Control and Cybernetics

vol. **42** (2013) No. 3

Novel robust control design for unstable systems with dual pole and dual zero*

by

Xianku Zhang and Guoqing Zhang

Dalian Maritime University,
Key Laboratory of Marine Simulation and Control,
Dalian 116026, P. R. China
zhangxk@dlmu.edu.cn, zgq_dlmu@163.com

Abstract: In this note, a novel robust control design for a class of unstable systems with dual pole and dual zero is addressed. In the algorithm, αfeedback node was moved from the interior of the controlled plant to its right-hand side. Then, the closed-loop gain shaping algorithm was used to design the feedback transfer function. Consequently, controller and the controlled plant have clear physical meaning and are easily tuned. Simulation results show that the control effects are better than for the previous modification, the system can have rapid settling time with no overshoot, zero steady-state error and satisfactory robust stability to model perturbation and disturbance. The method has the advantages of simple and efficient design, and it was also successfully applied to control static unstable missile.

Keywords: unstable process, robust control, improvement, closed-loop gain shaping, dual pole, dual zero

1. Introduction

Unstable processes are frequently encountered in control system design. In Garcia, Albertos & Hägglund (2006), robust analysis for unstable process based on Smith predictor was presented, with a rather compact expression in the controller structure. In parallel, Seshagiri Rao & Chidambaram (2006) developed a lead compensator in series with a PID controller for unstable processes with two unstable poles and a zero with time delay. The algorithm was developed in modeling an isothermal continuous stirred tank reactor, and significant improvement was obtained in practical engineering. The controller design method called Derivative State Constrained optimal H_2 control for unstable system was given in Pannil et al. (2009), the control scheme being based on the state-space approach with the quadratic derivatives of state variables in the standard performance index for the Linear Quadratic Gaussian as an extra weighting function.

*Submitted: May 2012; Accepted: March 2013

Control of unstable PDEs with input delays is also an interesting area that is just opening up for research. An example of a relevant effort was given in Krstic (2009). The analysis involved an interesting structure of PDEs, of parabolic and first-order hyperbolic types. In recent years, authors of this paper dealt with developing and applying new algorithms in the design of controllers for unstable plants. Several results are summarized below. Mirror-injection method provided a new tool to design the robust controller of unstable process (Zhang and Jia, 2000; Zhang, 2004, 2005). In this approach, the unstable zero-poles of the controlled plant are mirror-injected onto the left half plane so that stable zero-pole partners which are symmetric to the unstable zero-poles with respect to the imaginary axis are obtained. In such case, the unstable process is transformed into a minimum-phase system, and the robust controller can be designed by applying closed-loop gain shaping algorithm. The largest singular value curves of unstable process are identical before and after the mirror-injecting, and so the closed-loop gain shaping algorithm directly constructs the robust controller using parameters, which have the engineering sense, according to the shape of singular value curves with robust performance index, and the designed controller has equal robustness to the shaped unstable process. The method has the advantages of simple design procedure and clear physical meaning. Through the test of practice, this method was shown to be easy to use, but it could not be used directly for the pure unstable process, whose poles lay in the right half-plane. Therefore, zeros and poles in the left half-plane were added so that the loop-shaped controlled process was converted into a regular unstable process (Zhang, 2005), and then the robust controller could be constructed using the mirror-injection method. Simulation curves showed that the designed controller had good robustness. Furthermore, the method presented in Zhang (2008) was meant to design a robust controller for unstable process when there was a dual pole, $(s+\omega_1)(s-\omega_1)$, in the denominator of the controlled plant, and this robust controller was used into the maglev train with satisfactory control performance. Based on Zhang and Jia (2000), and Zhang (2004, 2005, 2008), the controller design method for unstable process with a dual pole in the denominator and a dual zero in the numerator was proposed in Zhang, Yang and Yi (2011) (i.e. $(s+\lambda_1)(s-\lambda_1)/[(s+\omega_1)(s-\omega_1)]$). The unstable process was divided into two parts: a pure unstable process and a general stable process. For the pure unstable process, root locus shaping method was used; while for the general stable process, closed-loop gain shaping algorithm was used to design the controller. The method is feasible and has the advantage of simple design procedure, which can avoid complex calculations and higher controller orders. However, it is necessary to derive the feedback signal from the interior of the controlled plant, whose engineering sense is not clear. In order to treat the original mathematical model as a whole, changes are made in the block diagram, at the cost of sacrificing some robustness.

