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Multi-ship Encounter Situation Analysis with the Integration of Elliptical Ship Domains and Velocity Obstacles

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ABSTRACT: With economic globalization, ships tend to be larger and faster, and the volume of maritime traffic is increasing. Ships sailing in waters with dense traffic flow are easy to fall into complicated multi-ship encounter situations and have a high risk of collision. Thus, it is crucial to conduct risk analysis in such situations. In this paper, a modified collision analysis method for detecting dangerous multi-ship encounters in ports and waterways is proposed. The velocity obstacle algorithm is utilized to detect encounters. The model of the elliptic ship domain was integrated into the algorithm as the criteria. The Boolean operation was also used in the multi-ship encounter. A case study is conducted to illustrate the efficacy of the improved model, and a comparison between the existing method and the formal model is also performed. The results indicate that with the integration of the ship domain, the proposed method can effectively detect the encounters of multiple ships which are dangerous to collide.

1 INTRODUCTION

Over 80% of global transportation goods are conducted by ship. [1]The prosperity and globalization of the world trade economy depend on the rapid development of the maritime shipping industry. However, the larger, and faster ships bring increased risk to the individual, societies, and environment, in terms of dire consequences such as loss of life, the economy lost, and environmental pollution. Assessing the risk of ship collision is therefore of great importance as it provides a cost-effective and practical way to mitigate risk.

To analyze the risk of ship collision, various research has been carried out from various perspectives. Among all the studies of quantitative risk analysis, indicator-based models and safety boundary models are quite popular, see [2]. Ship indicators are utilized in the modeling of indicator -based approaches, such as the Closest Point of Approach (CPA), etc. Many efforts have been taken to integrate multiple indicators into one, for a better and more accurate characterization of collision risk. CRI index [3]was put forward which integrates the TCPA and DCPA into one indicator. To combine more indicators such as relative speeds and bearing, the VCRO index is proposed[4]. Huang et al. [5] and Chen et al. [6]analyzed collision risk by projecting pair ships' distance into velocity space, which is another way to integrate distance and time into one indicator.

Most collision risk analysis methods are limited to the encounters of pair ships by now. However, in navigation practice, multi-ship encounters are quite common and often more dangerous. Therefore, it is surely necessary to analyze the collision risk of multi-ship encounters. To analyze that, a two-stage MC simulation algorithm was presented[7], and CPA was improved to estimate collision risk in multi-ship encounters. In[8], a negative exponential function was used to characterize the collision risk for each cluster of encounter vessels with DCPA and TCPA. The value of multi-ship collision risk was obtained by using fuzzy logic theory and Analytic Hierarchy Process (AHP) to incorporate the impact factors of DCPA, TCPA, etc., see[9]. Although several methods have been proposed to get the collision risk of multi-ship encounters, they are all depended on multiple separate metrics, like DCPA and TCPA. Chen et al.[10] applied a velocity obstacle-based risk measurement to measure the risk of collision between multiple ships from the velocity perspective, which provided an interpretable method that incorporated multi-ship encounters into the linear algorithm. In [11], the modified TD - NLVO algorithm with the integration of an elliptical ship domain was applied to detect the pair-ship encounter. Based on this, this paper aims to improve the collision risk detection model of multi-ship encounter, incorporate multi-ship encounter into the model of time-discrete and nonlinear velocity obstacles, and combine it with the elliptical ship domain to make it more practical.

In this paper, an improved Time Discrete Non-Linear Velocity Obstacle algorithm is used to combine the multi-ship encounter model with the ship domain model, to further improve the accuracy of the results. Firstly, the Non - linear velocity obstacle algorithm is introduced as the basic tool to detect multi-ship encounter situations from the perspective of the process; Then, the elliptical ship domain model is integrated into the algorithm to detect the candidate ships. A case study using actual AIS (Automatic Information System) data is conducted, together with compassion between the old and new algorithms. The arrangement of the article is as follows: Section 2 illustrates the methodology of this paper, followed by the design of the algorithm in Section 3. A case study is performed in section 4 to show the results of the algorithm and the comparison. Section 5 makes a conclusion.

