Adaptive robust simultaneous stabilization controller with tuning parameters design for two dissipative Hamiltonian systems

ZHONG CAO, XIAORONG HOU and WENJING ZHAO

This paper investigates the problem of adaptive robust simultaneous stabilization (ARSS) of two dissipative Hamiltonian systems (DHSs), and proposes a number of results on the controller parameterization design. Firstly, an adaptive H_∞ control design approach is presented by using the dissipative Hamiltonian structural for the case that there are both external disturbances and parametric uncertainties in two DHSs. Secondly, an algorithm for solving tuning parameters of the controller is proposed using symbolic computation. The proposed controller parameterization method avoids solving Hamilton-Jacobi-Issacs (HJI) equations and the obtained controller is easier as compared to some existing ones. Finally, an illustrative example is presented to show that the ARSS controller obtained in this paper works very well.

Key words: ARSS control, DHSs, symbolic computation, controller parameterization, tuning parameters.

1. Introduction

In practical control systems design, the simultaneous stabilization control problem has often to be considered because of system parameters' uncertainty, controller failure modes or systems with different modes of operation. It is a more economical approach to such problem that designing a single controller to stabilize all of the systems simultaneously. So far, there are many important results for simultaneous stabilization control in linear systems [1-4]. In general, designing simultaneous stabilization controller is

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very difficult for a class of nonlinear systems, but it is a work worth doing for many researchers. Some nice results were proposed for the existence of simultaneous stabilization controller of a set of nonlinear systems [5]. Through using Lyapunov function method, sufficient and necessary conditions for the existence of feedback time-invariant simultaneous stabilization controllers in single-input nonlinear systems were derived [6]. In the area of robust nonlinear systems control, the simultaneous stabilization control problem is one of important research topics, which has to be studied more and more. Sufficient conditions for simultaneous stabilization control with and without H_∞ performance were proposed for a class of state-dependent polynomial systems [7]. The simultaneous stabilization control problem for a set of port-controlled Hamiltonian systems was investigated and some results were proposed by using the Hamiltonian function method [8, 9]. Ref. [10] proposed a number of results on design of parallel simultaneous stabilization controllers for a set of port-controlled Hamiltonian systems with actuator saturation. An adaptive robust parallel simultaneous stabilization of two uncertain port-controlled Hamiltonian systems subject to input saturation had been presented [11].

As everyone knows, the control systems have two basic constraints, which are the internal stability and the external disturbance attenuation. However, it's crucial to satisfy some desired control objectives in designing a practical control system. Finding the parameterized controller is a sophisticated and efficient way for solving various control problems. Therefore, controller parameterization is a basic problem in the control theory, which has been received considerable attentions in recent decades [12-16]. Firstly, Ref. [12] proposed polynomial parameterization of stabilizing controllers. A family of nonlinear H_∞ controller via output feedback had been proposed [13]. Yung, Ref. [14] presented a family of H_∞ state-feedback controller for n-dimensional nonlinear system by extending the state-space formulas. A family of reliable nonlinear H_∞ controller had been proposed via solving the Hamilton-Jacobi-Issacs (HJI) inequality (or equations) [15]. These controllers obtained in [12-16] are intended to solve a class of HJI inequalities (or equations), which have actually imposed a considerable difficulty. Ref. [17] studied the generalized Hamiltonian system and proposed a family of parameterized controller in H_∞ control. An H_∞ controller with tuning parameters for polynomial Hamiltonian systems was proposed by using symbolic computation approach [18]. The methods in [17, 18], avoided solving HJI inequalities (or equations) through applying clear physical expression and good structure of dissipative Hamiltonian systems. But the parameterized controller is intended to solve the control problem for just one system. There are fewer works for ARSS control design of two DHSs.

Therefore, it is a challenging issue to design a controller with tuning parameters to solve ARSS problem for two DHSs. In this paper, a novel and convenient method to design an ARSS controller with tuning parameters for two DHSs is proposed. We add some items containing tuning parameters into the controller, and solve the tuning parameters ranges. Within parameters ranges, the controllers insure that the DHSs are ARSS control and can optimize the robustness for all or part of DHSs.

The remainder of this paper is organized as follows. In Section 2, the problem of ARSS for DHSs is formulated. The main contribution of this paper is then given in Sec-

tion 3, in which ARSS controller with tuning parameters and an algorithm for solving tuning parameters are provided, respectively. We present a numerical example for illustrating effectiveness and feasibility of controller in Section 4 and conclusions follow in Section 5.

2. Problem formulation

Consider the following two DHSs [8]:

$$\sum_1: \begin{cases} \dot{x} = [J_1(x, p_1) - R_1(x, p_1)]\dfrac{\partial H_1(x, p_1)}{\partial x} + g_1(x)u + \bar{g}_1(x)\omega \\ y = g_1^T(x)\dfrac{\partial H_1(x)}{\partial x} \end{cases} \tag{1}$$

$$\sum_2: \begin{cases} \dot{\xi} = [J_2(\xi, p_2) - R_2(\xi, p_2)]\dfrac{\partial H_2(\xi, p_2)}{\partial \xi} + g_2(\xi)u + \bar{g}_2(\xi)\omega \\ \eta = g_2^T(\xi)\dfrac{\partial H_2(\xi)}{\partial \xi} \end{cases} \tag{2}$$

where $x, \xi \in \Re^n$ and $y, \eta \in \Re^m$ are the states vector and the outputs of the two DHSs, respectively; p_1 and p_2 are constant unknown vectors in systems, which are sufficiently small; $u \in \Re^m$ is the controller with tuning parameters; $\omega \in \Re^s$ is the disturbance; $J_i(x,0) = J_i(x)$, $R_i(x,0) = R_i(x)$ and $J_i(x, p_i) = -J_i^T(x, p_i) \in \Re^{n \times n}$, $0 \leqslant R_i(x, p_i) \in \Re^{n \times n}$; $g_i(x) \in \Re^{n \times m}$ and $\bar{g}_i(x) \in \Re^{n \times s}$ are sufficiently smooth functions; $H_i(x,0) = H_i(x)$ and $H_i(x)$ is the Hamiltonian function which has a local minimum at the equilibrium $x_e^{(i)}$, $i = 1, 2$, $x_e^{(1)} = x_0$, $x_e^{(2)} = \xi_0$.

Assumption 1 $H_i\left(x^{(i)}\right) \in C^2$ *and the Hessian matrix* $Hess\left(H_i\left(x_0^{(i)}\right)\right) > 0$ *for system* (1) *and system* (2).