This paper, based on Zhang and Jia (2000), Zhang (2004, 2005, 2008) and Zhang, Yang and Yi (2011), gives the improved controller design for unstable process with a dual pole in the denominator and a dual zero in the numerator.

In section five, this method is examined by control of static unstable missile (Fan et al., 2008, 2010), and it is shown how well it is able to follow command and its engineering feasibility, while the actuator is the crucial limiting factor. Figs. 5 and 6 are particularly important for the comprehension of the approach proposed.

2. Problems of the original control design

The mathematical model of unstable process with a dual pole in the denominator and a dual zero in the numerator is presented in eq.(1) (see Zheng, 2002):

$$G(s) = \frac{(s-1.414)(s+1.414)}{s^2(s-2)(s+2)}. \tag{1}$$

According to the controller design method given in Zhang, Yang & Yi (2011), the control system is simulated using Matlab and Simulink toolbox, and the relevant parameters are consistent with the reference mentioned. The original simulation block diagram is shown in Fig. 1, it has good robustness and the simulation result is mathematically satisfactory. However, the controlled plant is divided into two parts, a feedback signal is derived from the interior of them (this feedback is called α feedback in the following), and it is hard to derive in practice. The physical meaning of this method is not clear.

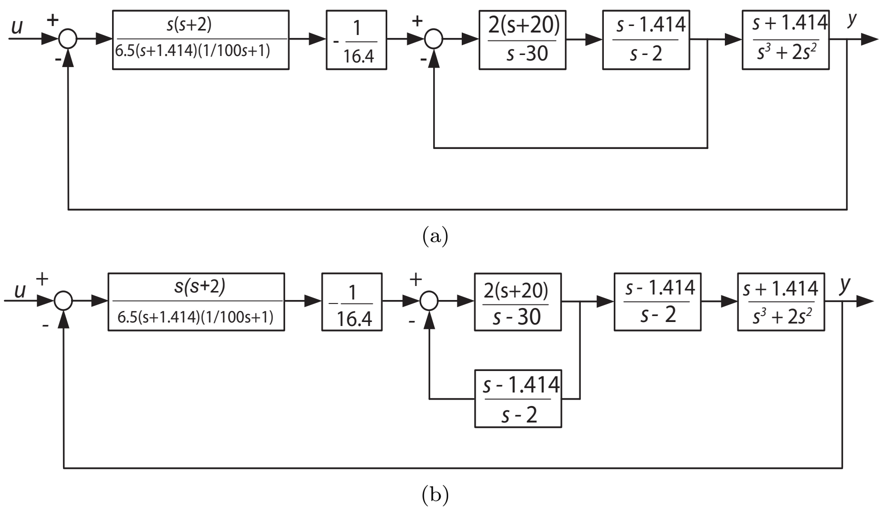


Figure 1. The original simulation block diagram of the system: (a) simulation block diagram and (b) equivalent block diagram

To deal with this defect, Zhang, Yang & Yi (2011) gave a method for equivalent conversion, the α feedback is moved to the left according to the block diagram algebra rules (Nagrath, 1982) to treat the original mathematical model as a whole (see Fig. 1(b)). Simulation results show that the controller design method shown in the equivalent system has satisfactory robustness to model perturbation of the stable part of unstable process, while has no robustness to model perturbation of the unstable part.

The major cause for partial robustness of equivalent system is that the corresponding zeroes in equivalent system and the unstable poles of unstable process are canceled, which further demonstrates that it is unreasonable to reduce block diagrams with unstable pole-zero cancellation. It requires utmost care to conduct block diagram reduction, if there are unstable blocks and unstable pole-zero cancellation in the forward path.