2 METHODOLOGY

Chen et al.[10] proposed a TD-NLVO algorithm integrated with Boolean operation on the individual NLVO, which provided an effective tool to detect multiple ship encounter situations using historical AIS data. Therefore, in this paper, the objective is to improve the criteria of the multi-ship encounter detection algorithm from circle to elliptical ship domain, which is a common domain model in maritime practice. The improved algorithm is then applied to determine the dangerous multi-ship encounters according to the violation of the own ship's domain through the process of the encounter. To do so, the TD - NLVO algorithm is adopted as the basic framework for collision analysis, and the elliptical ship domain model is integrated as the criteria.

3 DETECTION OF MULTI-SHIP ENCOUNTER

3.1 *TD - NLVO algorithm*

The velocity obstacle algorithm is a series of algorithms that can represent the potential of collision as it determines the range of velocities that will result in a collision with those obstacles based on geometrical calculations of the velocities of obstacles relative to an object. The velocity obstacle (VO) algorithm was originally used as a method for obstacle avoidance in robotics and autonomous systems [12]. It has been extensively studied and improved upon in the literature, with applications in various fields such as robotic navigation, ship collision avoidance, and autonomous systems. Assessing the risk of ship collision using the VO algorithm is a great idea. In maritime shipping area, this idea was first applied by Degre and Lefevre [13] in 1981 and further developed by Lenart [14] in 1983 into the Collision Threat Parameter Area (CTPA), also known as Linear Velocity Obstacle (LVO) analysis. Like the principle of radar ARPA, the principle of this algorithm is relatively simple, with a small calculation amount, and fast solution speed, and it can be calculated in real-time. Moreover, the model established based on collision maneuver is more in line with the maritime navigation, it can effectively reflect the collision risk of ships and visualize it. Currently, the common velocity obstacle algorithms used in collision avoidance models on safety support system for marine traffic include Linear Velocity Obstacle (LVO), Nonlinear Velocity Obstacle (NLVO) [5, 6], Probabilistic Velocity Obstacle (PVO)[15], Generalized Velocity Obstacle [16]et al. Those methods respectively considered different degrees of motion constraints, and the simulated ship encounter process tends to the encounters in actual navigation practice.

The Nonlinear Velocity Obstacle algorithm can adopt nonlinear trajectories of the robot and obstacles while maintaining real-time performance. Since the NLVO algorithm can update the motion state of the ship and the target ships, and the operation speed is fast enough with few constraints, it is suitable for detecting dangerous encounters during the process of multi-ship encounters. Therefore, just as the non - linear velocity obstacle algorithm was applied in [6] as the fundamental tool for collision candidate detection, this work utilizes the algorithm to detect collision candidates and analyze the collision risk level.

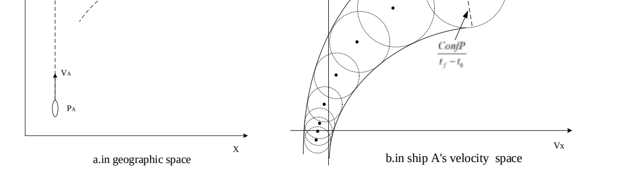

Figure 1. The basic illustration of the Non - linear Velocity Obstacle algorithm

As shown in Fig. 1, the spatiotemporal relationship between own ship A and target ship B was projected into the velocity space of ship A. *ConfP* are all the dangerous positions for ship A that may collide with ship B, defined as a circular area with radius R. NLVO is the set of all the dangerous velocity circles, as shown in Fig. 1(b), means the set of the velocity of ship A which may collide with ship B.

The basic theory of the NLVO algorithm is expressed as Eq.1, according to [5].

$$\begin{aligned}
&P_A\left(\mathrm{t}_0\right)+V_A\cdot\left(t_f-t_0\right)\in P_B\left(t_f\right)\oplus ConfP\\
&ConfP=\{\|P_B\left(t\right)-P_A\left(t\right)\|\leq R\}\\
&V_A\in\frac{P_B\left(t_f\right)-P_A\left(t_0\right)}{t_f-t_0}\oplus\frac{ConfP}{t_f-t_0}\\
&NLVO=\bigcup_{t_f}^{\infty}\left(\frac{P_j(t_f)}{t_f-t_0}\oplus\frac{ConfP}{t_f-t_0}\right)
\end{aligned}\tag{1}$$

where $P_i(t)$ means the position of ship i at time t, V_A means the velocity of own ship A which may collide with ship B, t_0 means the current detection time step, t_f means one time step in the time period of detection.