Given a disturbance attenuation level $\gamma > 0$, choose

$$z = \Lambda\left(g_1^T(x)\frac{\partial H_1(x)}{\partial x} + g_2^T(\xi)\frac{\partial H_2(\xi)}{\partial \xi}\right) \tag{3}$$

as the penalty function, where $\Lambda \in \Re^{s \times m}$ is a weighting matrix with full column rank and satisfying $\lambda\left(\Lambda^T \Lambda\right) \leqslant 1$, where $\lambda(\cdot)$ denotes the eigenvalue of a matrix. Then, our objective of this section is described as follows:

ARSS control: Design an L_2 feedback controller $u = \alpha(x, \xi)$, $(\alpha(x_0, \xi_0) = 0)$, such that

R1: The L_2 gain (from ω to z) of the closed-loop system is less than γ.

R2: System (1) and system (2) are simultaneously asymptotically stable when ω vanishes.

hold simultaneously.

Then, a lemma and a definition required in next section are proposed.

Lemma 1 *Consider a nonlinear system* [19]*:*

$$
\begin{cases}
\dot{x} = f(x) + g(x)\,\omega = T(x)\dfrac{\partial H}{\partial x} + g(x)\ \omega, \quad f(x_0) = 0 \\
y = h_1(x) \\
z = h_2(x)
\end{cases}
\tag{4}
$$

where $x \in \Re^n$ is the state vector, $\omega \in \Re^s$ is the disturbances, $z \in \Re^q$ is the penalty and $H(x)$ has a local minimum at the equilibrium x_0. If there exists the function $V(x) \geqslant 0$, $(V(x_0) = 0)$ such that HJI inequality

$$
\left(\frac{\partial V}{\partial x}\right)^T f(x) + \frac{1}{2\gamma^2}\left(\frac{\partial V}{\partial x}\right)^T g(x)g(x)^T\left(\frac{\partial V}{\partial x}\right) + \frac{1}{2}h(x)^T h(x) \leqslant 0
\tag{5}
$$

holds, it's implied that the L_2 gain of the closed-loop system (2) *(from ω to z) is bounded by γ, i.e.,*

$$
\int\limits_0^T \|z\|^2 dt \leqslant \int\limits_0^T \gamma^2 \|\omega\|^2 dt, \quad \forall \omega \in L_2[0, T]
\tag{6}
$$

where γ is a positive number.

Definition 1 [20] *System* (4) *is called zero-energy-gradient (ZEG) observable with respect to y if $y(t) = 0$ and $\omega(t) = 0$, $\forall t \geqslant 0$, implies $\nabla H(x(t)) = 0$, $\forall t \geqslant 0$; system* (4) *is called ZEG detectable with respect to y if $y(t) = 0$ and $\omega(t) = 0$, $\forall t \geqslant 0$, implies $\lim_{t \to \infty} \nabla H(x(t)) = 0$; system* (4) *is called generalized ZEG observable (detectable) if $y(t) = 0$, $z(t) = 0$, $\omega(t) = 0$, $\forall t \geqslant 0$, implies $\nabla H(x(t)) = 0$, $\forall t \geqslant 0$, $(\lim_{t \to \infty} \nabla H(x(t)) = 0)$.*

3. Main results

In this section, firstly, an adaptive H_∞ controller with tuning parameters for system (1) and system (2) is proposed. Then, we present an algorithm for solving tuning parameters.

Notation 1: $\nabla H_1 = \frac{\partial H_1(x)}{\partial x}$, $\nabla H_2 = \frac{\partial H_2(\xi)}{\partial \xi}$, $g_1 = g_1(x)$, $\bar{g}_1 = \bar{g}_1(x)$, $g_2 = g_2(\xi)$, $\bar{g}_2 = \bar{g}_2(\xi)$.

3.1. ARSS control of two DHSs

Assumption 2 *There exits an $n_1 \times m$ matrix Ψ such that*

$$[J_i (x_i, p_i) - R_i (x_i, p_i)] \Delta_{H_i} (x_i, p_i) = g_i \Psi^T \theta \tag{7}$$

where $\Delta_{H_i} (x_i, p_i) = \frac{\partial H_i}{\partial x_i} (x_i, p_i) - \frac{\partial H_i}{\partial x_i} (x_i)$ and $\theta \in \Re^{n_1}$ is a constant unknown parameter vector depending on p_i, $i = 1, 2$, $x = x_1$, $x_2 = \xi$.

Assumption 2 is a common assumption in the adaptive control of Hamiltonian systems. In most cases, we can find and such that (7) holds [20]. For the above ARSS control problem, we have the following result.

Theorem 1 *Considering system* (1) *and system* (2)*, with the penalty function* (3) *and the given disturbance attenuation level $\gamma > 0$, assume that system* (1) *and system* (2) *are generalized ZEG detectable (when $u = 0$). If*

- *i) There exists a symmetric matrix $K \in \Re^{m \times m}$, satisfying*

$$K \left(\Lambda^T \Lambda + \frac{1}{\gamma^2} I_m \right) = \left(\Lambda^T \Lambda + \frac{1}{\gamma^2} I_m \right) K$$

  such that

$$\begin{cases} \widetilde{R}_1 (x, p_1) = R_1 (x, p_1) + K_{11} (x, x) - \frac{1}{2\gamma^2} \bar{g}_1 \bar{g}_1^T - \frac{1}{2} g_1 \Lambda^T \Lambda g_1^T \geqslant 0 \\ \widetilde{R}_2 (\xi, p_2) = R_2 (\xi, p_2) - K_{22} (\xi, \xi) - \frac{1}{2\gamma^2} \bar{g}_2 \bar{g}_2^T - \frac{1}{2} g_2 \Lambda^T \Lambda g_2^T \geqslant 0 \end{cases} \tag{8}$$

  where $K_{ij} (x, \xi) = \frac{1}{2} g_i (x) K \left(\Lambda^T \Lambda + \frac{1}{\gamma^2} I_m \right) g_j^T (\xi)$, $i, j = 1, 2$;

- *ii)*

$$g_1 g_2^T = 0 \;\; and \;\; \bar{g}_1 \bar{g}_2^T = 0; \tag{9}$$

- *iii)*

$$\left[\nabla H_1^T g_1 + \nabla H_2^T g_2 \right] \Phi (x, \xi, \varphi) \leqslant 0; \tag{10}$$

hold simultaneously. Then

$$\begin{cases} u = \frac{1}{2} K \left(\Lambda^T \Lambda + \frac{1}{\gamma^2} I_m \right) \left(-g_1^T \nabla H_1 + g_2^T \nabla H_2 \right) + \Phi (x, \xi, \varphi) - \Psi^T \hat{\theta} \\ \dot{\hat{\theta}} = Q \Psi \left(g_1^T \nabla H_1 + g_2^T \nabla H_2 \right) \end{cases} \tag{11}$$

is an adaptive L_2 disturbance attenuation controller, such that both **R1** *and* **R2** *hold simultaneously for system* (1) *and system* (2)*, where $\Phi (x, \xi, \varphi)$ is a polynomial matrix with a set of tuning parameters, φ is a set of tuning parameters and I_m is an $m \times m$ unit matrix.*

