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Sensitivity analysis of the specific heat and thermal conductivity values on pore pressure and temperature distribution within embankment

Abstract

The paper presents the results of the sensitivity analysis of the variability of thermal conductivity and specific heat values on temperature field and pore pressure distribution during the flooding process simulation. The sensitivity analysis was carried out using data obtained from numerical modellings. During numerical modellings the simulation of flooding process included flooding and discharging the tank, with maximum water level equal to 3.0 m was performed. 2D model was realized using Fast Lagrangian Analysis of Continua (FLAC) 7.0 software, which is a two-dimensional explicit finite difference program for engineering mechanics computation. The results presented in this paper show that the sensitivity analysis method is powerful technique that can be used in detection of area within the embankment where the biggest impact on variability of thermal conductivity are presented. It may be helpful in selecting the best area for pore pressure and temperature sensors location.

Keywords: sensitivity analysis, thermal 2D numerical modeling, embankment, FLAC, ISMOP.

1. Introduction

Flooding is an example of natural phenomena which have great impact on both individuals and communities. It have serious social, economic and environmental consequences. The flooding consequences are both immediate and also causes long-term impacts such as disruptions to electricity, transport, communication, education or health care. Therefore effective warning system of increase flooding risk is especially important in the area where inland and costal flood appear. The ISMOP (polish abbreviation of Computer System for Monitoring River Embankments) project [1] is an example of cooperation between science and industry in the field of flood defense. The main aim of the ISMOP project is to research complex systems for embankment monitoring and threat forecasting [2], [3]. As a part of the project, an experimental embankment was constructed in order to conduct real experiment of the flooding process. The state of the experimental embankment was evaluated on the basis of the pore pressure and temperature values recorded by the sensors placed inside it. Consider the fact that experimental embankment is not made of a material having the same geotechnical and thermal parameters, several numerical models were conducted for different values of soil parameters. Then the sensitivity analysis was carried out in order to discover the sensitivity of the pore pressure and temperature distribution to the specific heat and conductivity values variability. A sensitivity analysis method was previously used for uncertainty analysis in a real geotechnical problem [4], [5] and to indicate the most significant parameters of flood waves in terms of embankment state [6].

2. Sensitivity analysis

The sensitivity analyses in general is applied to determine which parameters require additional research in order to 1) reduce the output uncertainty 2) determine which parameters are insignificant and can be eliminated from the final model 3) which inputs contribute most to output variability [7]. There are many different ways of conducting sensitivity analyses and the various analyses may not produce identical results [8].

In this work the sensitivity analysis was conducted in order to assess which of the two tested thermal parameters: thermal conductivity and specific heat are most highly correlated with the distribution of pore pressure and temperature within the embankment. It was also examined how strong is the impact and if so weather it is changing over the time passed.

The results presented in this work were obtained using sensitivity score and relative sensitivity value, which can be used for the parameters values determined by the intervals without any information over the probability distribution across them.

The value of sensitivity score is calculated on the basis of sensitivity ratio value η_{SR}. The sensitivity ratio η_{SR}* can be described as the percentage change in model output divided by the unit change of the input variable [9]:

$$\eta_{SR}* = \left| \frac{\dfrac{f(x*) - f(x_m)}{f(x_m)}}{\dfrac{x* - x_m}{x_m}} \right|, \tag{1}$$

where: $f(x_m)$ is the reference value of model output obtained after using the reference value x_m of the given parameter, often the mean value [9], and $f(x*)$ is the value of the output variable obtained after using the changing value of the given input parameter $x* = [x_G^{lo}, x_L^{lo}, x_L^{up}, x_G^{up}]$, which are lower (*lo*) and upper (*up*) range of the global (*G*) and local (*L*) category. Figure 1 illustrates construction of cumulative distribution function (Fig. 1a) and location of $x*$ components (Fig. 1b) from multiple sources of a given intervals $A_1..A_4$. The value of sensitivity score is calculated separately for each input variable.

