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Technical note

BUCKLING ANALYSIS OF AIRFRAME JOINTED PANEL UNDER COMBINED LOADING

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The complicated relationship of the high order static indeterminate structure will lead to a lot of calculation work. The strength analysis of the structure is very difficult. In aircraft design phase, a structural simplified method should be used to model the load characteristics of the structure. In the paper, the buckling analysis of airframe jointed panel is investigated under combined loading and the effect of jointed position to buckling load is also presented. For the buckling analysis of special joined structure, one new method which is better than traditional methods is described.

Key words: strength, buckling analysis, jointed panel, combined loading.

1. Introduction

Special aircraft structures such as the fuselage and wing are composed of many parts with fastening connections. Aircraft design engineers will have much calculation work to analyze these complicated structures. The special structures need the engineering experience of aircraft design engineers and special analysis method. To assess the structural strength and stiffness, the allowable stress or strain will be calculated to be compared with structural work stress or strain.

At present, the fuselage panel and the wing panel are thin-walled reinforced structure which have many failure forms, such as buckling instability. Now a lot of aircraft research and development institutions have used the structural analysis method and had many research achievements concerning buckling instability, involving the buckling analytic method, semi-analytic semi-empirical method, numerical analysis method and test method, etc. In the literatures [1-4], the analytical solutions of the elastic buckling for flat plates and curved plates under different boundary conditions and load conditions are given. The influence of plate buckling parameters is also assessed and summarized into graphs. In [5], the arc-length method is applied to track the whole equilibrium path of typical stiffened panels. The buckling effect of geometrical and material nonlinear factors is analyzed, meanwhile the buckling process and ultimate strength is also studied.

In [6] the buckling behavior for a class of the reinforced plate under compressive and shear loading, the different structural parameters and load ratio are numerically studied. The theoretical method is more

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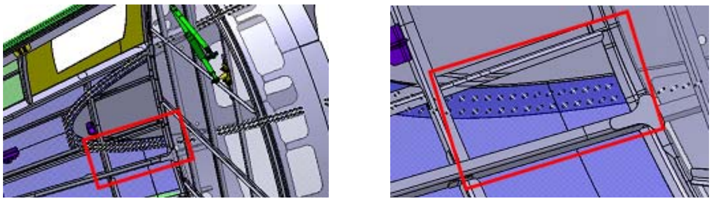
difficult than the semi-analytic semi-empirical method to engineering applications. In [7-9], the parameters of the semi-analytic semi-empirical buckling analysis are summarized into some charts. In the conceptual design phase, these methods are used to analyse buckling strength of the reinforced plate. However, these methods are not enough to exactly predict the buckling strength which needs verified test and corrected method in the subsequent detailed design phase. In this paper, the buckling method of joined flat panel is studied. Semi-analytic semi-empirical equation is given to assess this special structure. The effect of jointed position and lap thickness is assessed by the finite element analysis method, and the results could be used to direct the analysis of jointed panel.

2. Analysis model

Lap is used to joint two fuselage sections. In Fig.1, the fuselage and vertical tail are jointed with two fasten rows. The basic sizing of the special structure is shown in Fig.2. Both sides of the jointed plate are of different material and different thickness. One side is the aluminum alloy with *2.3 mm* and the other is the titanium alloy with *1.6 mm*.



(a) global configuration (b) local view

Fig.1. Jointed panel.

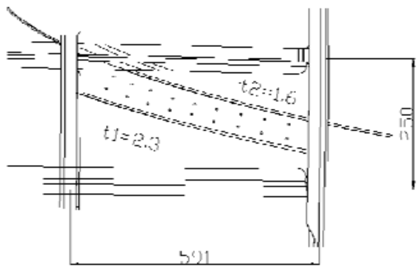
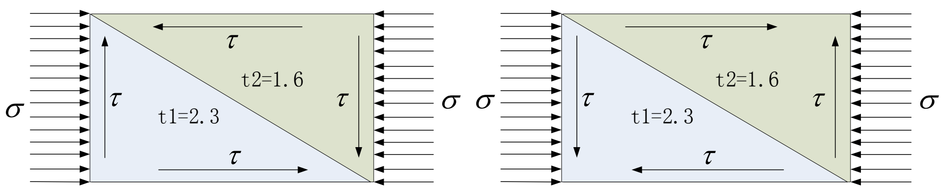


Fig.2. Structural sizing.

The jointed plate which consists of two materials and two thicknesses is different than the common flat panel. The detail structural configuration is necessary to study the buckling behavior under combined loading. There are two combined configurations, as shown in Fig.3. In order to consider the effect of the material and thickness exactly, fastening would be ignored in jointed position.