Proof Substituting controller (11) into system (1) and system (2), and Assumption 2 holds, we obtain the following closed-loop systems:

$$
\begin{cases}
\dot{x} = [J_1(x,p_1) - R_1(x,p_1)] \dfrac{\partial H_1(x,p_1)}{\partial x} \\
\quad + g_1 \left[\dfrac{1}{2} K \left(\Lambda^T \Lambda + \dfrac{1}{\gamma^2} I_m \right) \left(-g_1^T \nabla H_1 + g_2^T \nabla H_2 \right) + \Phi(x,\xi,\varphi) \right] - g_1 \Psi^T (\theta - \hat{\theta}) + \bar{g}_1 \omega \\
\dot{\xi} = [J_2(\xi,p_2) - R_2(\xi,p_2)] \dfrac{\partial H_2(\xi,p_2)}{\partial \xi} \\
\quad + g_2 \left[\dfrac{1}{2} K \left(\Lambda^T \Lambda + \dfrac{1}{\gamma^2} I_m \right) \left(-g_1^T \nabla H_1 + g_2^T \nabla H_2 \right) + \Phi(x,\xi,\varphi) \right] - g_2 \Psi^T (\theta - \hat{\theta}) + \bar{g}_2 \omega \\
\dot{\hat{\theta}} = Q \Psi \left(g_1^T \nabla H_1 + g_2^T \nabla H_2 \right)
\end{cases}
\tag{12}
$$

The system (12) can rewrite as:

$$
\begin{bmatrix} \dot{x} \\ \dot{\xi} \\ \dot{\hat{\theta}} \end{bmatrix} =
\begin{bmatrix}
J_1(x,p_1) - R_1(x,p_1) - K_{11}(x,x) & K_{12}(x,\xi) & -g_1 \Psi^T Q \\
-K_{21}(\xi,x) & J_2(\xi,p_2) - R_2(\xi,p_2) + K_{22}(\xi,\xi) & -g_2 \Psi^T Q \\
\left(g_1 \Psi^T Q \right)^T & \left(g_2 \Psi^T Q \right)^T & 0
\end{bmatrix}
$$
$$
\times \begin{bmatrix} \dfrac{\partial H_1(x,p_1)}{\partial x} \\ \dfrac{\partial H_2(\xi,p_2)}{\partial x} \\ -Q^{-1}(\theta - \hat{\theta}) \end{bmatrix} + \begin{bmatrix} g_1(x) \\ g_2(\xi) \\ 0 \end{bmatrix} \Phi(x,\xi,\hat{\theta}) + \begin{bmatrix} \bar{g}_1(x) \\ \bar{g}_2(\xi) \\ 0 \end{bmatrix} \omega
\tag{13}
$$

Let $X = \left[x^T, \xi^T, \hat{\theta}^T \right]^T$, $Y = \left[y^T, \eta^T \right]^T$,

$$
\bar{J}(X,p) = \begin{bmatrix}
J_1(x,p_1) & K_{12}(x,\xi) & -g_1 \Psi^T Q \\
-K_{21}(\xi,x) & J_2(\xi,p_2) & -g_2 \Psi^T Q \\
\left(g_1 \Psi^T Q \right)^T & \left(g_2 \Psi^T Q \right)^T & 0
\end{bmatrix},
$$

$$
\bar{R}(X,p) = \begin{bmatrix}
R_1(x,p_1) + K_{11}(x,x) & 0 & 0 \\
0 & R_2(\xi,p_2) - K_{22}(\xi,\xi) & 0 \\
0 & 0 & 0
\end{bmatrix}, \; G(X) = \begin{bmatrix} g_1(x) \\ g_2(\xi) \\ 0 \end{bmatrix},
$$

$$
\bar{G}(X) = \begin{bmatrix} \bar{g}_1(x) \\ \bar{g}_2(\xi) \\ 0 \end{bmatrix}, \bar{H}(X) = H_1(x) + H_2(\xi) + \tfrac{1}{2}(\theta - \hat{\theta})^T Q^{-1}(\theta - \hat{\theta}), \text{ where}
$$

$$
\frac{\partial \bar{H}(X)}{\partial X} = \begin{bmatrix} \dfrac{\partial \bar{H}(X)}{\partial x} \\ \dfrac{\partial \bar{H}(X)}{\partial \xi} \\ \dfrac{\partial \bar{H}(X)}{\partial \hat{\theta}} \end{bmatrix} = \begin{bmatrix} \nabla H_1 \\ \nabla H_2 \\ \nabla H_3 \end{bmatrix}, \quad p = \left[p_1^T, p_2^T \right]^T, \quad v = \Phi(x,\xi,\varphi).
$$

So, the systems (13) can be rewritten as:

$$\begin{cases} \dot{X} = [\bar{J}(X,p) - \bar{R}(X,p)] \dfrac{\partial \bar{H}(X)}{\partial X} + G(X)v + \bar{G}(X)\omega = f(X) + \bar{G}(X)\omega \\ Y = M(X) \dfrac{\partial \bar{H}(X)}{\partial X}. \end{cases} \tag{14}$$

Obviously, $\bar{J}(X,p)$ is a skew-symmetric matrix and $\bar{R}(X,p)$ is a positive semi-definite matrix. Thus, the system (14) is a dissipative Hamiltonian system (DHS).

The output of the system (1) and system (2) can be rewritten as:

$$Y = M(X) \frac{\partial \bar{H}(X)}{\partial X} \tag{15}$$

where $M(X) = \begin{bmatrix} g_1^T(x) & 0 & 0 \\ 0 & g_2^T(\xi) & 0 \\ 0 & 0 & 0 \end{bmatrix}.$

The penalty function can be expressed as:

$$z = \Lambda G^T(X) \frac{\partial \bar{H}(X)}{\partial X} := h(X). \tag{16}$$

Consider the candidate Lyapunov function $V(X) = \bar{H}(X) - c \geqslant 0$, where $c = \bar{H}(X_0)$. With Lemma 1 and the conditions of the Theorem 1, we have

$$\left(\frac{\partial V}{\partial X} \right)^T f(X) + \frac{1}{2\gamma^2} \left(\frac{\partial V}{\partial X} \right)^T \bar{G}(X)\bar{G}^T(X) \left(\frac{\partial V}{\partial X} \right) + \frac{1}{2} h^T(X) h(X)$$

$$= -\left(\frac{\partial \bar{H}}{\partial X} \right)^T \bar{R}(X,p) \frac{\partial \bar{H}}{\partial X} + \left(\frac{\partial \bar{H}}{\partial X} \right)^T G(X)v + \frac{1}{2\gamma^2} \left(\frac{\partial \bar{H}}{\partial X} \right)^T \bar{G}(X)\bar{G}^T(X) \left(\frac{\partial \bar{H}}{\partial X} \right)$$

$$+ \frac{1}{2} h^T(X) h(X)$$

$$= -\begin{bmatrix} \nabla H_1^T & \nabla H_2^T & \nabla H_3^T \end{bmatrix} \begin{bmatrix} R_1(x,p_1) + K_{11}(x,x) & 0 & 0 \\ 0 & R_2(\xi,p_2) - K_{22}(\xi,\xi) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \nabla H_1 \\ \nabla H_2 \\ \nabla H_3 \end{bmatrix}$$

