

## Application of the modified hybrid rule of mixture (ROHM) and Halpin–Tsai equation for predicting mechanical properties of wood/hemp/polymer composites

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**Abstract** *Application of the modified hybrid rule of mixture (ROHM) and Halpin–Tsai equation for predicting mechanical properties of wood/hemp/polymer composites.* An object of investigations was hybrid wood-polymer composite containing HDPE, hemp fibers and wood particles. The degree of addition of hemp and wood particles ranged from 0 to 60% of filler. The usefulness of the mathematical models ROHM and Halpin-Tsai to predict the Young's modulus of composites was tested. Additionally, experimental research was carried out. It was found that the hemp content in the wood-polymer composite significantly influences the growth of the composite module. The usefulness of mathematical models for predicting the Young's modulus of hybrid composites was also confirmed.

*Key words:* wood-plastic composite, WPC, hybrid rule of mixture, mechanical properties

### INTRODUCTION

Biodegradable composite materials are a new generation of composites in which fiber component comes from natural and renewable resources (plant biomass). They are a good alternative to traditional composite materials with comparable technical properties (Bolton 1995, Madsen et al. 2009, Madsen et al. 2011, Gozdecki et al. 2007, Gozdecki and Wilczyński 2015). One of the most interesting types of wood-polymer composites (WPC) are hybrid composites, especially those containing various types of natural raw material. The type of application of such a composite depends primarily on its mechanical properties and especially its elastic properties. Determining them can be done through a laborious experiment or theoretical considerations, which in most cases leads to solving the equations according to a selected mathematical model. Such a model is the rule of mixtures (ROM) and after some modifications (ROHM) it can be used to predict the elasticity constants of hybrid composite materials containing more than one type of filler (Fu et al. 2002, Mirbagheri 2007). This model is much easier to use than most of the more complex models for hybrid composites. However, the linear characteristics of the influence, for example, of the amount of filler on the elastic properties of the composite determined from the ROHM equation, generally does not correspond to the non-linear characteristics obtained on the basis of experimental results. Therefore, it was decided to examine the usefulness of ROHM for predicting the modulus of elasticity of a hybrid wood-polymer composite containing fibrous particles and wood particles.

### THEORETICAL APPROACH

The upper bound of the modulus of elasticity of a composite made of particles and a polymer matrix can be predicted according to Voight's hypothesis:

$$E_c = E_f V_f + E_p (1 - V_f) \quad (1)$$

And the lower one can be predicted using Reuss assumption:

$$E_c = \frac{E_f E_p}{V_p E_f + V_f E_p} \quad (2)$$

The actual value of the elastic modulus of elasticity of the composite lies between the predicted results of the equations. For simplifying but without losing accuracy, the modified rule of mixtures can be used to predict the elastic modulus of composites containing two forms of filler. This relation is as follows:

$$E_c = E_{c1} V_{c1} + E_{c2} V_{c2} \quad (3)$$

Where  $E_c$  is the modulus of elasticity of the hybrid composite.  $V_{c1}$  and  $V_{c2}$  are the relative hybrid volume fraction of the first and the second system, respectively and  $V_{c1} + V_{c2} = 1$ . Also  $V_{c1} = V_{f1} / V_t$  and  $V_{c2} = V_{f2} / V_t$ , where  $V_t$  is the total fraction of the reinforcement volume and is equal to  $V_{f1} + V_{f2}$ . In order to determine the Young's modulus of individual composite systems, that is  $E_{c1}$  and  $E_{c2}$ , the Halpin–Tsai equation was used:

$$E_{c,w} = \left( \frac{1 + \xi_w \eta_w V_w}{1 - \eta_w V_w} \right) E_p \quad (4)$$

$$E_{c,h} = \left( \frac{1 + \xi_h \eta_h V_h}{1 - \eta_h V_h} \right) E_p \quad (5)$$

$$\eta_w = \frac{\left( \frac{E_{L,w}}{E_p} \right) - 1}{\left( \frac{E_{L,w}}{E_p} \right) + \xi_w} \quad (6)$$

