NUKLEONIKA 2016;61(2):115–118
doi: 10.1515/nuka-2016-0020

ORIGINAL PAPER

DE GRUYTER
OPEN

Self-similar solution of laser-produced plasma expansion into vacuum with kappa-distributed electrons

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Abstract. The expansion of semi-infinite laser produced plasma into vacuum is analyzed with a hydrodynamic model for cold ions assuming electrons modeled by a kappa-type distribution. Self-similar analytic expressions for the potential, velocity, and density of the plasma have been derived. It is shown that nonthermal energetic electrons have the role of accelerating the self-similar expansion.

Key words: kappa distribution function • laser-produced plasmas • nonthermal electrons • plasma expansion • self-similar solution

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Received: 17 September 2015
Accepted: 9 November 2015

Introduction

Plasma expansion into vacuum is a basic physical problem with a variety of applications, ranging from space to laboratory plasmas [1–6]. Caused generally by electron pressure, it serves as an energy transfer mechanism from electrons to ions. The expansion process is often described under the assumption of Maxwellian electrons with velocities in local thermal equilibrium, known to be isotropically distributed around the average velocity. This assumption easily fails in the absence of collisions. Indeed, in many cases of astrophysical and laboratory plasma expansion, the electron distribution functions are non-Maxwellian and have more complicated shapes showing high-energy tails. The fundamental reason is that fast electrons collide much less frequently than slow ones. Indeed, their free path is very large because it is proportional to v^4, and cannot relax to a Maxwellian value [7].

In this paper, we analyzed the free expansion of semi-infinite plasma into vacuum assuming kappa distribution for electrons in order to show the significant modification of the ion acceleration process when this non-Maxwellian distribution is assumed. In laser-produced plasma, it was used to model ion acceleration and plasma expansion as it was described in reference [8] where the authors showed numerically that by increasing the population of energetic electrons, the expansion took place faster, the resulting electric field was stronger, and the ions accelerated to higher energy.

The present study of plasma expansion is useful for applications deduced from high-intensity laser--matter interactions. The ions can be accelerated by the charge separation in plasma created at the front side of the target [9] or at the back of the target [10]. The most interesting and efficient one is the electrostatic acceleration at the rear side of a thin dense target. In this case, electrons heated by the laser at the front surface of the target propagate through the solid material and form a space charge cloud in vacuum at the rear surface. The static electric field induced by this electron cloud is strong enough to fully ionize the material and accelerate ions perpendicularly to the rear surface up to multi-MeV energies [11].

Model equations

The plasma expansion model used in this study can be applied for both cases of expansion: a backward expansion from the target front and a forward expansion from the rear side of the target. The present study of plasma expansion is deduced from high--intensity laser-matter interactions with picoseconds pulse duration lasers of intensities above 10^{16} W/cm^2, when energetic electrons play a major role in the ion acceleration. Values of the initial electron and ion densities are of the order of 10^{19} cm^{-3}.

The process of the interaction is described in a one-dimensional (1D) geometry. The one-dimensional kappa distribution function for free electrons in the case they feel an electric potential φ is given by [12]:

$$f_e = \frac{n_{e0}}{\sqrt{\pi}}\frac{1}{\theta\kappa^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{\nu_e^2}{\kappa\theta^2}-\frac{2e\varphi}{m_e\kappa^2\theta}\right)^{-\kappa} \tag{1}$$

where $\theta = \left(\frac{2\kappa-3}{\kappa}\right)^{1/2}\left(\frac{T_e}{m_e}\right)^{1/2}$

is the average thermal velocity of the electrons, n_{e0}, m_e, ν_e and T_e are the initial electron density, the electron mass, the electron velocity, and the electron temperature, respectively; $\kappa \geq 3/2$ is the spectral index that measures the strength of the excess superthermality; φ is the electrostatic potential.

When κ tends to ∞, we retrieve the Maxwellian case.

The spatial electronic density is deduced from Eq. (1) as:

$$n_e = n_{e0}\left(1-\frac{\Phi}{\left(\kappa-\frac{3}{2}\right)}\right)^{-\kappa+½} \tag{2}$$

where $\Phi = e\varphi/T_e$.