$$+ \begin{bmatrix} \nabla H_1^T & \nabla H_2^T & \nabla H_3^T \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ 0 \end{bmatrix} \Phi(x,\xi,\varphi)$$

$$
+ \frac{1}{2\gamma^2} \left[\begin{array}{ccc} \nabla H_1^T & \nabla H_2^T & \nabla H_3^T \end{array} \right] \left[\begin{array}{c} \bar{g}_1 \\ \bar{g}_2 \\ 0 \end{array} \right] \left[\begin{array}{ccc} \bar{g}_1^T & \bar{g}_2^T & 0 \end{array} \right] \left[\begin{array}{c} \nabla H_1 \\ \nabla H_2 \\ \nabla H_3 \end{array} \right]
$$

$$
+ \frac{1}{2} \left[\begin{array}{ccc} \nabla H_1^T & \nabla H_2^T & \nabla H_3^T \end{array} \right] \left[\begin{array}{c} g_1 \\ g_2 \\ 0 \end{array} \right] \Lambda^T \Lambda \left[\begin{array}{ccc} g_1^T & g_2^T & 0 \end{array} \right] \left[\begin{array}{c} \nabla H_1 \\ \nabla H_2 \\ \nabla H_3 \end{array} \right]
$$

$$
= - \left(\nabla H_1^T (R_1 + K_{11}) \nabla H_1 + \nabla H_2^T (R_2 - K_{22}) \nabla H_2 \right) + \left(\nabla H_1^T g_1 + \nabla H_2^T g_2 \right) \Phi(x, \xi, \varphi)
$$

$$
+ \frac{1}{2\gamma^2} \left(\nabla H_1^T \bar{g}_1 \bar{g}_1^T \nabla H_1 + \nabla H_2^T \bar{g}_2 \bar{g}_1^T \nabla H_1 + \nabla H_1^T \bar{g}_1 \bar{g}_2^T \nabla H_2 + \nabla H_2^T \bar{g}_2 \bar{g}_2^T \nabla H_2 \right)
$$

$$
+ \frac{1}{2} \left(\nabla H_1^T g_1 \Lambda^T \Lambda g_1^T \nabla H_1 + \nabla H_2^T g_2 \Lambda^T \Lambda g_1^T \nabla H_1 \right.
$$

$$
\left. + \nabla H_1^T g_1 \Lambda^T \Lambda g_2^T \nabla H_2 + \nabla H_2^T g_2 \Lambda^T \Lambda g_2^T \nabla H_2 \right)
$$

$$
= - \nabla H_1^T \left(R_1 + K_{11} - \frac{1}{2\gamma^2} \bar{g}_1 \bar{g}_1^T - \frac{1}{2} g_1 \Lambda^T \Lambda g_1^T \right) \nabla H_1
$$

$$
- \nabla H_2^T \left(R_2 - K_{22} - \frac{1}{2\gamma^2} \bar{g}_2 \bar{g}_2^T - \frac{1}{2} g_2 \Lambda^T \Lambda g_2^T \right) \nabla H_2
$$

$$
+ \left(\nabla H_1^T g_1 + \nabla H_2^T g_2 \right) \Phi(x, \xi, \varphi) \leqslant 0
$$

$$
(17)
$$

According to Lemma 1, the L_2 gain of system (14) (from ω to z) is no more than γ and **R1** holds.

Next, we prove that system (14) is asymptotically stable when $\omega = 0$. When $\omega = 0$, it is easy to know from system (14) that

$$
\dot{V}(X) = \left(\frac{\partial V}{\partial X} \right)^T [\bar{J}(X, p) - \bar{R}(X, p)] \left(\frac{\partial V}{\partial X} \right) + \left(\frac{\partial V}{\partial X} \right)^T G(X) v
$$

$$
= - \left(\frac{\partial \bar{H}(X)}{\partial X} \right)^T \bar{R}(X, p) \left(\frac{\partial \bar{H}(X)}{\partial X} \right) + \left(\frac{\partial \bar{H}(X)}{\partial X} \right)^T G(X) v
$$

$$
= - \left[\begin{array}{ccc} \nabla H_1^T & \nabla H_2^T & \nabla H_3^T \end{array} \right] \left[\begin{array}{ccc} R_1(x, p_1) + K_{11}(x, x) & 0 & 0 \\ 0 & R_2(\xi, p_2) - K_{22}(\xi, \xi) & 0 \\ 0 & 0 & 0 \end{array} \right] \left[\begin{array}{c} \nabla H_1 \\ \nabla H_2 \\ \nabla H_3 \end{array} \right]
$$

$$
+ \left[\begin{array}{ccc} \nabla H_1^T & \nabla H_2^T & \nabla H_3^T \end{array} \right] \left[\begin{array}{c} g_1 \\ g_2 \\ 0 \end{array} \right] \Phi(x, \xi, \varphi)
$$

$$
\begin{aligned}
= & -\nabla H_1^T \left(R_1 + K_{11} - \frac{1}{2\gamma^2} \bar{g}_1 \bar{g}_1^T - \frac{1}{2} g_1 \Lambda^T \Lambda g_1^T \right) \nabla H_1 - \frac{1}{2\gamma^2} \nabla H_1^T \bar{g}_1 \bar{g}_1^T \nabla H_1 \\
& - \frac{1}{2} \nabla H_1^T g_1 \Lambda^T \Lambda g_1^T \nabla H_1 - \nabla H_2^T \left(R_2 - K_{22} - \frac{1}{2\gamma^2} \bar{g}_2 \bar{g}_2^T - \frac{1}{2} g_2 \Lambda^T \Lambda g_2^T \right) \nabla H_2 \\
& - \frac{1}{2\gamma^2} \nabla H_2^T \bar{g}_2 \bar{g}_2^T \nabla H_2 - \frac{1}{2} \nabla H_2^T g_2 \Lambda^T \Lambda g_2^T \nabla H_2 \\
& + \left(\nabla H_1^T g_1 + \nabla H_2^T g_2 \right) \Phi(x, \xi, \varphi) \leqslant 0
\end{aligned}
\tag{18}
$$

Thus, the solution of the closed-loop system converges to the largest invariant set contained in

$$
S = \left\{ X : \dot{V}(X) = 0 \right\} \subset \left\{ X : Y = M(X) \nabla \bar{H} \equiv 0, z = \Lambda G^T(X) \nabla \bar{H} \equiv 0, \forall t \geqslant 0 \right\}
$$