3.2 *Elliptical ship domain*

In the preceding section, the *ConfP* is defined as a circular area with a fixed radius, which is similar to the Collision Diameter defined by Fujii[17] and Pedersen[18]. However, this definition has some limitations in practice, as any violation of the area results in physical contact due to the small size of the circle. To overcome these limitations, this paper expands the *ConfP* and introduces the static elliptical ship domain model as a new criterion for detecting collision candidates. This model is based on the fundamental concept of the ship domain, which represents an area around a ship that must be kept clear of other vessels to avoid collisions. We respectively set the parameters of semi-major and semi-minor axes, according to the research by [19], at 8 and 4 times the own ship's length. Integration of this domain into the TD-NLVO requires a mathematical function, and variables can be obtained from own ship's information: length and course over the ground. This information can be acquired from historical AIS data. Overall, this paper replaces the circular *ConfP* with a static elliptical ship domain model and adopts the following mathematical function to integrate the model into the TD-NLVO.

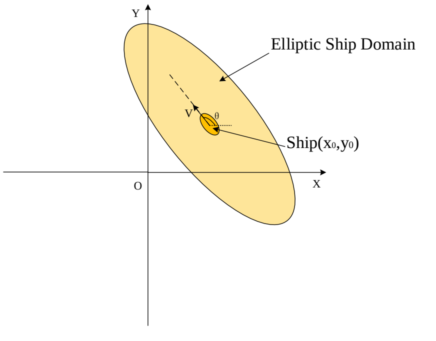

Figure 2. Illustration of elliptic ship domain

The parametric equations for oblique ellipse are shown as Eq.2:

$$\begin{aligned}
&Ax'^2+By'^2+Cy'^2+D=0\\
&A=a^2\sin^2\theta'+b^2\cos^2\theta'\\
&B=2(a^2-b^2)\sin\theta'\cos\theta'\\
&C=a^2\cos^2\theta'+b^2\sin^2\theta'\\
&D=-a^2b^2\\
&x'=x-x_0,\,y'=y-y_0,\,\theta'=\alpha-\theta
\end{aligned}\tag{2}$$

where x_0, y_0 means the horizontal and vertical coordinates of the centre of the ellipse, a, b means the length of semi-major and semi-minor axes, θ means the angle between the major and horizontal axes of the elliptical ship domain. Those equations were incorporated into the algorithm described in the next section.

3.3 *Collision risk detection of multi-ship encounter*

When ships go through busy waterways, such as ports and important water channels, the encounter of ships can be even more complicated because they may encounter multiple ships and may pose a pressing situation. The International Regulations for Preventing Collisions at Sea (COLREGs) only provide guidance on how to avoid collisions between two ships, leaving the officers responsible for evaluating and prioritizing the response to each target ship's collision risk. This can be particularly difficult in ports and waterways that are critical to commercial activities or national security. This work has been discussed in [10], and the TD-NLVO algorithm has been improved for multiple encounter situations. The principle is shown in Fig.3. Based on the TD-NLVO, the technique of Boolean operation on polygons[20] has been introduced into constructing the dangerous velocity set combined with multiple target ships, as is shown in Figure 3(b). Combine multiple VO of each obstacle to form the own ship's reachable velocity space (RVO), which defines the range of velocities that the own ship can take while avoiding obstacles.

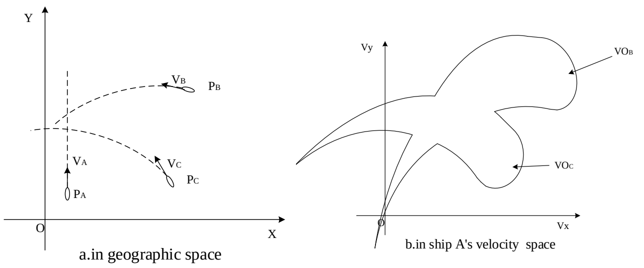

Figure 3. Illustration of multiple encounter situation and Boolean operation on polygons.