Ions of density n_i and velocity ν_i are governed by the equations of continuity and momentum in the cold fluid approximation:

$$\frac{\partial n_i}{\partial t}+\nu_i\frac{\partial(n_i\nu_i)}{\partial x}=0 \tag{3}$$

$$\frac{\partial \nu_i}{\partial t}+\nu_i\frac{\partial \nu_i}{\partial x}+\frac{e}{m_i}\frac{\partial\varphi}{\partial x}=0 \tag{4}$$

The characteristic dimension L of inhomogeneities in the plasma is assumed large compared with the plasma Debye length

$$\lambda_D = \sqrt{\frac{\varepsilon_0 T_e}{n_e(x)e^2}} \tag{5}$$

in such a way the plasma remains quasi-neutral during all the expansion, that is to say:

$$n_e = n_i \tag{6}$$

Self-similar solution

In the presence of free boundary associated to plasma expansion, it is a hard task to solve the hydrodynamic Eqs. (3)–(6) describing the expansion, numerically. But under certain assumptions, these partial differential equations can be reduced to ordinary differential equations that greatly simplify the problem. This transformation is based on the assumption that we have a self-similar solution, i.e., every physical parameter distribution preserves its shape during expansion and there is no scaling parameter in the equations and in the initial conditions [13]. Under such conditions, all the variables depend only on the spatial coordinate x and on the time t, through the combination x/t. A self-similar solution has been built using the following normalized variables: $\tilde{n}_i = n_i/n_{i0}$, $\tilde{\nu}_i = \nu_i/c_s$, c_s, is the ionic sound velocity given by $c_s^2 = T_e/m_e$ and n_{i0} ($n_{i0} \approx 10^{19}$ cm^{-3}) is the initial ion density [14]. Using the dimensionless self-similar variable $\xi = x/c_st$, we obtain the system of normalized ordinary differential equations to be solved.

$$(\nu_i-\xi)\frac{dn_i}{d\xi}+\ {}_i\frac{d\ \ _i}{d\xi}= \tag{7}$$

$$(\nu_i-\xi)\frac{d\tilde{\nu}_i}{d\xi}+\frac{d\Phi}{d\xi}=0 \tag{8}$$

Differentiating Eq. (2) with respect to ξ and using the equation of quasi-neutrality $n_e = n_i$, we obtain

$$\frac{d\tilde{n}_e}{d\xi}=\frac{d\tilde{n}_i}{d\xi}=\frac{\left(\kappa-\frac{1}{2}\right)}{\left(\kappa-\frac{3}{2}\right)}\left(1-\frac{\Phi}{\left(\kappa-\frac{3}{2}\right)}\right)^{-1}\tilde{n}_i\frac{d\Phi}{d\xi} \tag{9}$$

then Eq. (8) becomes

$$\frac{\left(\kappa-\frac{3}{2}\right)}{\left(\kappa-\frac{1}{2}\right)}\left(1-\frac{\Phi}{\left(\kappa-\frac{3}{2}\right)}\right)\frac{d\tilde{n}_i}{d\xi}+\tilde{n}_i(\nu_i-\xi)\frac{d\nu_i}{d\xi}=0 \tag{10}$$

If we treat all the derivative terms as independent variables and the resulting set of equations as algebraic ones, then the nontrivial solution to the system of Eqs. (7) and (10) requires that the determinant of their coefficients must vanish [15], i.e.,

$$\tilde{\nu}_i-\xi=\sqrt{\frac{\left(\kappa-\frac{3}{2}\right)}{\left(\kappa-\frac{1}{2}\right)}\left(1-\frac{\Phi}{\left(\kappa-\frac{3}{2}\right)}\right)} \tag{11}$$

In this work, the positive root has been chosen to correspond to an expansion in the $+x$ direction and a velocity increasing with increasing x. Initially, the metal target surface is located at $x = 0$, and the target is in the $x < 0$ region. Laser radiation is switched on in the $-x$ direction and the plasma starts expanding.

Due to the expansion, the plasma density will decrease and hence a rarefaction wave will propagate in the $-x$ direction [16].