From the fact that system (14) is generalized ZEG detectable, we know that

$$
\lim_{t \to \infty} \nabla \bar{H}(X(t)) = \lim_{t \to \infty} \begin{pmatrix} \nabla H(x(t)) \\ \nabla H(\xi(t)) \\ -Q^{-1}(\theta - \hat{\theta}(t)) \end{pmatrix} = \begin{pmatrix} \nabla H(\lim_{t \to \infty} x(t)) \\ \nabla H(\lim_{t \to \infty} \xi(t)) \\ \lim_{t \to \infty} \left[-Q^{-1}(\theta - \hat{\theta}(t)) \right] \end{pmatrix} = 0,
\tag{19}
$$

$$
\forall X \in S
$$

Hence, the largest invariant set contains only one point, i.e., $X_0 = \left[x_0^T, \xi_0^T, \hat{\theta}^T \right]^T$ which is the equilibrium point. From LaSalle's invariance principle, the closed-loop system (14) and controller (11) is asymptotically stable at its equilibrium and R2 holds. This completes the proof. □

Remark 1

1. In Theorem, when all of tuning parameters φ in controller (11) are zero (i.e., $\Phi(x, \xi, \varphi) = 0$), the controller is same to the controller proposed [7], which is a controller without parameter. However, the controller (11) obtained in this paper is a parameterized controller, which has a set of tuning parameters.

2. The condition (8) and (9) in Theorem are not restrictive, and can be easily satisfied in many systems.

3. $\Phi(x, \xi, \varphi)$ is a polynomial vector with tuning parameters. We can obtain the tuning parameters range of via solving condition (10).

4. The proposed parameterization method can be used for a nonlinear control system, and of course the first step in applying the method is to express the nonlinear system as a DHS based on dissipative Hamiltonian realization methods [21, 22].

3.2.   Solving of tuning parameters (STP) algorithm

From condition (8), we can obtain the γ^*. Let $\gamma \geqslant \gamma^*$ such that condition (8) holds. Then we propose an algorithm to find tuning parameters ranges of controller (11) via solving tuning parameters of $\Phi(x,\xi,\varphi)$ in condition (10). The STP algorithm now proceeds as follows.

STP Algorithm

INPUT: A set parameters of system $Sys = \{x_{i_N}, N_1, N_2, r, \nabla H_1, \nabla H_2, G_1, G_2\}$. $//x_{i_N}$ is the state variable of two systems, N_1 is the number of the state variable of system one, N_2 is the number of state variable of system two and r is the highest order polynomial for tuning parameters.

OUTPUT: A set of Tuning parameters' ranges U. $//U$ is empty set initially.

Step 1: Initialization

For $i = 1$ to l $//l = N_1$

For $k = 1$ to r

$$\{t = c(l+k-1,k); k = k_1 + k_2 + \cdots + k_l; P_{il} = \sum_{j=1}^{t} \nu_{jl} x_{i1}^{k_1} x_{i2}^{k_2} \cdots x_{il}^{k_l};$$

$$\Phi_a = \Phi_a + P_{il}; \}$$

$//c(l+k-1,k)$ is the combinatorial function.

For $i = 1$ to l $//l = N_2$

For $k = 1$ to r

$$\{t = c(l+k-1,k); k = k_1 + k_2 + \cdots + k_l; P_{il} = \sum_{j=1}^{t} \nu_{jl} x_{i1}^{k_1} x_{i2}^{k_2} \cdots x_{il}^{k_l};$$

$$\Phi_b = \Phi_b + P_{il}; \}$$

$$\Phi(X_1, X_2, \nu) = [\Phi_a, \Phi_b]^T$$

Step 2: Main body

Begin

$$S = - \left[\nabla H_1^T G_1 + \nabla H_2^T G_2\right] \Phi(X_1, X_2, \nu);$$

For $j = 1$ to $nops(S)$; $//nops(S)$ is the number of terms of polynomial S.

   If $(deg(S)$ is odd number$)$; $//deg(S)$ is degree of polynomial S.

      $\Theta_1 = \{coef(S, deg(S) = odd) = 0\}$;   //Choose some terms, which degree is

      odd number $(deg(S) = odd)$ from S and let the coefficients of such terms be

      zero. We can obtain a set of equations Θ_1.

   End If

$U_1 = \{CylindricalAlgebraicDecompose(\Theta_1)\}$;   //Solve the Θ_1 by using cylindri-

cal algebraic decompositions (CAD) algorithm [23] and obtain the solution set U_1

of the tuning parameters.

$S' = subs(U_1, S)$; // Substitute solution set U_1 into S and obtain a new polynomial S'.

$M = Matrix(S')$;   // Rewrite S' as coefficient matrix M.

$\Theta_2 = \{Determinant(M) \geqslant 0\}$;   //All principal minors of M must be positive

semi-definite [24] and we can obtain a set of inequalities Θ_2.

$U_2 = \{CylindricalAlgebraicDecompose(\Theta_2)\}$;   // Solve the Θ_2 by using CAD and

choose one solution set of tuning parameters ranges U_2.

$U = U_1 \cup U_2$ and Return (U);

End. This completes the algorithm.

Remark 2

1. The STP algorithm starts from $r = 1$ normally.

2. The CAD algorithm is given in Semi-Algebraic-Set-Tools of Regular-Chains in Maple 16.

3. It is merely to simplify computation that we let some tuning parameters be zero before using CAD algorithm. However, these tuning parameters are not necessarily zero. The solution set of tuning parameters obtained by STP algorithm is a subset of solutions.

4.   Numerical experiments

Consider the following two DHSs with external disturbances and parametric uncertainties described as:

$$\begin{cases} \dot{x} = [J_1(x, p_1) - R_1(x, p_1)]\dfrac{\partial H_1(x, p_1)}{\partial x} + g_1(x)u + \bar{g}_1(x)\omega \\ y = g_1{}^T(x)\dfrac{\partial H_1(x)}{\partial x} \end{cases} \tag{20}$$

where

$$J_1(x, p_1) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -2 \\ 0 & 2 & 0 \end{bmatrix}, \quad g_1 = \begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}, \quad \bar{g}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$R_1(x, p_1) = Diag\{1 + p_1, 1, 2\}, \quad H_1(x, p_1) = \frac{1}{2}x_1{}^2 + x_2{}^2 + \frac{1}{2}(1 + p_1)x_3{}^2, \quad |p_1| < 1.$$

$$\begin{cases} \dot{\xi} = [J_2(\xi, p_2) - R_2(\xi, p_2)]\dfrac{\partial H_2(\xi, p_2)}{\partial \xi} + g_2(\xi)u + \bar{g}_2(\xi)\omega \\ \eta = \bar{g}_2^T(\xi)\dfrac{\partial H_2(\xi)}{\partial \xi} \end{cases} \tag{21}$$

where

$$J_2(\xi, p_2) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad g_2 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 0 \end{bmatrix}, \quad \bar{g}_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$R_2(\xi, p_2) = Diag\{1, 2, 1 + p_2\}, \quad H_2(\xi, p_2) = (1 + p_2)\xi_1{}^2 + 2\xi_2{}^2 + \xi_3{}^2, \quad |p_2| < 1$$

4.1.   Controller design and solving tuning parameters

From system (20) and system (21), it is easy to get

$$\text{Hess}\,(H_1\,(x_0)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1+p_1 \end{bmatrix} > 0, \quad \text{Hess}\,(H_2\,(\xi_0)) = \begin{bmatrix} 2+2p_2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} > 0.$$

So Assumption 1 holds.