$$\eta_h = \frac{\left( \frac{E_h}{E_p} \right) - 1}{\left( \frac{E_h}{E_p} \right) + \xi_h} \quad (7)$$

$$\xi_w = 2 \left( \frac{l}{d} \right) + 40 V_w^{10} \quad (8)$$

$$\xi_h = 2 \left( \frac{l}{d} \right) + 40 V_h^{10} \quad (9)$$

$$\xi_w = 2 \left( \frac{l}{d} \right) + 40 V_w^{10} \quad (10a) \quad \xi_w = 2 \left( \frac{l}{d} \right) \quad (10b)$$

$$\xi_h = 2 \left( \frac{l}{d} \right) + 40 V_h^{10} \quad (11a) \quad \xi_h = 2 \left( \frac{l}{d} \right) \quad (11b)$$

where:  $E_c$  is Young's modulus of the composite,  $V_w$  wood content,  $V_h$  hemp content,  $E_{L,w}$  Young's modulus of wood in the  $L$  direction,  $E_h$  Young's modulus of hemp,  $\xi_w$ ,  $\xi_h$  coefficients of reinforcement (yet, the formulas of 10a and 11a are used provided that the fiber content  $\geq 40\%$ ),  $\eta_w$ ,  $\eta_h$  functions depending on the ratio of fiber and polymer moduli and the reinforcement coefficient.

## MATERIALS AND METHODS

In the studies two types of wood raw material, hemp fiber (HF) and industrial soft wood particles (WPs) were used. Technical raw hemp fiber (length 25mm, dimension 0,25mm) obtained from BAVE – Badische Faserveredlung GmbH German. The WPs were screened

through an analytical LAB-11-200/UP sieve shaker using the 18 and 35 mesh sieves to obtain the sizes: WP 1–2 mm and length 6-8mm. As a matrix was used the high density polyethylene (HDPE) Tipelin 550-13 in a granule form obtained from Basell Orlen Poliolefins (Plock, Poland) with the density of 956 kg/m<sup>3</sup> and melt flow index of 0.35 g/10 min (190°C/2.16 kg). The HF were treated with 5% NaOH to remove surface impurities, then washed using distilled water and dried to a moisture of 6%. Young's modulus of materials used in tests is for hemp 43,35GPa, HDPE 1,58GPa, and wood along the fibers 11,56GPa.

The raw materials were mixed in proper proportions (Table 1), then poured into metal frames using a trapezoidal gutter to direct wood particles and HF. Composite boards (thickness 16mm) were produced by the pressing method (total pressing time 1230s, temperature range 185°C, pressure 8,5 MPa), and the samples in a form of strips were cut out of the boards prepared in a described way.

**Table 1.** Composition of tested formulations

No	Cod	content (%)		
		HDPE	WPs	HF
1	HDPE	100	0	0
2	W60H0	40	60	0
3	W40H20	40	40	20
4	W20H40	40	20	40
5	WOH60	40	0	60

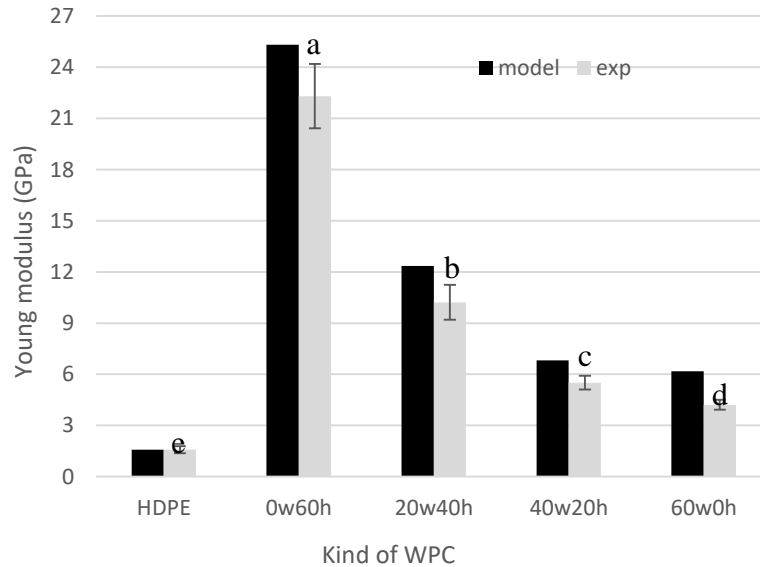
## RESULTS

The obtained data were statistically analyzed using the Statistica 13. The one-way analysis of variance (ANOVA) was conducted to determine the significance of the effect of the WPC composition on Young's modulus of it (Table 2).

