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Synthesis and optimization of sequencing operation algorithm

Abstract

Synthesis and optimization ways of sequencing operation applied in computer system, are described in the paper. The ways are general, and use sequencing and eliminating operations of algorithm algebra. They allow for automated synthesis of the sequencing operations. Optimization of algorithm formulas has been made on the basis of the properties of sequencing operations.

Keywords: algebra of algorithms, operation of sequencing, synthesis of algorithm formula, optimization of algorithm formula.

1. Introduction

There is possible to describe non-associative algorithms by means of algebra of algorithms and its modifications [1, 2, 3], unlike the system of algorithmic algebras and its modifications [4, 5, 6].

In algebra of algorithms the operator sequences are described by operation of sequencing of two variables. Two operators of the operation of sequencing are separated by a comma (,) or a semicolon (;). Comma separator is used in cases when the operation is commutative. If the operation is non-commutative the separator of operators is a semicolon.

The operation can have horizontal (⌒) or vertical (() orientation. The horizontal and vertical orientations are used in order to get a more compact and better visual formula. The choice of horizontal or vertical orientation is determined by the user in the process of formulas synthesis.

In algebra of algorithms and its modifications there is used an empty operator that is marked with an asterisk (*). In a computer system this symbol has been marked as a gray rectangle (▮), which is supposed to be an abstract graphic operator.

Sequencing operation symbol is a graphic object of a complex form. Its geometric size depends on the size of pins and fonts, as well as the length of operators or formulas. The algorithm to automate the design processes of the operation of sequencing there is synthesized and optimized in the paper.

2. Models of the Operation of Sequencing

Taking into account the horizontal and vertical orientations, and the presence or absence of operators [1, 2] the operation of sequencing can have some models.

The model of the operation of sequencing with two operators and horizontal orientation has the form as below:

$$\overset{\frown}{O_i : O_j},$$

where O_i – is any i-th abstract or concrete operator, O_j – is any j-th abstract or concrete operator, : – is the separator of operators, and it can be a comma when the operation of sequencing is commutative, or a semicolon when the operation of sequencing is non-associative.

This sequence is denoted by S_1.

All other possible models of the operation of sequencing are following:

– Sequence S_2 of horizontal orientation with first abstract or concrete operator, and second abstract graphic operator (▮):

$$\overset{\frown}{O_i : \blacksquare};$$

– Sequence S_3 of horizontal orientation with first abstract graphic operator and second abstract or concrete operator:

$$\overset{\frown}{\blacksquare : O_i};$$

– Sequence S_4 of horizontal orientation with two abstract graphic operators,

$$\overset{\frown}{\blacksquare : \blacksquare};$$

– Sequence S_5 omitting the sequence formation of horizontal orientation,

– Sequence S_6 of vertical orientation with two abstract or concrete operators:

$$\left(\begin{matrix} O_i \\ : \\ O_j \end{matrix}\right.;$$

– Sequence S_7 of vertical orientation with first abstract or concrete operator, and second abstract graphic operator (▮),

$$\left(\begin{matrix} O_i \\ : \\ \blacksquare \end{matrix}\right.;$$

– Sequence S_8 of vertical orientation with first abstract graphic operator and second abstract or concrete operator:

$$\left(\begin{matrix} \blacksquare \\ : \\ O_i \end{matrix}\right.;$$

– Sequence S_9 of vertical orientation with two abstract graphic operators:

$$\left(\begin{matrix} \blacksquare \\ : \\ \blacksquare \end{matrix}\right.;$$

– Sequence S_{10} omitting the sequence formation of vertical orientation.

3. Synthesis of Sequences

We synthesize the sequence (S_1) of formation of the operations of horizontal orientation with two abstract or concrete operators, from:

- the variable formation operators (F_1),
- setting the coordinates of the first operator (F_2),
- forming the first operator (F_3),
- processing (calculating coordinates, taking into account regional features, set font and size) of the separator of operators (F_4),
- processing the second operator (F_5),
- and the operator of omission of the operation forming (R).

Operators of sequence are executed sequentially. To describe the sequence of the operators we use the operation of sequencing. Commutability of operators is unacceptable.