To detect the collision candidates from multi-ship encounters, this paper used the TD-NLVO algorithm integrated with improved criteria (as shown in Eq.3). The dangerous encounters were determined by the violation of the combined TD-NLVO of the target ships, which criteria are determined by domain of elliptic. The available range of speed was also considered in the construction of the TD-NLVO.

$$
\begin{aligned}
&NLVO_{A|ship_{Bt_f}} = \bigcup_{t_f}^{\infty} \left(\frac{P_j(t_f)}{t_f - t_0} \oplus \frac{ConfP_{DOE}}{t_f - t_0} \right) \\
&ConfP_{DOE} = \{ \| P_B(t) - P_A(t) \| \leq R_{\text{ellipse}} \} \\
&NLVO_{A|allship_{Bt_f}} = \bigcup_{j=1}^{n} NLVO_{A|ship_{Bt_f}} \\
&NLVO_{DOE} = (NLVO_{A|allship_{Bt_f}}) \bigcap V_{region|A}
\end{aligned}
\tag{3}
$$

where $NLVO_{A|ship_{Bt_f}}$ denotes the dangerous velocity set of ship A calculated by the TD-NLVO algorithm induced by the target ship B, and the elliptic ship domain mentioned in Section 3.2 was integrated into the algorithm replacing the *ConfP* mentioned in Section 3.1 as $ConfP_{DOE}$. $NLVO_{A|allship_{Bt_f}}$ denotes the union of all the velocity sets of ship A induced by all target ships in the multi-ship encounter. $NLVO_{DOE}$ means the polygon intersection of $NLVO_{A|allship_{Bt_f}}$ and $V_{region|A}$ which contains all the adoptable velocity range of own ship A.

This research is discussing the development of an improvement of the encounter detection method for multiple ships, which integrates an improved TD-NLVO and elliptical ship domain model. This new version method for multi-ship encounters focuses on detecting through the entire process of the encounter, rather than just at a certain time interval during the encounter. To achieve this, the trajectory data of ships in the area is reconstructed using their MMSI and then divided into subsets to speed up computation. The design of the new model is shown in Figure 4.

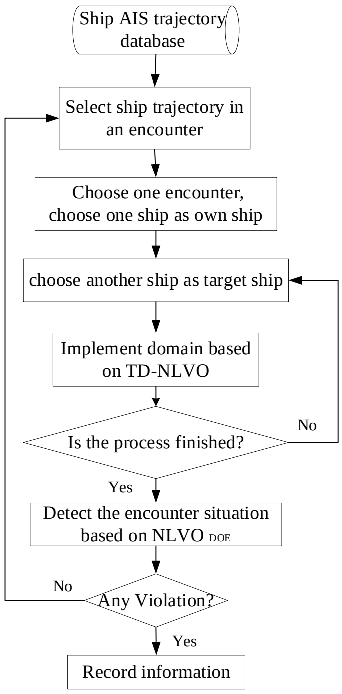

Figure 4. Flow chart of the ship domain-based encounter detection model

4 CASE STUDY

In this section, two case studies on implementing the domain-based TD-NLVO were illustrated. Each case is an encounter situation involving three ships. Historical AIS data provided by the Wuhan University of Technology was utilized as the test datasets. The case study was performed to verify the capability of the proposed method for detecting the encounter process of multiple ships.

We introduced two sets of AIS data on May 20, 2019, in the East China Sea, at the estuarine waters of the Yangtze River as the test dataset. To apply the proposed algorithm according to the actual waterway and traffic, the semi-major and semi-minor axes of the safety region are respectively set as 1000m and 500m. The parameter $T_{threshold}$ is set to be 10s, T_{scan} is set to be 20mins. The own ship's available speed is set to be within 20 m/s.

Case one is a multi-ship encounter situation for 5mins from “13:15:05” to “13:20:13” between ship “413XXX250”, ship “413XXX080”, and ship “413XXX480”. Ship “413XXX250” was chosen as the research object (Own ship). Based on the proposed method, a three-ship encounter between the own ship “413XXX250” and target ship1“413XXX480”, target ship2“413XXX080” was detected, which are shown with their trajectories and encounter situation in velocity space at a certain time step. Fig.5 shows their trajectories and relative distance from “13:15:05” to “13:20:13”, on May 20, 2019.

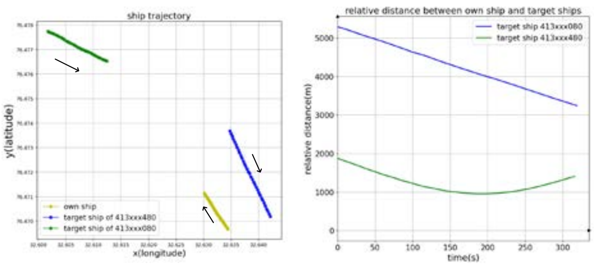

Figure 5. Trajectory and relative distance between ships-case 1

From the information of ship trajectories, we can see that at the encounter process, own ship “413XXX250” were in “head on” situation with target ship1“413XXX480”, and in “crossing” situation with target ship2“413XXX080” at last.