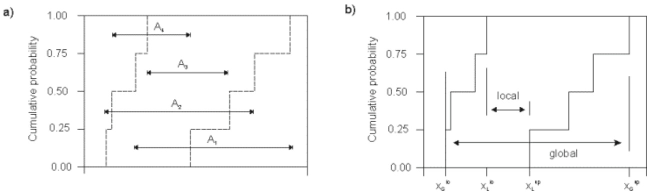

Fig. 1. Construction of cumulative distribution function for a given intervals *A*1..*A*4 (a) and location of the upper and lower range of the local and global input parameters used in sensitivity score calculation. Modified after [10]

Sensitivity ratio was then used in sensitivity score η_{SS} calculation. Sensitivity score is more robust method of evaluating the uncertainty of a given model then sensitivity ratio. It is obtained by normalizing and weighting the sensitivity ratio value (Eq. 2).

$$\eta_{SS} = \eta_{SR} \frac{(\max x_G - \min x_G)}{x_m}, \quad \eta_{SR} = \sum_{i=1}^{4} \eta_{SR,i}^{*} . \tag{2}$$

The normalization procedure makes the sensitivity score value independent of the input values units of the given parameter. It also allows to compare results of sensitivity analysis obtained for different parameters. Sensitivity score $\eta_{SS,\,i}$ is calculated for all *N* number of the parameters considered.

The second parameter calculated during sensitivity analysis of the thermal parameters was total relative sensitivity (x_i) value (Eq. 3).

$$\alpha(x_i) = \frac{\eta_{SS,i}}{\sum_{i=1}^{N} \eta_{SS,i}}. \tag{3}$$

Results of all the field computed during numerical modelling (in this case pore pressure and temperature distribution within experimental embankment) are taken into account. The value of total relative sensitivity if often used to determine which parameters have great impact on the model state if the threshold value is determined. If the value of total relative sensitivity of the given parameter is less then threshold value it may indicate that this parameter is insignificant and can be eliminated from the final model.

3. Uncertainty of specific heat and thermal conductivity values

The uncertainty of specific heat and thermal conductivity values is caused by significant variation in the values of geothermal parameters (inner ISMOP report) used for experimental embankment construction. Only the intervals instead of exact values of the input parameters were obtained. Therefore, to reduce ambiguity additional source of information based on estimated values was found [11]. The interval values of specific heat was assumed after laboratory measurements (data set I in Tab. 1) and after measurements made in situ (data set II in Tab. 1). Thermal conductivity values were derived from geotechnical measurements (data set I) and from literature and estimated values (data set II). In the Tab. 1 intervals for two materials with different thermal parameters are presented. Those materials were used in construction of experimental embankment and in numerical modellings used for sensitive analysis presented in this article. In the next section detail description of experimental embankment geometry and material used in its construction is presented.

Tab. 1. Specific heat and thermal conductivity intervals used in sensitivity analysis

Material	Data set number	Specific heat, J/(kg·K)	Thermal conductivity W/(m·K)
A	I	1194 – 1267	0.6 – 2.3
	II	950 – 1133	1.6 – 2.3
D	I	1095 – 1196	1.0-3.3
	II	910 - 1005	1.5 –3.1

4. Numerical modelling of pore pressure and temperature distribution

Numerical modelling of pore pressure and temperature distribution within embankment during the simulated flood was performed using FLAC, a two-dimensional explicit finite difference program. The software allows the performing of the coupled mechanical-fluid and flow-thermal processes which are used in modelling the impact of the flooding wave on the experimental embankment [12].

Numerical calculation were conducted for the central cross section (Fig. 2a) of the experimental embankment build model of the ISMOP project (Fig. 2b).

The experimental embankment was built using material with different geotechnical properties, named A – D (Fig. 2). The use of inhomogeneous material in embankment construction allows to observe and draw conclusion of the behavior of the embankment build with material with different geotechnical parameters.

Material parameters used in numerical calculation are presented in the table below (Tab. 2).