Given a disturbance attenuation level $\gamma > 0$, choose

$$z = \Lambda\left(g_1^T \nabla H_1 + g_2^T \nabla H_2\right). \tag{22}$$

Suppose $K = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\Lambda = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$. We have $K^T K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\Lambda^T \Lambda =$

$\begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$, $K\left(\Lambda^T\Lambda + \frac{1}{\gamma^2}I_m\right) = \left(\Lambda^T\Lambda + \frac{1}{\gamma^2}I_m\right)K = \begin{bmatrix} \frac{1}{4}+\frac{1}{\gamma^2} & 0 \\ 0 & -\frac{1}{4}-\frac{1}{\gamma^2} \end{bmatrix}$. A straightfor-

ward computation shows that when $\gamma \geqslant \frac{2\sqrt{15}}{5}$,

$$\widetilde{R}_1\,(x) = \begin{bmatrix} 1+p_1 & 0 & 0 \\ 0 & \frac{3}{4}-\frac{1}{\gamma^2} & -\frac{1}{4} \\ 0 & -\frac{1}{4} & \frac{7}{4} \end{bmatrix} \geqslant 0, \quad \widetilde{R}_2\,(\xi) = \begin{bmatrix} \frac{3}{4}-\frac{1}{\gamma^2} & \frac{1}{4}+\frac{1}{\gamma^2} & 0 \\ \frac{1}{4}+\frac{1}{\gamma^2} & \frac{7}{4}-\frac{1}{\gamma^2} & 0 \\ 0 & 0 & 1+p_2 \end{bmatrix} \geqslant 0.$$

Thus, the condition (8) holds. From system (20) and system (21),

$$g_1 g_2^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{g}_1 \bar{g}_2^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The condition (9) holds.

Let $\Phi\,(x,\xi,\varphi) = \begin{bmatrix} \Phi_1\,(x,\varphi_1) \\ \Phi_2\,(\xi,\varphi_2) \end{bmatrix}$. We know that $n = 6$ in system (20) and system (21).
Let $r = 1$, We have $\Phi_1\,(x,\varphi_1) = a_1 x_1 + a_2 x_2 + a_3 x_3$, $\Phi_2\,(\xi,\varphi_2) = b_1 \xi_1 + b_2 \xi_2 + b_3 \xi_3$,
where φ_1 is the tuning parameters a_i and φ_2 is the tuning parameters b_i, $i = 1,2,3$.

From system (20) and system (21), we obtain that

$$\left[\nabla H_1^T g_1 + \nabla H_2^T g_2\right] = \begin{bmatrix} 2x_2 + x_3 + 2\xi_1 - 4\xi_2 & -2x_2 - x_3 + 2\xi_1 - 4\xi_2 \end{bmatrix}.$$

Let $S = -\left[\nabla H_1^T g_1 + \nabla H_2^T g_2\right]\Phi\,(x,\xi)$, we have

$$S = -2a_1 x_1 x_2 - a_1 x_1 x_3 - 2a_1 x_1 \xi_1 + 4a_1 x_1 \xi_2 - 2a_2 x_2^2 - (a_2 + 2a_3)x_2 x_3 - (2a_2 - 2b_1)x_2 \xi_1$$

$$+ (4a_2 + 2b_2)x_2 \xi_2 + 2b_3 x_2 \xi_3 - a_3 x_3^2 - (2a_3 - b_1)x_3 \xi_1 + (4a_3 + b_2)x_3 \xi_2 + b_3 x_3 \xi_3$$

$$- 2b_1 \xi_1^2 + (4b_1 - 2b_2)\xi_1 \xi_2 - 2b_3 \xi_1 \xi_3 + 4b_2 \xi_2^2 + 4b_3 \xi_2 \xi_3.$$

S is a quadratic form and can be rewritten as a coefficient matrix (multiply constant 2 for simplified calculation).

$$M = \begin{bmatrix} 0 & -2a_1 & -a_1 & -2a_1 & 4a_1 & 0 \\ -2a_1 & -4a_2 & -2a_3-a_2 & -2a_2+2b_1 & 4a_2+2b_2 & 2b_3 \\ -a_1 & -2a_3-a_2 & -2a_3 & -2a_3+b_1 & 4a_3+b_2 & b_3 \\ -2a_1 & -2a_2+2b_1 & -2a_3+b_1 & -4b_1 & 4b_1-2b_2 & -2b_3 \\ 4a_1 & 4a_2+2b_2 & 4a_3+b_2 & 4b_1-2b_2 & 8b_2 & 4b_3 \\ 0 & 2b_3 & b_3 & -2b_3 & 4b_3 & 0 \end{bmatrix}$$

All the principal minors of M must be positive semi-definite. We have inequalities B from M. From B, we can easily obtain that $a_3 \leqslant 0$, $b_1 \leqslant 0$.

Substituting $U_1 = \{a_1 = 0, b_3 = 0, a_2 = 2a_3, b_2 = -2b_1\}$ into inequalities B for simplify computation, we obtain simplified inequalities B'. Solving inequalities B' by using CAD algorithm, we obtain a series of sets. Choose some sets, which satisfy inequalities B', and organize them. We have

$$[U = \left\{a_3 \leqslant 0, b_1 \leqslant 0, 12a_3b_1 - 4a_3^2 - b_1^2 \geqslant 0\right\} \cup U_1. \tag{23}$$

From system (20) and system (21), we have

$$\left[-g_1^T \nabla H_1 + g_2^T \nabla H_2\right] = \left[\begin{array}{cc} -2x_2 - x_3 + 2\xi_1 - 4\xi_2 & 2x_2 + x_3 + 2\xi_1 - 4\xi_2 \end{array}\right]^T,$$

$$\left[g_1^T \nabla H_1 + g_2^T \nabla H_2\right] = \left[\begin{array}{cc} 2x_2 + x_3 + 2\xi_1 - 4\xi_2 & -2x_2 - x_3 + 2\xi_1 - 4\xi_2 \end{array}\right]^T.$$

Substituting them and U into controller (11), we obtain the controller:

$$\begin{cases} u = \left[\begin{array}{c} \left(\frac{1}{8} + \frac{1}{2\gamma^2}\right)(-2x_2 - x_3 + 2\xi_1 - 4\xi_2) \\ \left(-\frac{1}{8} - \frac{1}{2\gamma^2}\right)(2x_2 + x_3 + 2\xi_1 - 4\xi_2) \end{array}\right] + \left[\begin{array}{c} 2a_3x_1 + a_3x_3 \\ b_1\xi_1 - 2b_1\xi_2 \end{array}\right] - \Psi^T \hat{\theta} \\ \dot{\hat{\theta}} = Q\Psi \left[\begin{array}{c} 2x_2 + x_3 + 2\xi_1 - 4\xi_2 \\ -2x_2 - x_3 + 2\xi_1 - 4\xi_2 \end{array}\right] \end{cases} \tag{24}$$

where $a_3 \leqslant 0, b_1 \leqslant 0, 12a_3b_1 - 4a_3^2 - b_1^2 \geqslant 0$.