(a) type Ⅰ (b) type Ⅱ

Fig.3. Two combined configurations.

3. Panel buckling

For a single material panel, the effect of panel sizing, loading and support on the buckling load is given in many existing references. The buckling deflection of flat plates set on the central region, and the deflection is zero in supported edges. The support of adjacent parts can make the panel carry more load and the buckling load is usually smaller than other failure load. In [2] and [3], the critical buckling load of a single material panel is given. The critical load analysis process under the axial load is introduced in detail.

Under the axial compressive loading, the four sides of the panel are simply supported. The deflection can be represented by the double trigonometric series

$$\omega = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{3.1}$$

where ω is the deflection of the panel; m is the number of half wave on the x axis; n is the number of half wave on the y axis; a is the long size of the panel on the x axis; b is the short size on the y axis which also is loaded edge; A_{mn} is unknown coefficients which would satisfy the differential equation of deflection curve and boundary conditions (see the [5]).

Total potential energy equation of the panel is

$$U + V = \frac{1}{2}\int_0^a\int_0^b \left[D\left\{ \left(\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2} \right)^2 - 2(1-\upsilon)\left[\frac{\partial^2\omega}{\partial x^2}\frac{\partial^2\omega}{\partial y^2} - \left(\frac{\partial^2\omega}{\partial x\partial y} \right)^2 \right] \right\} - N_x \left(\frac{\partial\omega}{\partial x} \right)^2 \right] dxdy\,, \tag{3.2}$$

in which U is the total strain energy; V is the total potential energy; D is the bending stiffness of the panel; N_x is the compressive loading on the x axis.

By using the deflection equation, the total potential energy equation become

$$U + V = \frac{\pi^4 abD}{8}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 - \frac{\pi^2 b}{8a} N_x \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m^2 A_{mn}^2\,. \tag{3.3}$$

With the principle of invariable potential energy, we immediately obtain

$$\frac{\partial(U+V)}{\partial A_{mn}} = \frac{\pi^4 abD}{4} A_{mn} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2 - \frac{\pi^2 b}{4a} N_x m^2 A_{mn} = 0\,. \tag{3.4}$$

Then we have a special solution

$$N_{x,cr} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2. \tag{3.5}$$

Equation (3.5) is an increased function with m and n, so it will get the minimum value at $n = 1$. The critical load equation can be written as

$$N_{x,cr} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2}+\frac{1}{b^2}\right)^2 . \tag{3.6}$$

With the definition of the buckling coefficient

$$k_c = \left(\frac{mb}{a}+\frac{a}{mb}\right)^2 . \tag{3.7}$$

We simplify Eq.(3.6) as

$$N_{x,cr} = \frac{k_c \pi^2 D}{b^2}, \tag{3.8}$$

$N_{x,cr}$ is the critical load under axial compressive loading (unit *N/mm*). We use force flow for the analysis which is traditionally calculated by stress[8, 9].

Under shear loading, the total potential energy equation of panel is

$$U+V = \frac{1}{2}\int_0^a\int_0^b \left[D\left\{ \left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)^2 - 2(1-\upsilon)\left[\frac{\partial^2\omega}{\partial x^2}\frac{\partial^2\omega}{\partial y^2} - \left(\frac{\partial^2\omega}{\partial x\partial y}\right)^2\right]\right\} - 2N_{xy}\frac{\partial\omega}{\partial x}\frac{\partial\omega}{\partial y}\right] dxdy . \tag{3.9}$$

The solving process is the same as the process under axial compressive loading. We get the critical shear load

$$N_{xy,cr} = \frac{k_s \pi^2 D}{b^2} \tag{3.10}$$

where $N_{xy,cr}$ is the critical load under shear loading (unit *N/mm*). The curve of k_s is illustrated in [2] and [3].

4. Solution and analysis

4.1. Engineering analysis

The critical buckling load of the panel depends on the size ratio of the panel, the thickness and the boundary conditions. The size and thickness of the jointed panel are shown in Fig.2. The elastic modulus of the aluminum alloy is *71000* MPa and the Poisson's ratio is *0.33*. The elastic modulus of the titanium alloy material is *110.000* MPa and the Poisson's ratio is *0.3*. The axial compressive loading of the analysis zone is *20 N/mm* and the shear flow is *20 N/mm*.

In the engineering analysis, the stability method of the panel [9] is usually used to calculate the safety margin of the panel buckling under the combined loading. In the calculation, the jointed panel with different materials and different thickness is assumed for the whole panel of a single material. According to the general engineering experience, the equivalent method with extensional stiffness *EA* is usually used to deal with the configuration of two cross-sections.