Numerical modelling of the flooding phenomena was realized by progressively increase of the water level to the height of 4 meters for a period of 3 days. Then, modelling of a high water level maintained for a period of 2 days was carried out. The last stage of numerical modelling assumed simulated decreasing of the water level from the height of 4 meters to dry state lasted 5 days. In the Fig. 4 the plot of the changing water level in time assumed during numerical modelling is presented. The results of the pore pressure values and temperature field obtained for all the computational nodes were recorded during numerical simulations. The sensitivity analysis of the thermal conductivity and specific heat value was performed for the time of the highest water level just before decreasing of the water level has started ("H" in Fig. 3) and after complete regression of the water ("L" in Fig. 3).

Tab. 2. Material parameters assumed in calculation (after [14])

	Material A	Material D	Sub-surface	Silty	Sand
Density, kg/m^3	1900	1940	2100	1890	1830
Cohesion, kPa	12.5	16.11	10.3	13.7	9.75
Friction, deg	30.04	31.42	32.9	22.5	35.8
Bulk module, MPa	8.53	7.25	7.25	16.2	36.3
Shear module, MPa	3.27	3.35	3.43	6.63	21.8
Porosity, %	37	32	27	40	35
Permeability, m/s	1.2×10^{-5}	3.9×10^{-5}	1.49×10^{-4}	1.32×10^{-4}	5.50×10^{-4}

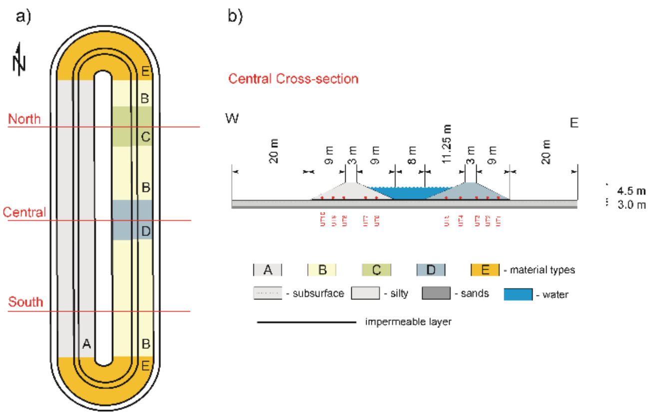

Fig. 2. Experimental embankment plan together with marked three cross section (a) and geometry of the geological model assumed for the numerical calculation (b). After [13], modified

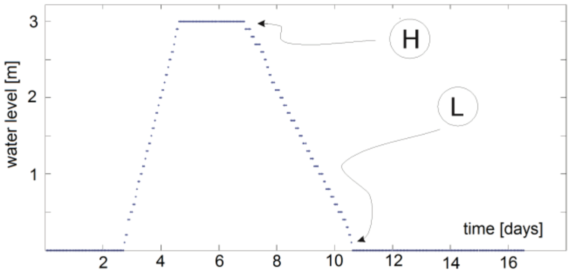

Fig. 3. Plot of water level assumed during the numerical modelling of flood. The time stamps of the flooding process for which the sensitivity analysis were performed are highlighted with the letters "H" and "L"

5. Results of sensitivity analysis

Results of sensitivity analysis obtained for all computational nodes, indicating the impact of the thermal conductivity and specific heat values to pore pressure and temperature distribution within experimental embankment are presented in Fig. 4, 6, 8, 10. Plots correspond to the value of sensitivity score (Eq. 2) described above. The results of sensitivity analysis conducted for temperature and pore pressure distribution that correspond to the total relative sensitivity (Eq. 3) are also presented (Fig. 5, 7 and 9, 11).

5.1. Sensitivity results after the highest water level of flood wave

The results of the sensitivity analysis obtained after maintaining of the height water level for pore pressure distribution are presented in Fig. 4, 5.