The result is the following formula:

$$S_1 = \overparen{F_1; \overparen{F_2; \overparen{F_3; \overparen{F_4; F_5}}}}.$$

The following operation formation sequences of horizontal sequencing are given as below:
– missing the second operator:

$$S_2 = \overparen{F_1; \overparen{F_2; \overparen{F_3; \overparen{F_4; *}}}};$$

where * – is an empty operator;
– missing the first operator:

$$S_3 = \overparen{F_1; \overparen{F_2; \overparen{*; \overparen{F_4; F_5}}}};$$

– missing both operators:

$$S_4 = \overparen{F_1; \overparen{F_2; \overparen{*; \overparen{F_4; *}}}};$$

– omitting the formation of the operation of horizontal sequencing:

$$S_5 = \overparen{F_1; R}$$

The sequence of forming of operation of vertical sequencing (S_6) with two abstract or concrete operators contains:
- the operators of variables formation (F_1),
- setting the coordinates of the first operator (F_6),
- forming the first operator (F_7),
- forming the separator of operators (F_8),
- forming of the second operator (F_9)
- and the operator of formation omitting of vertical sequencing (D). The sequence S_6 has a form:

$$S_6 = \overparen{F_1; \overparen{F_6; \overparen{F_7; \overparen{F_8; F_9}}}}.$$

The following operation formation sequences of vertical sequencing are given as below:
– missing a lower operator:

$$S_7 = \overparen{F_1; \overparen{F_6; \overparen{F_7; \overparen{F_8; *}}}};$$

– missing an upper operator:

$$S_8 = \overparen{F_1; \overparen{F_6; \overparen{*; \overparen{F_7; F_8}}}};$$

– missing upper and lower operators:

$$S_9 = \overparen{F_1; \overparen{F_6; \overparen{*; \overparen{F_7; *}}}};$$

– omitting of the operation formation of the vertical sequencing:

$$S_{10} = \overparen{F_1; D}.$$

4. Synthesis of elimination expressions

First of all we synthesize the eliminations of horizontal sequencing operation.

Sequences S_1 and S_2, S_3 and S_4 are eliminated with the condition u_4 – denoting the presence or absence of second operator of the sequencing operation. This is given by the formulas:

$$E_1 = \overline{S_1; S_2; u_4\text{-?}} \quad \text{and} \quad E_2 = \overline{S_3; S_4; u_4\text{-?}}.$$

We apply the operation of elimination to the received eliminations E_1 and E_2, with the condition u_3 denoting the presence or the absence of the first operator. As a result we get the expression:

$$E_3 = \overline{E_1; E_2; u_3\text{-?}}.$$

We apply to elimination E_3 and sequence S_5 the operation of elimination with the condition u_2 denoting the formation of the operation of sequencing. As a result we get the expression:

$$E_4 = \overline{E_3; S_5; u_2\text{-?}}.$$

Now we go to the synthesis of eliminations of sequencing operation of vertical orientation.

Sequences S_6 and S_7, S_8 and S_9 are eliminated with the condition u_6, denoting the presence or absence of both operators of the operation of vertical sequencing, giving the following formula

$$E_5 = \overline{S_6; S_7; u_6\text{-?}} \quad \text{and} \quad E_6 = \overline{S_8; S_9; u_6\text{-?}}.$$

We apply the elimination operation to the received eliminations with the condition u_5, denoting the presence or the absence of the first operator. As a result we get the expression:

$$E_7 = \overline{E_5; E_6; u_5\text{-?}}.$$

We apply to E_7 and sequence S_{10} the operation of elimination with the condition u_2, denoting the formation of the sequencing operation of vertical orientation. As a result we get the expression:

$$E_8 = \overline{E_7; S_{10}; u_2\text{-?}}.$$

Sequencing operation expressions of horizontal (E_4) and vertical (E_8) orientations are next eliminated with the condition u_1, denoting the presence or the absence of the horizontal orientation. We get the formula:

$$E_9 = \overline{E_4; E_8; u_1\text{-?}}.$$

Substituting in E_9 the expressions E_4 and E_8 we get the formula:

$$E_9 = \overline{\overline{E_3; S_5; u_2\text{-?}}; \overline{E_7; S_{10}; u_2\text{-?}}; u_1\text{-?}}.$$

Substituting the E_3 and E_7 to the expression we get the formula:

$$E_9 = \overline{\overline{\overline{E_1; E_2; u_3\text{-?}}; S_5; u_2\text{-?}}; \overline{\overline{E_5; E_6; u_5\text{-?}}; S_{10}; u_2\text{-?}}; u_1\text{-?}}.$$

Finally with such substituting we get the non-optimized formula of forming of the sequencing operation.