3. Improved method for controller design

Eq.(1) is divided into two parts according to Zhang, Yang & Yi (2011):

$$G(s) = G_1(s)\,G_2(s) = \frac{s-1.414}{s-2}\frac{s+1.414}{s^2(s+2)} \tag{2}$$

It is obvious that an unstable system cannot perform the control task. Therefore, while analyzing a given system, the very first investigation that needs to be made is whether the plant is stable. The stability is directly related to the location of the closed-loop poles of system. Root locus technique provides a graphical method of plotting the locus of the roots in the s-plane as a given system parameter is varied over the complete range of values.

The unstable part $G_1(s)$ has only one zero and one pole in the right half-plane, it can be converted to stability through the root locus shaping technique. The essence of the root locus shaping algorithm is to set roots of the closed-loop feedback system in the suitable place of left half-plane by shaping the open-loop transfer function of the system, so that the system is stable and has good dynamic performance. For $G_1(s)$, an unstable pole which is far away from the original pole in the right half-plane and a zero which is far away from the original zero in the left half-plane are added to carry out the root locus shaping (Hu, 2001). Only in this way can it be ensured that every root locus has a branch in the left half-plane. The shaping function is chosen to be $K_2(s) = (s+20)/(s-30)$, and the root locus of the unstable part after shaping is shown in Fig. 2. If we adjust the root locus gain K^* to the value 2.0, the characteristic equation after shaping is $s^2+1.724s+1.147=0$, and the two roots are $-0.862 \pm 0.635\mathrm{i}$ (two black spots in Fig. 2). The closed-loop Bode diagram after shaping is given in Fig. 3, natural frequency of the system is $\omega_n = 1.071$, damping ratio is $\zeta = 0.805$, the system can quickly stabilize to a value with no overshoot, the setting time is $t_s = 4.06$ s, and the stable value is -16.4. So, the coefficient $-1/16.4$ should be multiplied by the shaped root locus, and then the unstable part $G_1(s)$ can be stabilized to the unit value 1. It is finally shaped as eq.(3). Unit step response of the unstable part after shaping is shown in Fig. 4.

$$G_1'(s) = -\frac{1}{16.4}K^*G_1(s)K_2(s)/(1+K^*G_1(s)K_2(s)) =$$
$$-\frac{1}{16.4}\frac{2(s+20)(s-1.414)}{(s-30)(s-2)+2(s+20)(s-1.414)} =$$
$$-\frac{1}{24.6}\frac{(s+20)(s-1.414)}{(s^2+1.724s+1.147)} \quad (3)$$