In the case of the same plate width, the equivalent method with extensional stiffness *EA* is simplified to the equivalent method with *Et*. So the thickness of the aluminum alloy plate is expressed as

$$t' = \left(\frac{E_1 t_1 + E_2 t_2}{2E_1} \right) = 2.38\ mm\ ,$$

$$D = \frac{E_1 t'^3}{12\left(1 - \upsilon_1^{\,2}\right)} = 87653\ N \cdot mm\ .$$

Table 1. Buckling analysis when thickness is *2.38mm*.

case	buckling coefficient *k*	critical load [*N/mm*]	Working load [*N/mm*]	the ratio of load	safety margin
shear	*6.24*	*86.4*	*20*	*0.23*	*1.1*
compression	*4*	*55.37*	*20.1*	*0.36*	

In order to evaluate the analysis results of the equivalent thickness, the stability of the aluminum alloy plate whose thickness is *2.3 mm* and the titanium alloy plate whose thickness is *1.6 mm* are analyzed, respectively. The results are shown in Tab.2 and Tab.3.

Table 2. Buckling analysis when thickness is *2.3mm*.

case	buckling coefficient *k*	critical load [*N/mm*]	Working load [*N/mm*]	the ratio of load	safety margin
shear	*6.24*	*77.98*	*20*	*0.26*	*0.9*
compression	*4*	*49.97*	*20*	*0.40*	

Table 3. Buckling analysis when thickness is *1.6mm*.

case	buckling coefficient *k*	critical load [*N/mm*]	Working load [*N/mm*]	the ratio of load	safety margin
shear	*6.24*	*40.67*	*20*	*0.49*	*-0.01*
compression	*4*	*26.06*	*20*	*0.77*	

From the values in these tables, the safety margin of jointed panel should be analyzed between *-0.01* and *0.9*. The analysis results of the single aluminum alloy panel are not reasonable, so the conventional equivalent method with extensional stiffness *EA* is not suitable for the stability analysis of the jointed panel.

When a rectangular panel is loaded by combined loading, the middle of the panel will produce stress. If the loading is large enough, it will increase the bending. Using the equivalent method of bending stiffness *EI*, the jointed panel is equivalent to the aluminum alloy panel. In the case of the same panel width, the equivalent method with bending stiffness *EI* is simplified to the equivalent method with Et^3. so the thickness is calculated as

$$t = \left(\frac{E_1 t_1^3 + E_2 t_2^3}{2E_1} \right)^{1/3} = 2.1\ mm\ ,$$

$$D = \frac{E_1 t^3}{12\left(1 - \upsilon_1^{\,2}\right)} = 59357.4\ N \cdot mm\ .$$

Table 4. Buckling analysis when thickness is *2.1mm.*

case	buckling coefficient k	critical load [N/mm]	Working load [N/mm]	the ratio of load	safety margin
shear	*6.24*	*58.51*	*20*	*0.342*	*0.42*
compression	*4*	*37.49*	*20.1*	*0.536*	

From above analysis results, the equivalent method with bending stiffness EI is better than conventional EA equivalent method to analyze the stability of the jointed panel.

4.2. FEM analysis

The buckling mode of the jointed panel is different from the mode of general thin-walled reinforced panel. Under combined loading, the stiffness distribution of the panel is affected by jointed position, In order to facilitate the engineering buckling analysis, it is necessary to study the effect of the jointed position and the loading state.

According to the finite element method (FEM), the jointed panel is modeled to calculate the buckling strength. There are some simulation software programs, such as MSC. Patran and MSC. Nastran[10]. In the finite element model where one edge is simply supported and the outside displacement of non-study region is restrained, the axial compressive load is *20 N/mm* and the shear load is *20 N/mm*. The finite element model is shown in Fig.4.

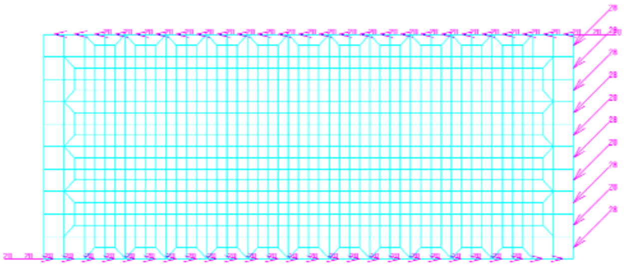
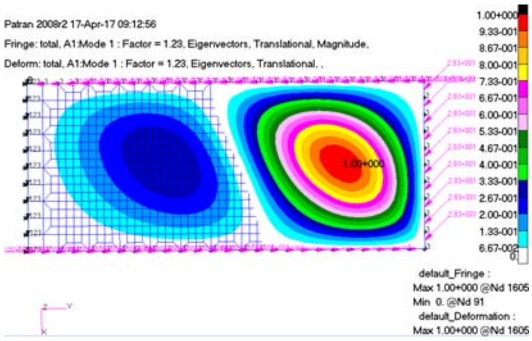