**Table 2.** Statistical parameter

Effect	Statistical parameter			
	SS	MS	F	P
kind of WPCs	1330,77	332,693	274,58	0,0001
error	24,233	1,212		

The significance of differences between the mean values of a given property was evaluated by Tukey's HSD test and presented in Figure 1. The same letters indicate that there is no significant difference (at  $\alpha = 0,05$ ) for a given property between different kinds of composites compared. The results of investigations into the effect of the composition (HDPE/HF/WP) on Young's modulus of the examined composite are presented in Figure 1. The results of investigations into the effect of a composition (HDPE/HF/WP) on Young's modulus of the examined composite are presented in Figure 1. In the same figure there is the value of Young's modulus of composites with different compositions being counted using models (ROHM and Halpin-Tsai).



**Figure1.** Effect of HF and WPs filling on Young's modulus of WPC

The effect of the filling of HDPE by HF and WPs on the WPC Young's modulus is strongly noticeable especially for WPC with the 60% content of hemp. The modulus of this composite is more than 13 times higher than that of HDPE. Similarly, Young's modulus of W40H20 is on average about 5,5 times, W20H40 on average about 2,5 times and W0H60 more than 1,7 times higher than that of HDPE. It is important to note that in each case the composite containing HF has higher elastic properties than the one containing WPs. Young's modulus of the composite 0w60h is higher on average by 6,9 times than that of the composite 60w0h, while hemp modulus is only on average about 3 times higher than the modulus of wood. This phenomenon may be due to poor adhesion of a polymer matrix and wood. In the case of a HF, this connection is better because hemp fibers were treated with NaOH. Comparing the results obtained in the experimental and theoretical ways (ROHM and Halpin-Tsai) one can see generally similar values of the modulus of composites. The lowest difference between the modulus obtained experimentally and the one obtained theoretically can be noticed for the composite 0w60h, and it is about 11%. These differences increase to about 30% along with increasing the content of WPs in the composite. The better fit of ROHM and Halpin-Tsai models during the Young's modulus prediction occurs when the composite contains a large amount of hemp. This is due to the greater slenderness of HF. This results in a more desirable fibers arrangement in the composite.

## CONCLUSIONS

1. In general, the use of ROHM and Halpin-Tsai models to predict the Young's modulus of composites containing hemp and wood particles brings good results.
2. Better fit of ROHM and Halpin-Tsai models is observed during the Young's modulus prediction occurs when the composite contains a large amount of hemp.
3. The effect of a filling HDPE with hemp and wood particles on Young's modulus of the WPC is strongly noticeable especially for composites with hemp.
4. Filling HDPE with hemp in 60% results in higher Young's modulus (on average by 6,9 times) than the composite containing 60% wood particles.

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**Streszczenie:** *Zastosowanie zmodyfikowanej hybrydowej regóły mieszalin (ROHM) oraz równania Halpin – Tsai do przewidywania właściwości mechanicznych kompozytów drewno/konopie/polimer. Przedmiotem badań był hybrydowy kompozyt drewno-polimerowy zawierający HDPE, włókna konopi oraz cząstki drzewne. Stopień dodatku konopi i cząstek drzewnych wynosił od 0 do 60%. Sprawdzono przydatność modeli matematycznych ROHM i Halpin-Tsai do przewidywania modułu Younga kompozytów. dodatkowo przeprowadzono badania eksperymentalne. Stwierdzono, że zawartość konopi w kompozycie drewno-polimerowym znacznie wpływa na wzrost modułu kompozytu. Potwierdzono również przydatność modeli matematycznych do przewidywania modułu Younga kompozytów hybrydowych.*

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