Differentiating Eq. (11) gives:

$$\frac{d\nu_i}{d\xi} = 1 - \frac{1}{2}\frac{\Phi}{\sqrt{\left(\kappa-\frac{3}{2}\right)}}\frac{\sqrt{\left(\kappa-\frac{1}{2}\right)}}{(\kappa-1)}\left(1-\frac{\Phi}{\left(\kappa-\frac{3}{2}\right)}\right)^{-0.5}\frac{d\Phi}{d\xi} \tag{12}$$

and using Eqs. (8), (11) and (12), we found the equation to solve for the electrostatic potential as:

$$\frac{d\Phi}{d\xi} = -\frac{\sqrt{\left(\kappa-\frac{3}{2}\right)\left(\kappa-\frac{1}{2}\right)}}{(\kappa-1)}\sqrt{\left(1-\frac{\Phi}{\left(\kappa-\frac{3}{2}\right)}\right)} \tag{13}$$

The plasma under consideration is expanding into vacuum at $t > 0$. The initial time $t = 0$ in our case corresponds to an unperturbed plasma with initial parameters $\tilde{n}_0 = 1$ and $\tilde{\nu}_0 = 0$. As a consequence, we require that there should exist a point ξ_0 at $t \le 0$ for which the plasma is unperturbed and at rest, such that $\tilde{\nu}(\xi_0) = 0$, $\tilde{n}(\xi_0) = 1$ at $\Phi(\xi_0) = 0$ [17, 18].

$$\xi_0 = -\sqrt{\frac{\left(\kappa-\frac{3}{2}\right)}{\left(\kappa-\frac{1}{2}\right)}} \tag{14}$$

One can derive a self-similar solution for the system of Eqs. (7), (10), and (13) under the assumption of charge quasi-neutrality, for the potential, the velocity, and the density of the ions:

$$\Phi_{ss} = -\frac{1}{(\kappa-1)^2}\left(\begin{aligned}&\tfrac{1}{4}\left(\kappa-\tfrac{1}{2}\right)\xi^2 + \left(\kappa-\tfrac{1}{2}\right)^{3/2}\left(\kappa-\tfrac{3}{2}\right)^{1/2}\xi\\ &+\left(\kappa-\tfrac{3}{4}\right)\left(\kappa-\tfrac{3}{2}\right)\end{aligned}\right) \tag{15}$$

$$\tilde{\nu}_{i,ss} = \left(\frac{1}{\kappa-1}\right)\left(\left(\kappa-\tfrac{1}{2}\right)\xi + \sqrt{\left(\kappa-\tfrac{1}{2}\right)\left(\kappa-\tfrac{3}{2}\right)}\right) \tag{16}$$

$$\tilde{n}_{i,ss} = \left(\frac{1}{2}\frac{\sqrt{\left(\kappa-\frac{1}{2}\right)}}{(\kappa-1)\sqrt{\left(\kappa-\frac{3}{2}\right)}}\xi + \frac{\left(\kappa-\frac{1}{2}\right)}{(\kappa-1)}\right)^{-2\kappa+1} \tag{17}$$

Results and discussion

The expansion of an ionized gas can result from the combination of two effects, the self-consistent electric field created by lighter particles, which first leave the bulk of the plasma, and the thermal pressure.

The plasma dynamics is governed by ion motion. To investigate the self-similar expansion into vacuum, in Figs. 1 through 3, we plot ion densities normalized to their initial values, ion velocities normalized to sound velocity and normalized electrostatic potential, respectively, vs. the self-similarity variable ξ for different values of the nonthermal parameter κ.

It is important to know that the curves are plotted using conditions at ξ_0 instead of $\xi = 0$, showing that initial conditions depend on nonthermal effects.

Figure 1 shows that for a Maxwellian distribution ($\kappa \sim \infty$), the density of the plasma is decreasing monotonically during expansion. It is the phenomenon of free expansion into vacuum. While κ is decreasing (energetic electron population increasing), in addition of the gradient pressure effect, there is the effect of the electrostatic potential that becomes more important due to the presence of fast electrons driving the ions to ensure quasi-neutrality: this phenomenon has the role of more pronounced depletion of the ion density with decreasing κ.

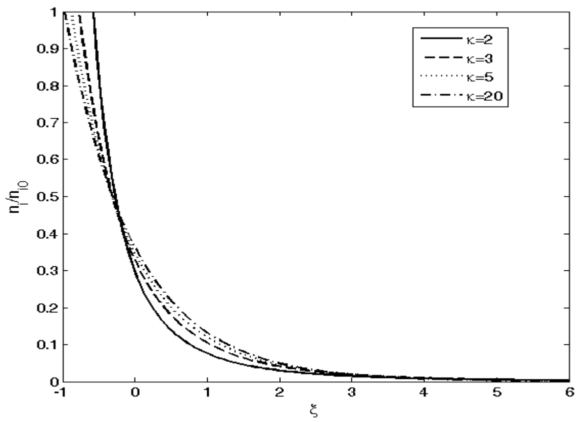

Fig. 1. Densities normalized to their initial values as function of the similarity variable for different values of κ.