Taking the encounter situation at 10:15:05, and 10:16:15 AM as examples, the spatiotemporal relationships between two target ships in the own ship’s velocity space are represented. To be more specific, a snapshot of the positions of ships, individual NLVO, and combined NLVO are illustrated in Fig. 6, 7.

From Fig. 6 (b), (c), one can see that the combined NLVO is made by two different individual NLVOs, and the violated NLVO induced by ship “413XXX480” is part of the combined NLVO. The results indicate that the own ship only has a violation with ship “413XXX480”, which indicates that at “10:15:05” AM own ship has a potential collision only with ship “413XXX480” in 5mins if keeping the current velocity. The NLVO of the other targets is “combined” into the large NLVO during the Boolean operation “Union” and the velocity of the own ship does not have a violation with it. In Fig. 7 (b), (c), we can see that the own ship’s velocity didn’t violate any NLVO, which illustrates the encounter was safe if keeping the current velocity. Over time, the position and encounter situation of the two ships changed, and the detection result of the encounter had changed from dangerous to safe, which reflected the collision risk in the future period.

With this design of algorithm, the encounter process can be detected in the following: firstly, determine if there is a violation of combined NLVO at each step, and if so, determine which individual ship is responsible for the violation with the own ship. In addition, since the modified TD - NLVO considered course information in domain modelling, the coverage of velocity obstacles during the multi-ship encounter process varies as the courses of target ships change constantly. This allows a detailed analysis of the encounter.

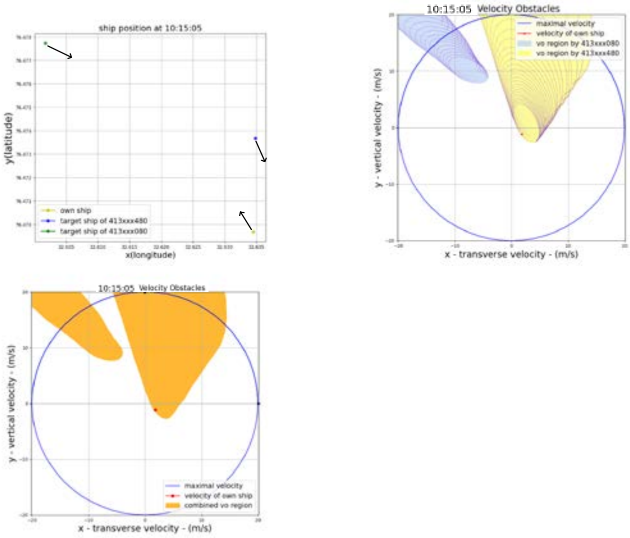

Figure 6. Positions and Vos of ships at “10:15:05” –case 1.

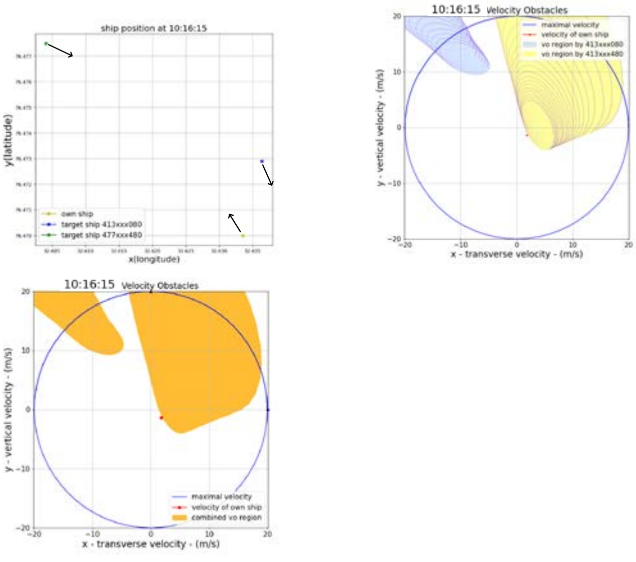

Figure 7. Positions and Vos of ships at “10:16:15” –case 1.

Case two is another three-ship encounter for 8mins, ship “413XXX220”, ship “413XXX350” and ship “477XXX900” are involved in the duration. Ship “413XXX220” was chosen as the research object (Own ship). The detection results of the three-ship encounter between the own ship “413XXX220” and target ship1“413XXX350”, target ship2“477XXX900” were shown with their trajectories and encounter situation in velocity space at a certain time step. Fig.8 shows their trajectories from “13:16:35” to “13:24:23” and the relative distance between the two target ships and the own ship, on May 20, 2019. We can see that at the beginning of the encounter process, the own ship “412XXX450” and two target ships respectively were in a “crossing” situation.