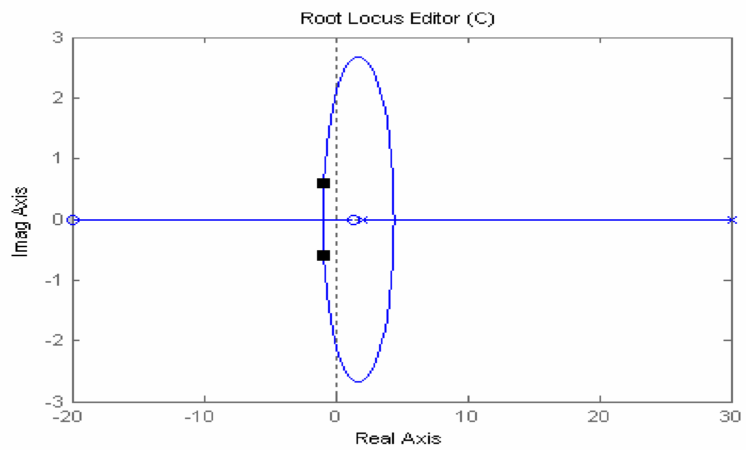


Figure 2. Root locus of the unstable part after shaping

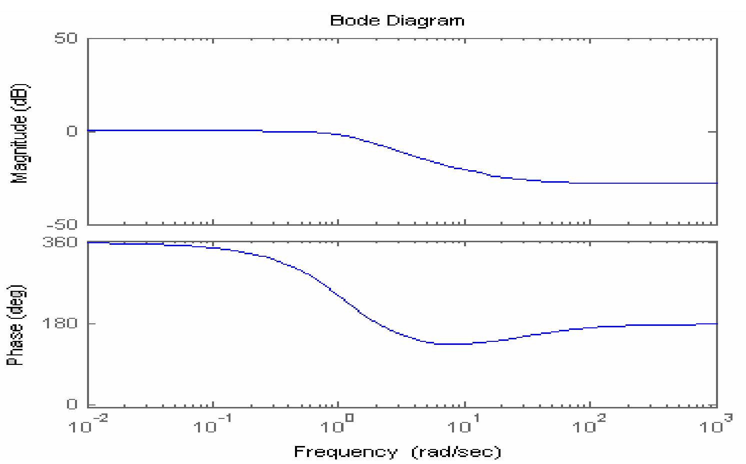


Figure 3. Closed–loop Bode diagram of the unstable part after shaping

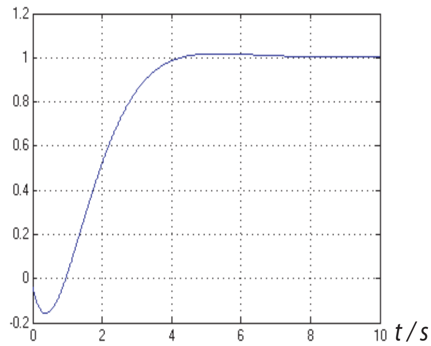


Figure 4. Response curve to unit step of the unstable part after shaping

Using the first-order closed-loop gain shaping algorithm to design a robust controller for $G_2(s)$, the controller$K_1(s)$ can be written as eq.(4). T_1 is the parameter to be adjusted and it can be chosen according the reciprocal of system bandwidth.

$$\frac{1}{T_1 s+1} = \frac{G_2 K_1}{1+G_2 K_1}$$
$$G_2 K_1 (T_1 s+1) = 1+G_2 K_1$$

$$K_1(s) = \frac{1}{G_2(s) T_1 s} = \frac{s(s+2)}{T_1(s+1.414)}. \tag{4}$$

The order of numerator in eq.(4) is higher than the order of denominator, so (4) is not a real function. In order to get the real function of a controller, a pole far away from the other ones, such as $(1/100s+1)$, can be added to the denominator. The robust controller $K_1(s)$ for the stable part $G_2(s)$ is designed as:

$$K_1(s) = \frac{1}{G_2(s) T_1 s\,(1/100s+1)} = \frac{s\,(s+2)}{T_1\,(s+1.414)\,(1/100s+1)}. \tag{5}$$

Fig. 5 shows the block diagram of the unstable control system, the constant coefficients are incorporated into $K_1$and K_2 for convenience of theoretical analysis. The open-loop transfer function of system is $G_2(G_1K_2/(1+G_1K_2))K_1$, $G_1K_2/(1+G_1K_2)$ asymptotically stabilizes to the unit value 1, so $G_2(G_1K_2/(1+G_1K_2))K_1$ asymptotically stabilizes to G_2K_1. The feedback system, includingK_1andG_2, is robust, thus the whole system is stable with robustness.

From the analysis of the operation of equivalent conversion given in Zhang, Yang & Yi (2011), α feedback node, which is between the unstable part $G_1(s)$ and the stable part $G_2(s)$of unstable process, is moved to the left according to the block diagram algebra rules in the classical control theory, in order to get

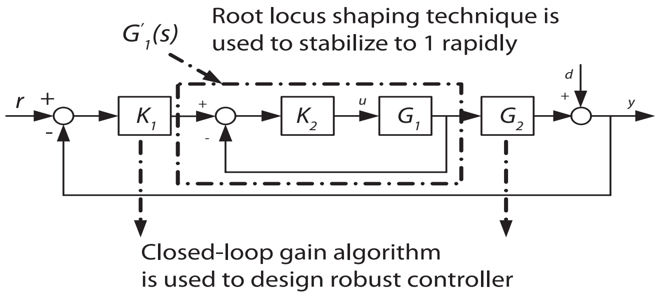


Figure 5. Block diagram of the unstable control system

the original mathematical model together and achieve the physical meaning of the controlled plant. However, authors further find that the block that the α feedback node moves across is an unstable process.