We have the controller with tuning parameters for system (20) and system (21). The controller (24) has a rather simple form.

4.2.   Simulations and results

In order to evaluate the robustness of the controller (24), we set the disturbance attenuation level of system (20) and system (21) as: $\gamma = \sqrt{3}$, and the tuning parameters

of controller as: $a_3 = -1$, $b_1 = -1$. Choose

$$\Psi = \begin{bmatrix} -\xi_1 & 0 & -x_3 \\ -\xi_1 & 0 & x_3 \end{bmatrix}^T, \quad \theta = \begin{bmatrix} p_2 & 0 & p_1 \end{bmatrix}^T,$$

we have

$$[J_1(x, p_1) - R_1(x, p_1)] \Delta_H(x, p_1) = g_1 \Psi^T \theta = \begin{bmatrix} 0 & -2p_1 x_3 & -2p_1 x_3 \end{bmatrix}^T,$$

$$[J_2(\xi, p_2) - R_2(\xi, p_2)] \Delta_H(\xi, p_2) = g_2 \Psi^T \theta = \begin{bmatrix} -2p_2 \xi_1 & 2p_2 \xi_1 & 0 \end{bmatrix}^T.$$

The Assumption 2 holds for system (20) and system (21). Suppose $Q = Diag\{1, 1, 1\}$ and we have the controller (25), which has a rather simple form.

$$\begin{cases} u_1 = -\dfrac{31}{12}x_2 - \dfrac{31}{24}x_3 + \dfrac{7}{12}\xi_1 - \dfrac{7}{6}\xi_2 + \xi_1\hat{\theta}_1 + x_3\hat{\theta}_3 \\ u_2 = -\dfrac{7}{12}x_2 - \dfrac{7}{24}x_3 - \dfrac{19}{12}\xi_1 + \dfrac{19}{6}\xi_2 + \xi_1\hat{\theta}_1 - x_3\hat{\theta}_3 \\ \dot{\hat{\theta}}_1 = -4\xi_1^2 + 8\xi_1\xi_2 \\ \dot{\hat{\theta}}_3 = -4x_2x_3 - 2x_3^2 \end{cases} \tag{25}$$

To illustrate the effectiveness of controller (25), we carry out some numerical simulations with the following choices: $x(0) = [0.6, 0.2, 1]^T$, $\xi(0) = [0.5, 1, 0.2]^T$. To test the robustness of the controller with respect to external disturbances, a square disturbance $\omega = [2, 3]^T$ is added to systems in the time duration $[2 \sim 2.5s]$. The parameter perturbation $p = [0.5, 0.5]^T$. The simulation results are shown in Figures 1-4, which are the responses of the state and control signal, respectively.

From Figures 1-4, we know that the controller (25) is very effective in ARSS control for system (20) and system (21).

In order to evaluate the robustness optimization of the systems by adjusting the tuning parameters of controller (24), we choose the parameters of system (20) and system (21) as: $a_3 = -1$, $b_1 = -100$. We have the following controller:

$$\begin{cases} u_1 = -\dfrac{31}{12}x_2 - \dfrac{31}{24}x_3 + \dfrac{7}{12}\xi_1 - \dfrac{7}{6}\xi_2 + \xi_1\hat{\theta}_1 + x_3\hat{\theta}_3 \\ u_2 = -\dfrac{7}{12}x_2 - \dfrac{7}{24}x_3 - \dfrac{1207}{12}\xi_1 + \dfrac{1207}{6}\xi_2 + \xi_1\hat{\theta}_1 - x_3\hat{\theta}_3 \\ \dot{\hat{\theta}}_1 = -4\xi_1^2 + 8\xi_1\xi_2 \\ \dot{\hat{\theta}}_3 = -4x_2x_3 - 2x_3^2 \end{cases} \tag{26}$$

To illustrate the effectiveness of controller (26), the initial value of systems and parameter perturbation are same to above. We impose an external disturbance $\omega = [2, 3]^T$ on system (20) and system (21) during the time period 2 2.5s.
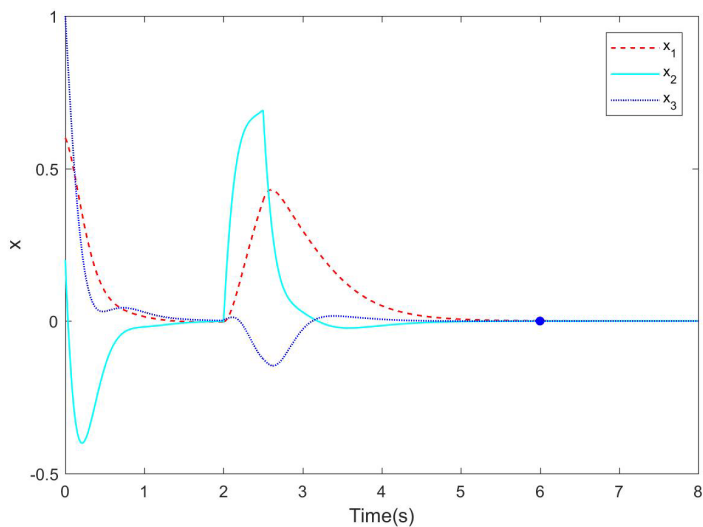
Figure 1: Swing curves of *x*
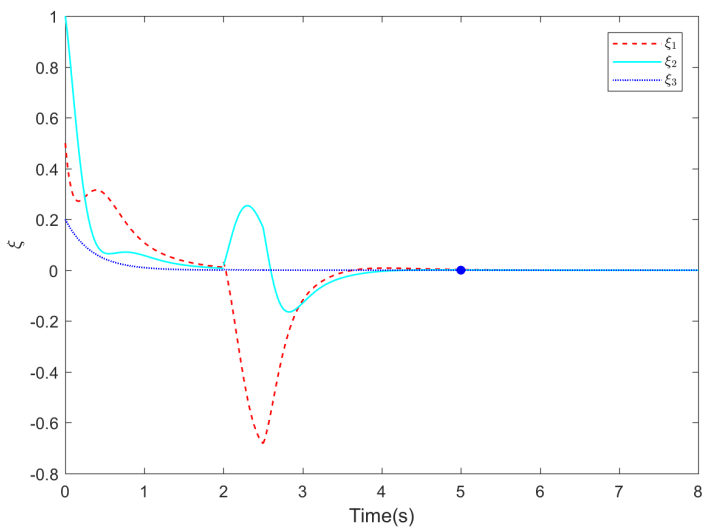
Figure 2: Swing curves of ξ

The simulation results are shown in Figure 5 and Figure 6, which are the response of the state of system (20) and system (21) under controller (26), respectively.