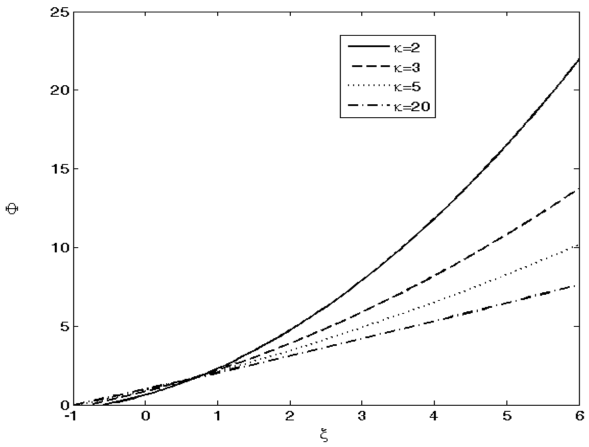

Fig. 2. Velocities normalized to the sound velocity as function of the similarity variable for different values of κ.

Moreover, it is also pointed out that the end of the expansion corresponds to vanishing density. It is represented by a limit of the self-similar parameter ξ_{Lim} determined by the quasi-neutrality condition $L = c_s t = \lambda_D$.

In Fig. 2, for any value of κ, the ion velocities are increasing almost linearly. The expansion of the plasma is similar to that in vacuum where the plasma expands in a way similar to a supersonic expansion with a free linear behavior. With non-Maxwellian distributions having superthermal electrons, the ambipolar electric field, which ensures plasma quasi-neutrality and zero electric current, is greater and a higher electrostatic potential is needed to be accelerate ions and produce larger final speeds. As κ is decreasing, the ion acceleration is more pronounced [19].

In Fig. 3, it is seen that while the electrostatic potential varies linearly with respect to the parameter ξ for the case of Maxwell electron distribution, this dependence becomes quadratic for non-Maxwellian distributions and as κ is decreasing, the potential is more and more enhanced, indicating ion acceleration.

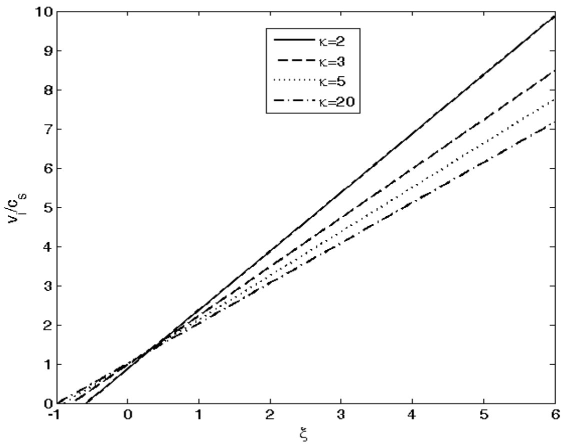

Fig. 3. Normalized electrostatic potential as function of the similarity variable for different values of κ.

Conclusions

The main scope of this work was the investigation of the effects of energetic electrons produced by intense laser-plasma interactions on semi-infinite plasma expansion in vacuum. For this purpose, the kappa distribution function has been employed to describe the electrons with a high-energy tail. In this distribution, decreasing the spectral index κ, the superthermal electrons population increases and the tail of electron distribution becomes more significant. The self-similar solution, obtained using initial self-similar value depending on the nonthermal parameter, shows that the density and velocity profiles depend not only on the nonthermal parameter but also on the limiting self-similar parameter. Depending on the self-similar range, the ion depletion is related to the decrease or increase of the nonthermal parameter. It is found that the presence of fast electrons enhances the ion expansion.

In conclusion, we emphasize that the results of the present investigation will greatly contribute to the understanding of the expansion plasma in laboratory experiments in which the plasma environments are non-Maxwellian ones.

This work was performed at the Centre de Développement des Technologies Avancées in Alger.

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