5. Optimization of the formula of forming of sequencing operation

On the basis of the properties of algorithm algebra operations we perform the optimization of the received in the fourth section formula, taking the number of operators as a criterion of optimization.

Substituting sequences S_1 and S_2 in the expression E_1 we get:

$$E_1 = \left[\begin{array}{l} F_1; F_2; F_3; F_4; F_5 \\ ; \\ F_1; F_2; F_3; F_4; * \\ ; \\ u_4\text{-?} \end{array}\right.$$

On the basis of the feature of shifting of the operator outside the elimination operation [1, 2, 3] we derive the formula:

$$E_1 = F_1; \left[\begin{array}{l} F_2; F_3; F_4; F_5 \\ ; \\ F_2; F_3; F_4; * \\ ; \\ u_4\text{-?} \end{array}\right. .$$

Applying three times to E_1 the feature of operator removal outside the operation of elimination we have:

$$E_1 = F_1; F_2; F_3; F_4; \overline{F_5; *; u_4\text{-?}} .$$

The same transformations are done to the formula E_2, E_5 and E_6, which gives:

$$E_2 = F_1; F_2; *; F_4; \overline{F_5; *; u_4\text{-?}} ,$$

$$E_5 = F_1; F_6; F_7; F_8; \overline{F_9; *; u_6\text{-?}} ,$$

$$E_6 = F_1; F_2; *; F_4; \overline{F_5; *; u_6\text{-?}} .$$

Substituting the received formulas E_1 and E_2 into E_3, as well as E_5 and E_6 into E_7 and applying the rule of the operator removal outside the elimination operation, we get:

$$E_3 = F_1; F_2; \left[\begin{array}{l} F_3 \\ ; \\ * \\ ; \\ u_3\text{-?} \end{array}\right. ; F_4; \left[\begin{array}{l} F_5 \\ ; \\ * \\ ; \\ u_4\text{-?} \end{array}\right. ,$$

$$E_7 = F_1; F_6; \left[\begin{array}{l} F_7 \\ ; \\ * \\ ; \\ u_5\text{-?} \end{array}\right. ; F_8; \left[\begin{array}{l} F_9 \\ ; \\ * \\ ; \\ u_6\text{-?} \end{array}\right. .$$

Substituting two last formulas into the expressions E_4 and E_8, respectively, we derive the formulas:

$$E_4 = \overline{F_1; F_2; \left[\begin{array}{l} F_3 \\ ; \\ * \\ ; \\ u_3\text{-?} \end{array}\right. ; F_4; \left[\begin{array}{l} F_5 \\ ; \\ * \\ ; \\ u_4\text{-?} \end{array}\right. ; F_1; R; u_2\text{-?}} ,$$

$$E_8 = \overline{F_1; F_6; \left[\begin{array}{l} F_7 \\ ; \\ * \\ ; \\ u_5\text{-?} \end{array}\right. ; F_8; \left[\begin{array}{l} F_9 \\ ; \\ * \\ ; \\ u_6\text{-?} \end{array}\right. ; F_1; D; u_2\text{-?}}$$

The received expressions are substituted into the expression E_9, and having removed the operator F_1 outside the elimination operation, we get:

$$E_9 = \left(F_1; \overline{ \left(\begin{array}{l} F_2; R; u_2\text{-?}; \\ ; \\ \left(\begin{array}{l} F_3; *; u_3\text{-?}; \\ ; \\ \left(\begin{array}{l} F_4 \\ ; \\ F_5; *; u_4\text{-?} \end{array}\right. \end{array}\right. \end{array}\right. \left(\begin{array}{l} F_6; D; u_2\text{-?}; \\ ; \\ \left(\begin{array}{l} F_7; *; u_5\text{-?}; \\ ; \\ \left(\begin{array}{l} F_8 \\ ; \\ F_9; *; u_3\text{-?} \end{array}\right. \end{array}\right. \end{array}\right. u_1\text{-?} } \right. .$$

This expression is an optimized form of sequencing operation formula, of minimized number of operators.

Before optimization the formula contained 38 operators, and after the optimization only 14. So, the result of optimization, performed on the basis of the algorithm algebra, minimizes the operator number by 2.7 times.

6. Conclusions

Algebra of algorithms provides the means of synthesis and optimization of formulas of algorithm, taking into account the algorithm operator number.

The number of operators of algorithm formulas can be significantly smaller than in original formulas thus reducing the cost of the practical implementation of algorithms.

7. References

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