Considering that the stable part of unstable process is in the right hand side of the α feedback node, we try to move the α feedback node to the right, as shown in Fig. 6: the α feedback signal is derived from the output of the controlled plant, adding α feedback transfer function to ensure that the α signal is unchanged. Thus, the key to solve the problem is to determine the α feedback transfer function.

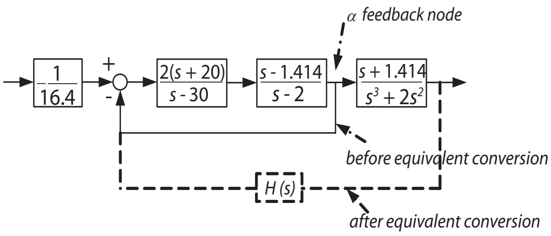


Figure 6. Improvement of the equivalent system

The α feedback transfer function, shown in eq. (6), can be obtained using the method of pole cancellation.

$$H_1(s) = \frac{s^2(s+2)}{s+1.414}. \tag{6}$$

Since the stable part $G_2(s)$ belongs to critical stable component, including two integrators $1/s^2$, we cannot directly use $H_1(s)$ as α feedback transfer function, $H(s) \neq H_1(s)$. Besides, the order of numerator in eq.(6) is higher than the order of denominator, so eq.(6) is not a real transfer function. In order to get the real transfer function $H(s)$ of α feedback, two poles far away from the others, such as $(1/600s+1)^2$, can be added to the denominator based on the closed-loop gain shaping algorithm. Eq.(7) gives the α feedback transfer function $H(s)$.

The value1/600 is added as coefficient of s, so that the gain of the system is not altered.

$$H(s) = \frac{s^2 (s+2)}{(s+1.414)(1/600s+1)^2}. \tag{7}$$

4. System simulation

In this paper, Matlab and Simulink toolbox are used in system simulation of both original and improved controller. Firstly, the original system is simulated again, using the controller design method given in Zhang, Yang & Yi (2011); then the improved system is simulated based on the method given in this paper (Fig. 7 gives the improved block diagram). T_1 in eq.(5) is adjusted to the value 6.5, and then we get the simulation results of the system shown in Fig. 8.

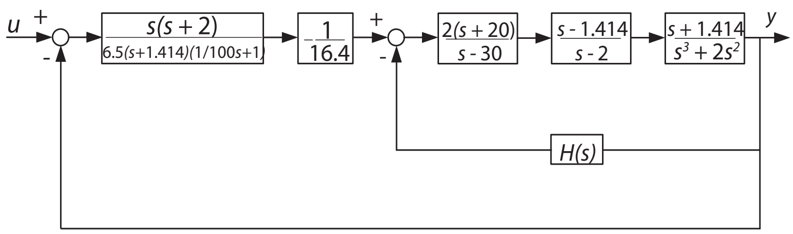
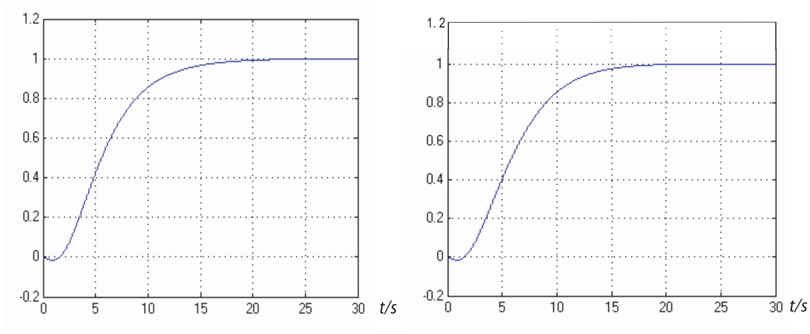