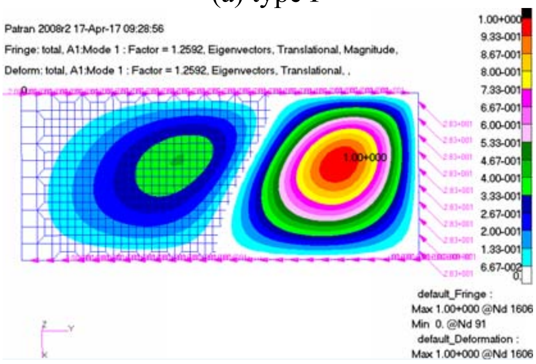
Fig.4. The finite element model.

For the two types in which the jointed position is different, the buckling modes are near the side where the bending stiffness is smaller. In Fig.5, the modal eigenvalue of the step Ⅰ is 1.23 and the eigenvalue of the step II is 1.259. The critical load is the product of the eigenvalue and the applied load, so the safety margin of two types are *0.23* and *0.259*, respectively. Based on the results of the finite element analysis, the buckling modes of the two types are similar.

The buckling mode of the panel of a single material is in the middle or symmetric in the middle. If the equivalent thickness of the panel is calculated by the extensional stiffness, the eigenvalue is 2.11. If the equivalent thickness of the panel is calculated by the bending stiffness, the eigenvalue is 1.45. For the buckling analysis of the jointed panel, the results with the equivalent method of bending stiffness are closer to the results of the finite element analysis.

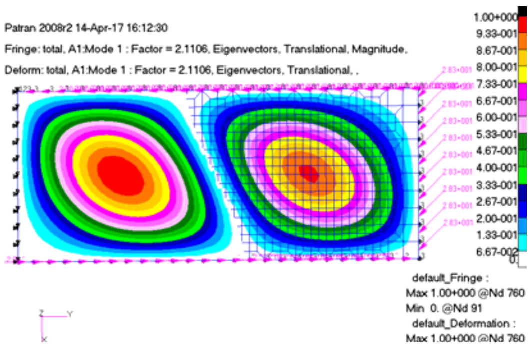


(a) type I

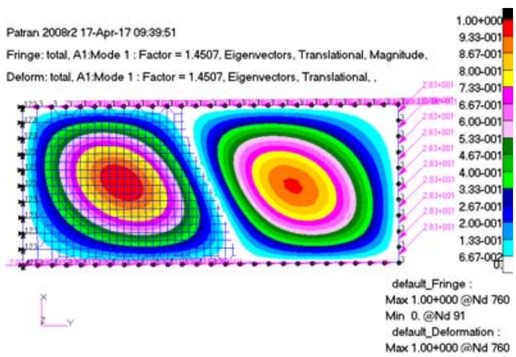


(b) type II

Fig.5. First-order buckling mode of two types.



(a) equivalent thickness is *2.38 mm*



(b) equivalent thickness is *2.1 mm*

Fig.6. First-order buckling modes with two thicknesses.

5. Conclusions

For the buckling analysis of th jointed panel which has different materials and different thicknesses, the buckling critical load equation is given in Eq.(3.8.) In this paper, the effect of jointed configuration and thickness to buckling result is considered in engineering analysis and FEM analysis. The following main conclusions are obtained:

(1) In the buckling analysis of the jointed panel, the equivalent thickness method with the bending stiffness is better than the traditional equivalent thickness method with extensional stiffness. The results based on the equivalent method of bending stiffness are closer to the results of the jointed panel.

(2) Under combined loading, the panel is subjected to diagonal tension. The lap configuration which defines the stiffness distribution could affect the buckling mode of the jointed panel. Through the buckling analysis of different configurations, we see that the jointed position could affect the buckling load, but the degree is very small. In the conceptual design phase, the consideration of special jointed position is not needed to assess the stability of the jointed panel.

In the era of the rapid development and wide use of computer technology, the approximate numerical method is easier and more effective than the others. Engineering empirical equations and FEM method are used to assess the structural strength before the structural verified test. The finite element numerical simulation could make up the shortcoming of conventional engineering methods.

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