observed the channel capacity increases with $\overline{\gamma}$ and also there is a shift in upwards direction with increase in value of $m$ parameter. At high value of $m$, NL shadowing fading moves towards deterministic, which results into better channel capacity.

## 7. Conclusion

In this paper, the closed-form expressions for PDF of instantaneous SNR, amount of fading, outage probability, and average channel capacity of the Nakagami-lognormal fading and combined (time shared) NL shadowing and unshadowing (Rician) fading are derived. The approach uses the Holtzmanian approximations to estimate the closed-form of PDF of NL fading. The resulting Holtzman approximation for NL fading has the advantage of being in closed-form, thereby facilitating the performance evaluation of communication links over combined NL shadowing and unshadowing fading channel.

# Appendix 1

$I_0(z)$ from [19]

$$I_0(z) = \sum_{n=0}^{\infty} \frac{\left(\frac{z}{z}\right)^{2n}}{n1^2}.$$

For any $z$ [20]:

$$\int z^n \mathrm{e}^{-az}\mathrm{d}z = a^{-n-1}(-1)^n.\Gamma(1+n,-z)$$

For any $z$ [20]:

$$\mathrm{e}^{-z} = \mathrm{G}_{0,1}^{1,0}\left[z\Big|\,0\right].$$

For any $z$ [20]:

$$\ln(1+\gamma) = \mathrm{G}_{2,2}^{1,2}\left[z\Big|\,{1,1 \atop 1,0}\right].$$

Meijer's integral from two G functions [20]:

$$\int_0^{\infty} z^n \mathrm{G}_{2,2}^{1,2}\left[z\Big|\,{1,1 \atop 1,0}\right]\mathrm{G}_{0,1}^{1,0}\left[z\Big|\,0\right]\mathrm{d}z = \mathrm{G}_{2,3}^{3,1}\left[a\Big|\,{-n,1-n \atop 0,-n,-n}\right].$$

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