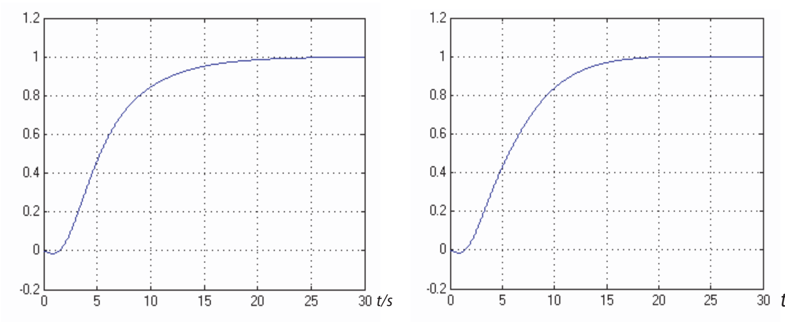
Figure 7. The improved simulation block diagram of system



(a) (b)

Figure 8. Simulation results of the control system: (a) before controller improvement (b) after controller improvement

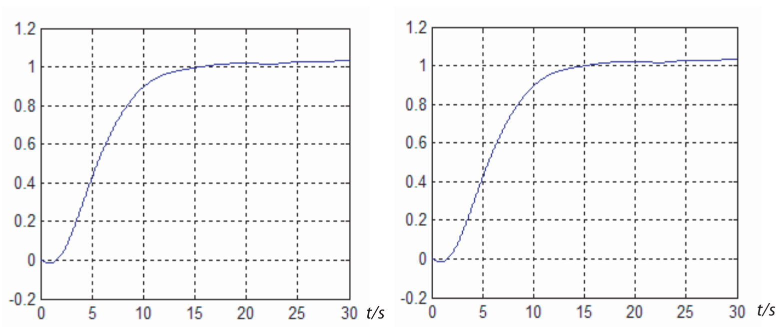
Fig. 9 shows the simulation results for unit step response before and after controller improvement when all parameters of the controlled plant are increased by 10% but the controller remains the same as before. The simulation results



(a) (b)

Figure 9. Simulation results for model perturbation increased by 10%: (a) before controller improvement and (b) after controller improvement

show that the controller displays satisfactory robustness to parameter perturbation. In Fig. 10, the simulation results are given when load disturbance is applied (band-limited white noise with the magnitude of 0.025) entering the plant along with the input variable. Even though it is fairly common that the unstable system output would be divergent for the large disturbance, as is the case with most of control schemes (Shiu, 1998; Seshagiri Rao, 2007), some potential limitations should be pointed out. For some load disturbances acting along with the input in the inner loop, the control system may not effectively stabilize the plant, which will narrow the field of application of the method.



(a) (b)

Figure 10. Simulation results when suffering from a load disturbance entering the plant along with the input variable: (a) before controller improvement and (b) after controller improvement

A practical control application to static unstable missile using the proposed method will be discussed in the next section.

5. Control of static unstable missile

In this section, this robust controller design will be used to control the static unstable airframes, compared with nonlinear control strategy based on back-stepping approach. Written in differential equation notation the basic unstable plant is

$$\begin{cases} \dot{\alpha} = -b_\alpha \alpha + \dot{\vartheta} - b_\delta \delta \\ \ddot{\vartheta} = -a_\alpha \alpha - a_\omega \dot{\vartheta} - a_\delta \delta \\ \dot{\theta} = b_\alpha \alpha + b_\delta \delta \end{cases}$$
$$a_y = V\theta \tag{8}$$

where α is angle of attack (rad) a_y is normal acceleration (m/s^2) $V$is velocity of missile (m/s) ϑ, $\dot{\vartheta}$, $\ddot{\vartheta}$ are, respectively, angle of pitch (rad), angular velocity of pitch (rad/s), angular acceleration of pitch (rad/s^2), θ, $\dot{\theta}$ are, respectively, angle of trajectory (rad), angular velocity of trajectory (rad/s). The parameters of eq.(8) are shown in Table 1.

From eq.(8), the airframe acceleration transfer function is obtained, which is approximately an unstable process with dual pole and dual zero.