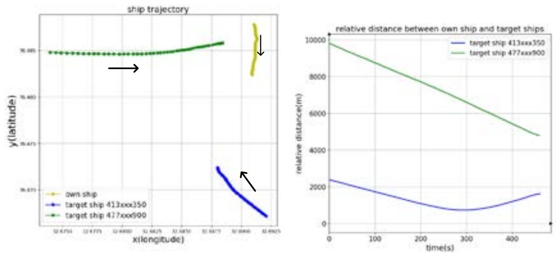

Figure 8. Trajectory and relative distance between ships-case 2

Fig. 9, 10 illustrate the encounter situation respectively at “10:18:15”, “10:19:05” AM by a snapshot of the positions of ships, individual NLVO and combined NLVO. The two individual NLVOs has an intersection, and the area of the combined NLVO is smaller than the sum of the two individual areas. In Fig. 9 (a) we can see target ship “413XXX350” was close to own ship at 10:18:15 AM, which may be dangerous. From Fig. 9 (b) and (c), we can see that there is no violation of any NLVO, which indicates that own ship was safe but need pay attention if a turn was needed. From Fig. 10 (b) and (c), we can see that own ship’s velocity violated the individual NLVO induced by the ship “413XXX350”, which indicates that own ship may collide with the target ship in 8 mins if keeping this velocity. The detection results changed during this period, reflecting the change of collision risk in this period.

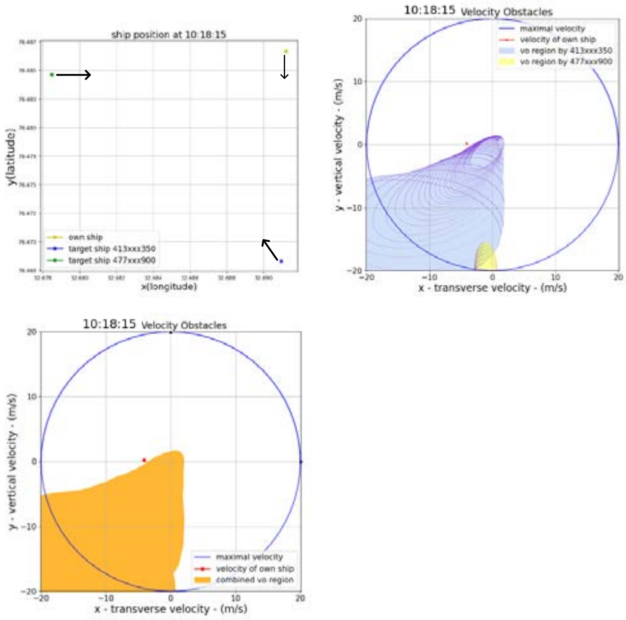

Figure 9. Positions and Vos of ships at “10:18:15” –case 2.

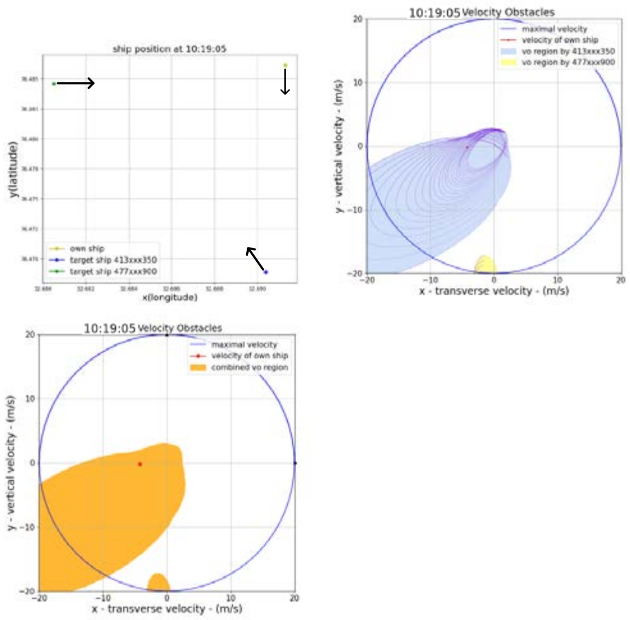

Figure 10. Positions and Vos of ships at “10:19:05” –case 2.