From Figure 5 and Figure 6, we know that the controller (26) is very effective in ARSS control for system (20) and system (21). From Figure 5, we can clearly see that
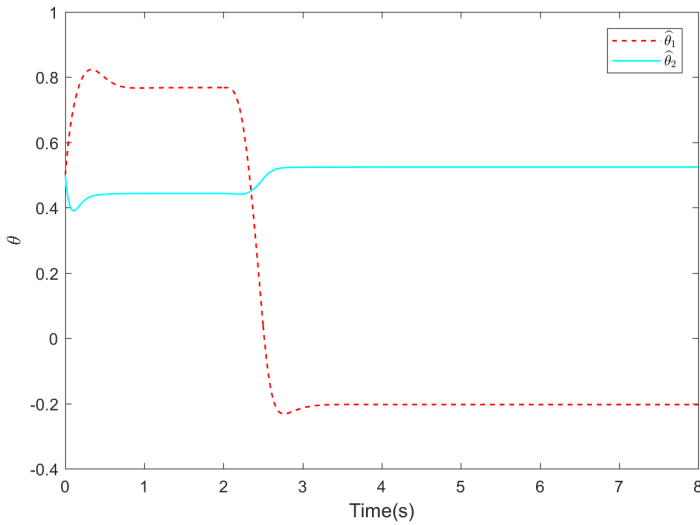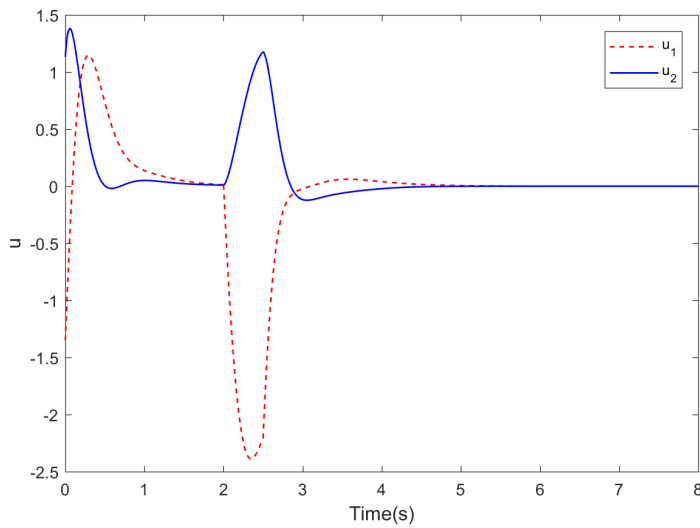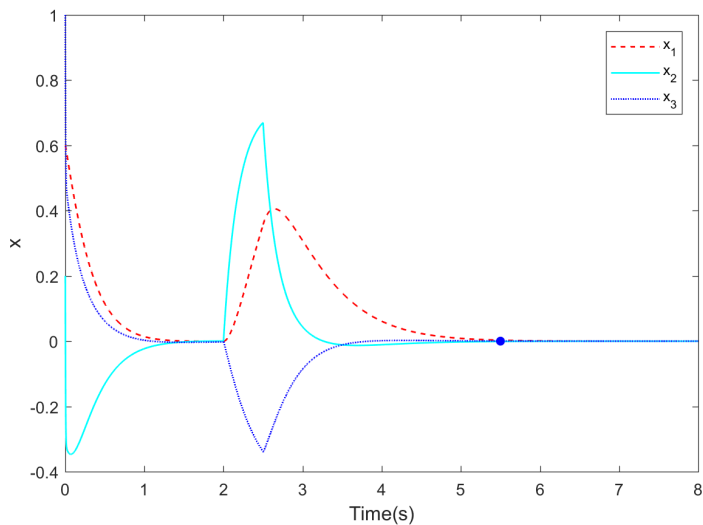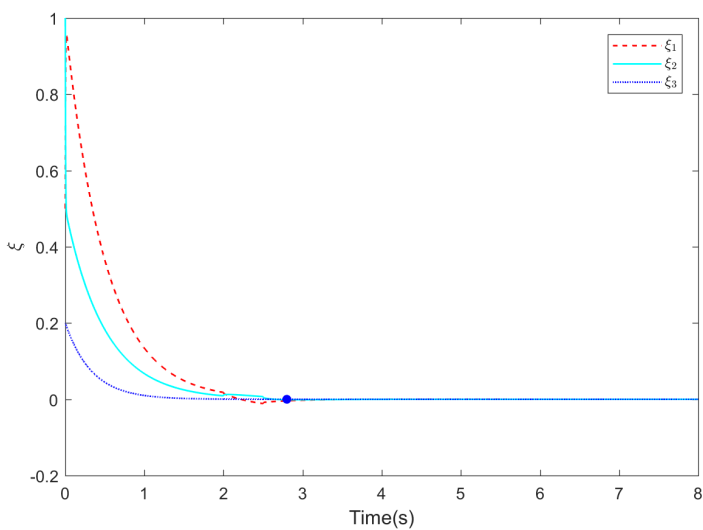
Figure 3: Swing curves of $\hat{\theta}$

Figure 4: Swing curves of u

under controller (26), it takes 5.5 seconds for the system to back to the equilibrium point (circle point in Figures), while under controller (25), it takes about 6.0 seconds in Figure 1. The controller (26) is a little better in robust performance for system (20) than controller (25). However, from Figure 6, we can clearly see that under controller

Figure 5: Swing curves of x

Figure 6: Swing curves of ξ

(26), it takes 2.8 seconds for the system to back to the equilibrium point, while under controller (25), it takes about 5.0 seconds in Figure 2. The time of state ξ back to the equilibrium point is much shorter in Figure 6 than in Figure 2. With the adjustment control parameters, the system achieved a more robust effect.

Simulation shows that the controller with tuning parameters is effective as it can optimize H_∞ control by adjusting the tuning parameters values. Within the tuning parameters ranges, the controllers insure that the dissipative systems are adaptive robust simultaneous stabilization control and can optimize the robustness for all or part of Hamiltonian systems and insure that the others complex control objectives by adjusting the tuning parameters values.

5.   Conclusions

In this paper, we have investigated the ARSS problem for two DHSs, and proposed an H_∞ ARSS controller with tuning parameters design method. A controller with tuning parameters has been obtained using Hamiltonian function method and an algorithm for solving tuning parameters of the controller has been proposed with symbolic computation. The proposed controller parameterization method avoids solving HJI equations and the obtained controller is easier as compared to some existing ones. The numerical experiment and simulations show that the ARSS controller is strongly robust in disturbances and unknown parameters, and meanwhile the ARSS controller can optimize the robustness of the system by adjusting tuning parameters.

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