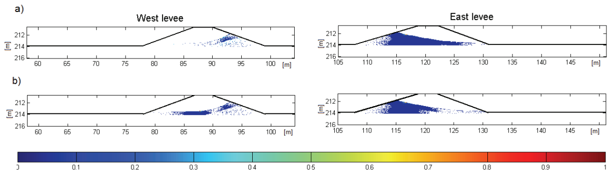

Fig. 4. Sensitivity score values of thermal conductivity (a) and specific heat (b) variability for the pore pressure distribution after height water level period

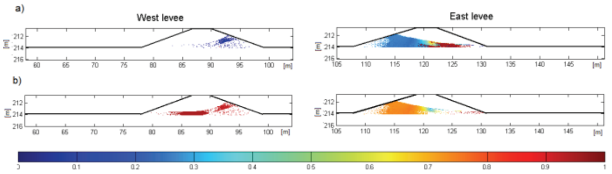

Fig. 5. The total relative sensitivity values of thermal conductivity(a) and specific heat variability (b) for the pore pressure distribution after height water level period

Distribution of sensitivity score values obtained for pore pressure field differ significantly for both wings of the experimental embankment. The east wing of embankment, which is constructed from material "D", is more sensitive to the variability of thermal conductivity and specific heat values than the west wing of embankment, built from material "A" (Tab. 2). This is reflected by larger area of no zero sensitive score value in the east wing of embankment than the same area detected in the west wing of embankment (Fig. 4). The plot of total relative sensitivity value obtained for pore pressure distribution (Fig. 5) presents even stronger differentiation between west and east wings of embankment. West and east embankment wing show the change in the impact of thermal conductivity and specific heat variability to the value of total relative sensitivity. Such strong differences is caused by are various porosity values of the material "A" and "D". The thermal conductivity of rocks is highly dependent upon porosity [15]. Although many different empirical equation are used to describe this dependency [16], all relationships assume negative correlation between porosity and thermal conductivity. The porosity of the material "A" used in construction of the west wing of the embankment is 5% higher than the porosity of the material "D", which in turn implies less impact of the variability of thermal conductivity value obtained for the west wing of embankment.

Sensitive analysis of temperature distribution (Fig. 6 and 7) shows the same pattern for both types of materials.

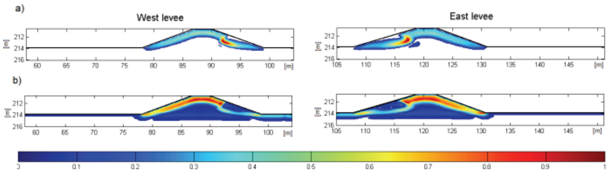

Fig. 6. Sensitivity score values of thermal conductivity (a) and specific heat variability (b) for the temperature distribution after height water level period

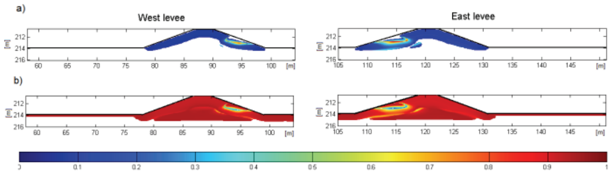

Fig. 7. The total relative sensitivity values of thermal conductivity (a) and specific heat variability (b) for the temperature distribution after height water level period

The size of area of increased sensitivity (Fig. 6), slightly larger for east wing of embankment is due to the higher permeability of material "D" (Tab. 2). The specific heat variability has the greatest impotence for the area where daily temperatures changing occurs. In contrary for the sensitivity analysis conducted for pore pressure distribution, the impact of the variability of thermal conductivity and specific heat values carried out for temperature distribution, remains the same for both wings of the embankment (Fig. 7).

5.2. Sensitivity results after fall of flood wave

The results of the sensitivity analysis obtained after fall of flood wave using the sensitivity score for pore pressure and temperature distribution and total relative sensitivity are presented in Fig. 8-11.

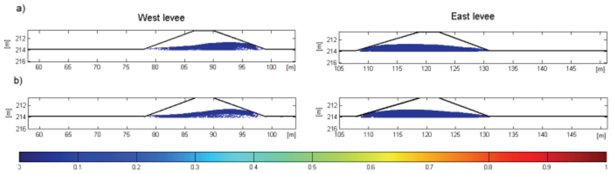

Fig. 8. Sensitivity score values of thermal conductivity (a) and specific heat variability (b) for the pore pressure distribution after fall of flood wave