5 DISCUSSION

In this section, a comparison between the original TD - NLVO (M1) [10]and ship domain - based TD - NLVO (M2) is conducted. The comparison has two components: 1) comparison between results from M1 and M2 and 2) analysis of the detection results by two different methods. The AIS dataset of encounters utilized is the same as the two cases in the case study. The parameter setting between the two methods is shown in the Table 1.

Table 1. Parameter setting of the two methods

Parameter	M1	M2
$T_{threshold}$	60s	60s
$T_{scanning}$	20mins	20mins
Available V	±20m/s	±20m/s
Criteria	Circular safety region	Elliptical ship domain
Radius	800m	Semi - major: 1000m Semi - minor: 500m

The two methods were applied to the same AIS data, and the danger of the same multi-ship encounters was detected at the same time step. In the case at the same time steps, the detection results are not always the same. For case one, at 10:15:05 AM, the velocity of the own ship just violated the NLVO induced by one target ship, which is the same in M1 and M2, as shown in Fig.11. At 10:16:15 AM, the velocity of the own ship violated the individual NLVO in M1, while in M2 there was no violation, as shown in Fig.12. The detection results of two methods at every time step of case one are shown in Appendix Ⅰ.

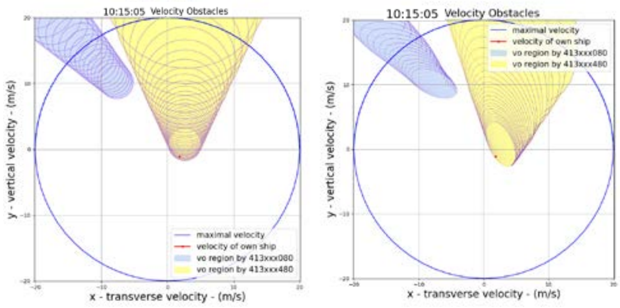

Figure 11. Illustration of NLVO with M1 and M2 at “10:15:05”-case 1

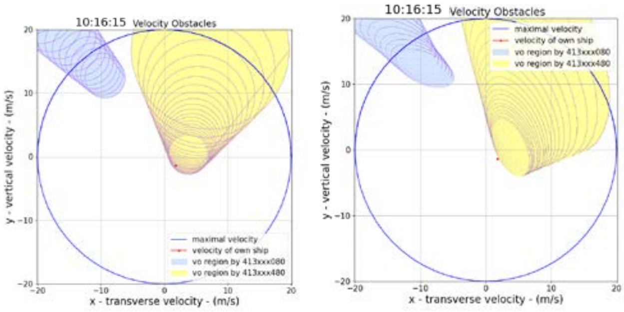

Figure 12. Illustration of NLVO with M1 and M2 at “10:16:15”-case 1

For case two, at 10:18:15 AM the velocity of the own ship just violated the NLVO induced by one target ship in M1, but in M2 there was no violation, see Fig.13. At 10:19:05 AM, the velocity of the own ship violated individual NLVO in both M1 and M2, see Fig.14. The detection results of the whole encounter process are shown in Appendix Ⅱ.