Table 1: Parameters of the missile model

V(m/s)	a_α(s^2)	a_δ(s^2)	a_ω(s^{-1})
914.4	-250	280	1.5
b_α(s^{-1})	b_δ(s^{-1})	C(m)	
1.6	0.23	0.681	

$$G_{\mathrm{m}}(s) = \frac{a_y(s)}{\delta_z(s)} = -V\frac{-b_\delta s^2 - a_\omega b_\delta s + a_\delta b_\alpha - a_\alpha b_\delta}{s^2 + (a_\omega + b_\alpha)s + a_\alpha + a_\omega b_\alpha} = \frac{210.3120(s+47.6370)(s-46.1370)}{(s+17.3615)(s-14.2615)} = \frac{s-46.1370}{s-14.2615}\frac{210.3120(s+47.6370)}{s+17.3615} \tag{9}$$

The robust control topology is shown in Fig. 11. Eq.(10) gives the transfer function of *Fin serve* and *Accelerometer*.

$$G_{\mathrm{Fin}}(s) = \frac{220^2}{s^2 + 2\times 0.65\times 220s + 220^2}$$
$$G_{\mathrm{Acce}}(s) = \frac{300^2}{s^2 + 2\times 0.65\times 300s + 300^2} \tag{10}$$

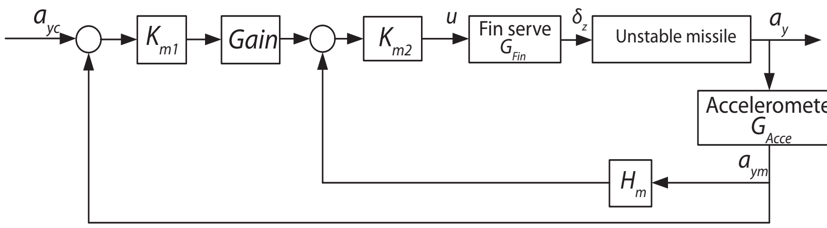


Figure 11. Robust control topology

The robust control law obtained by the proposed method could thus be expressed as

$$\begin{aligned} K_{\mathrm{m1}}(s) &= \tfrac{s+17.3615}{210.312(s^2+47.6370s)(T_1^2 s+2T_1)} \\ K_{\mathrm{m2}}(s) &= \tfrac{s-100}{s+220} \\ H_{\mathrm{m}}(s) &= \tfrac{s+17.3615}{210.312(s+47.6370)} \\ \mathrm{Gain} &= \tfrac{1}{3.125} \\ T_1 &= 0.1 \end{aligned} \quad . \tag{11}$$

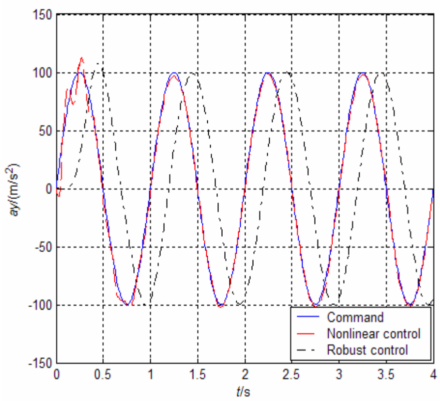
Note that the controller $K_{\mathrm{m1}}(s)$ is obtained by using the second-order closed-loop gain shaping algorithm.

For comparison, simulation results for a static unstable missile are shown in Fig. 12, where the nonlinear control algorithm applied is backstepping method as used in Fan et al. (2010).

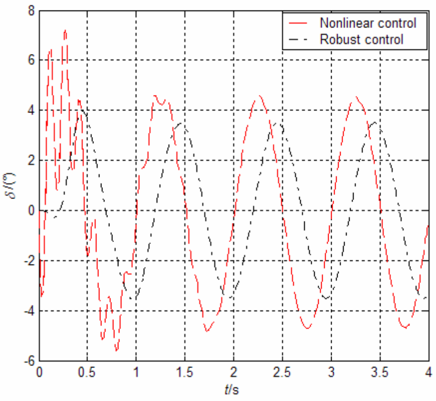
Based on these comparisons, the conclusions are:

1. It is obvious that the performance of the robust controller is slightly inferior to backstepping method, but the corresponding fin deflection is also smaller.
2. The algorithm presented in this paper has advantages of simple design procedure and effectiveness, avoiding higher order controller, while backstepping method is so complex that its engineering implementation will be a true challenge. Moreover, there is no simple way to determine the control parameters. Thus, it could hardly secure an optimal result.
3. The robustness of the robust control law is also improved by introducing closed-loop gain shaping algorithm, and the simulation results validate the feasibility.