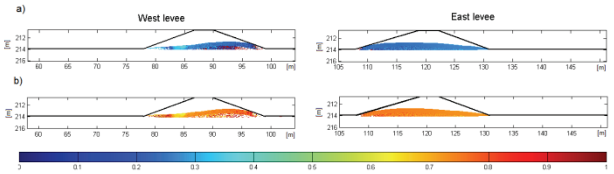

Fig. 9. The total relative sensitivity values of thermal conductivity (a) and specific heat variability (b) for the pore pressure distribution after fall of flood wave

In this case, area that are sensitive to thermal conductivity and specific heat values for pore pressure distribution in both wings of embankment (Fig. 8) are bigger then the sensitive area obtained after analysis carried out for high water level (Fig. 4). In contrary to the previous results, the impact of the thermal conductivity and specific heat variability shows the same pattern for both wings of embankment (Fig. 5 and 9 respectively). Furthermore, in both wings the impact of specific heat variability on pore pressure distribution is bigger than impact of variability of thermal conductivity value. Within detected area with increased sensitivity in the east wing of embankment almost constant proportion among thermal conductivity and specific heat variability is presented. However in the west wing of embankment the proportional influence of thermal conductivity and specific heat variability varies noticeably (Fig. 9).

The size of area of increased sensitivity of temperature distribution (Fig. 10), localized as a result of sensitivity analysis carried out after the fall of water, are bigger than the area detected after the stage of height water level (Fig. 6). The bigger size of sensitive area is associated with longer time of daily temperature influence and the larger embankment area subjected to that changes. However, just like in the case of sensitivity analysis conducted for high water level, the impact of the variability of thermal conductivity and specific heat values carried out for temperature distribution, remains the same for both wings of the embankment (Fig. 11).

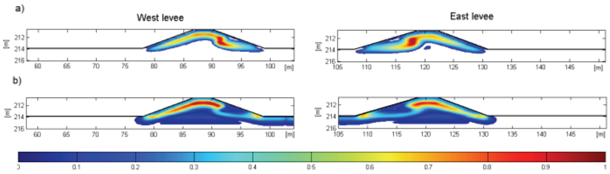

Fig. 10. Sensitivity score values of thermal conductivity (a) and specific heat variability (b) for the temperature distribution after fall of flood wave

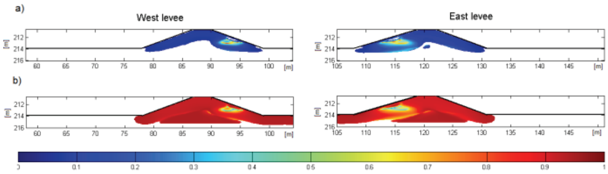

Fig. 11. The total relative sensitivity values of thermal conductivity (a) and specific heat variability (b) for temperature distribution after fall of flood wave

6. Conclusions

In this paper, sensitivity analysis was applied to the problem of variability of specific heat value and thermal conductivity to the distribution of pore pressure and temperature values within embankments during a flood.

The main differences between sensitive analysis conducted for different material are due to the different porosity of the material "A" and "D". Thermal conductivity of the material is highly dependent upon porosity. It is especially visible on results of sensitivity analysis on pore pressure distribution conducted after the state of high water level. The sensitivity analysis of the variation of thermal conductivity and specific heat values on pore pressure distribution carried out after the fall of water shows no differences among both wings of embankment.

The sensitivity analysis conducted for time steps corresponding to the highest water level and to the time of fall flood level allowed the same conclusions to be drawn about the significance of thermal conductivity and specific heat values to the temperature distribution. Both analysis (the highest and the lowest water level during the flood) conducted for both types of materials show strong impact of the thermal conductivity values to temperature distribution in the areas where the water infiltrated inside the shaft during flood. The longest infiltration process lasted, the biggest area of significant can be spotted. As it was presented in this paper, precise determination of thermal conductivity values, especially in the area of infiltration is crucial for using temperature field variation as an indicator of existence of infiltration processes within the embankment.

This work is financed by the National Centre for Research and Development (NCBiR), Poland, project PBS1/B9/18/2013 - (no 180535) and by AGH –UST grant no 11.11.140.613.

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Received: 22.11.2016 *Paper reviewed* *Accepted: 02.02.2017*

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