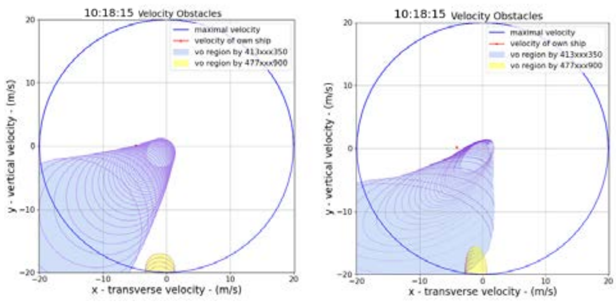

Figure 13. Illustration of NLVO with M1 and M2 at “10:18:15”-case 2

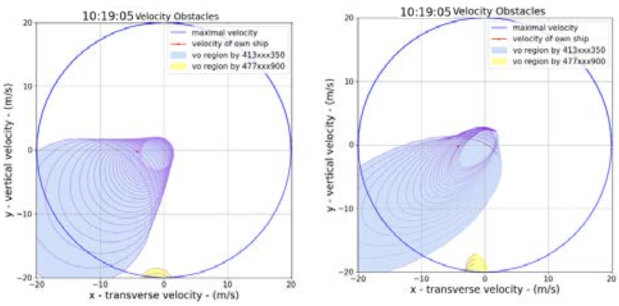

Figure 14. Illustration of NLVO with M1 and M2 at “10:19:05”-case 2

The difference can be explained by the difference in criteria choices. From the comparison, we found out that M2 was more sensitive to the course of target ships. The NLVO area located longitudinally on the target ship covers a larger area than in the lateral direction. In a result, any change in the course angle of the target ship will change the shape and area of the NLVO region and may make a difference to the detection results of the multi-ship encounter. Compared with the criteria of the circular region, integrating the elliptic region into TD - NLVO is more reasonable in this area, because it has the preference of coverage on different directions around the ship based on various aspects, e.g., the experience of the officers on watch, ship maneuverability, etc. What's more, compared with the detection model for pair-ship encounter in[11], we can find that the criteria of the algorithm were adjusted according to the different waterway and traffic situations. If more ships are involved in an encounter, a larger ship domain for the safe meeting may be required.

6 CONCLUSIONS

This paper proposes a modified non-linear velocity obstacle algorithm for the multi-ship encounter by integrating an elliptical ship domain. The algorithm includes a case study that shows its effectiveness in comparison to the existing VO algorithm. The integration of an elliptical ship domain helps in identifying collision candidates that consider the course and length of ships, which is shown in the discussion. The maritime transport system is the backbone of the global transportation system. This method presents a fresh perspective for considering multi-ship encounter cases. Port authorities and maritime safety administration can utilize this method to appreciate the collision risk in the region and facilitate the decision-making process of safety measures. However, the choice of parameters, such as the ship domain parameters, can have a significant impact on the detection results of collision candidates. Further efforts can be dedicated to determining the criteria considering the traffic characteristics of the region, such as the distribution of ship length. The parameter of the ship domain used in different waters needs to be adjusted according to the traffic data. Another aspect that requires further work is how to improve data quality to avoid underestimation of the results, as data can have missing information about ship length. In future studies, the method will be used to quantify the collision risk of ships in encounters to further quantify the risk and ensure the relative safety of all ships within their domain.

ACKNOWLEDGMENT

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APPENDIX I

DETECTION RESULTS OF BOTH MODELS

Detection Case	Detection Time	Violation Numbers in M1	Violation Numbers in M2
Case one	10:15:05	1	1
	10:15:15	0	1
	10:15:25	0	1
	10:15:35	0	1
	10:15:45	0	1
	10:15:55	0	1
	10:16:05	0	1
	10:16:15	0	1
	10:16:25	0	1
	10:16:35	0	1
	10:16:45	0	1
	10:16:55	0	1
	10:17:05	0	1
	10:17:15	0	1
	10:17:25	0	1
	10:17:35	0	1
	10:17:45	0	0
	10:17:55	0	0
	10:18:05	0	0
	10:18:15	0	0
	10:18:25	0	0
	10:18:35	0	0
	10:18:45	0	0
	10:18:55	0	0
	10:19:05	0	0
	10:19:15	0	0
Total		1	16

"Violation numbers" means the numbers of violation of Individual NLVO.

APPENDIX II

DETECTION RESULTS OF BOTH MODELS

Detection Case	Detection Time	Violation Numbers in M1	Violation Numbers in M2
Case two	10:16:35	0	0
	10:16:45	0	0
	10:16:55	0	0
	10:17:05	0	0
	10:17:15	0	0
	10:17:25	0	0
	10:17:35	0	0
	10:17:45	0	0
	10:17:55	0	0
	10:18:05	0	0
	10:18:15	0	0
	10:18:25	0	0
	10:18:35	0	1
	10:18:45	1	1
	10:18:55	1	1
	10:19:05	1	1
	10:19:15	1	1
	10:19:25	1	1
	10:19:35	1	1
	10:19:45	1	1
	10:19:55	1	1
	10:20:05	1	1
	10:20:15	1	1
	10:20:25	1	1
	10:20:35	1	1
	10:20:45	1	1
	10:20:55	1	1
	10:21:05	1	1
	10:21:15	1	1
	10:21:25	1	1
	10:21:35	1	1
	10:21:45	1	1
	10:21:55	1	1

	10:22:05	1	1
	10:22:15	0	0
	10:22:25	0	0
	10:22:35	0	0
	10:22:45	0	0
	10:22:55	0	0
Total		21	22

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