Fan et al. (2010) give also a linear control design strategy for airframes, based on optimal control. The robustness of the control law is guaranteed by the classic frequency constraint. The practical application shows that the performance of the robust controller in this article is approximately the same as that of the linear method, but the method here proposed is easy to implement.



(a)



(b)

Figure 12. Comparison of the robust and nonlinear controllers: (a) sine command response and (b) actuator deflection

6. Conclusions

Aiming at the unstable process with a dual pole in the denominator and a dual zero in the numerator, an improved controller design method is given in this paper. The α feedback node, which is between the unstable part and the stable part of unstable process, is moved to the right, and the α feedback transfer function is added to ensure that the α signal is unchanged, according to the algorithm of closed-loop gain shaping. The controller and the controlled plant after the operations proposed have clear physical meaning. It is shown that the method proposed has a simple design procedure. The control performance is better than before, and the method was also successfully applied to the control of static unstable missile.

Even so, there exist potential limitations for this control scheme as pointed out in at the end of Section 4, such that may narrow the field of application of the method.

Acknowledgments

This work was supported in part by National Natural Science Foundation of China (Grant No. 50979009), a grant from the Major State Basic Research Development Program of China (973 Program) (Grant No. 2009CB320805). The authors would like to thank Jia Xinle for his insightful remarks on this note, and anonymous reviewers for their valuable comments leading to improvement of the quality of this paper.

References

Seshagiri Rao A., Chidambaram M. (2006) Control of unstable processes with two RHP poles, a zero and time delay. *Asia-Pacific Journal of Chemical Engineering* **1**(1), 63-69.

Zheng D. (2002) *Linear System Theory (Version 2).* Tsinghua University Press, Beijing.

Fan J., Lin D., Qi Z., Zhang H. (2008) Design and analysis of two-loop autopilot. *Systems Engineering and Electronics* **30**(12), 2447-2450.

Fan J., Lin D., Su Zh., Li Q. (2010) Control of static unstable airframes. *Systems Engineering and Electronics* **21**(6), 1063-1071.

Krstic M. (2009) Control of an unstable reaction-diffusion PDE with long input delay. *Systems & Control Letters* (58), 773-782.

Nagrath I. J. (1982) *Control Systems Engineering.* Wiley.

Garcia P., Albertos P., Hägglund T. (2006) Control of unstable non–minimum-phase delayed system. *Journal of Process Control* **16**(10), 1099-1111.

Pannil P., Tirasesth K., Ukakimaparn P. & Trisuwannawat T. (2009) Derivative State Constrained Optimal H_2 Control For Unstable Systems. *International Journal of Innovative Computing, Information and Control* **5**(10(B)), 3541-3552.

Shiu Sh. J. (1998) Automatic tuning of systems with one or two unstable poles. *Chemical Engineering Communications* **167**(1), 51-72.

Hu Sh. (2001) *Automatic Control Elements.* Science Press, Beijing.

Zhang X. (2005) New method on design of robust controller for unstable process. *ICMLC2005[C]*, 643-648.

Zhang X. (2008) Robust control of unstable process with dual pole. *System Engineering and Electronics* **30**(5), 898-900.

Zhang X. (2004) Solving robust controller of pure unstable process using mirror-injection method. *System Engineering and Electronics* **26**(10), 1466-1467.

Zhang X., Jia X. (2000) Solving robust controller of unstable process using mirror-injection method. *System Engineering and Electronics* **22**(4), 10-12.

Zhang X., Yang Y., Yi Ch. (2011) Controller design of unstable process with dual pole and dual zero. *ICIC Express Letters* **5**(6), 1961-1967.