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APPLICATION OF FACTORING AND TIME-SPACE SIMULATION METHODS FOR ASSESSMENT OF THE RELIABILITY OF WATER-PIPE NETWORKS

ZASTOSOWANIE METOD FAKTORYZACJI ORAZ SYMULACJICZASOWO-PRZESTRZENNEJ DO OCENY NIEZAWODNOŚCI SIECI WODOCIĄGOWYCH*

This article presents a method for determining the reliability of water-pipe networks through the application of factoring algorithms. This is a method based on graph theory and graph reduction, making it possible to calculate the reliability of a system with a specific structure of connections between its elements without determining its reliability structure. The impact of damage to individual pipeline segments on a network's overall reliability was also determined. In water-pipe networks, it is also particularly important to ensure that the appropriate parameters of water are maintained. Values of the water supply conditions index (WSCI) in the entire analysed network and changes resulting from damage to selected pipeline segments were determined by means of time-space simulation. The presented factoring and time-space simulation methods for determining WSCI index values are mutually complementary in the assessment of reliability. They make it possible to improve the credibility of reliability assessment and may be used to conduct rational usage of water-pipe networks.

***Keywords**: reliability of network systems, water-pipe network, factoring algorithm, time-space simulation.*

W artykule przedstawiono sposób wyznaczenia niezawodności sieci wodociągowych przy wykorzystaniu algorytmu faktoryzacji. Jest to metoda oparta na teorii grafów i ich redukcji, umożliwiająca obliczenie niezawodności układu o określonej strukturze połączeń między elementami ale bez wyznaczania jego struktury niezawodnościowej. Dla wybranej sieci wyznaczono wpływ uszkodzenia poszczególnych odcinków rurociągów na jej niezawodność. W sieciach wodociągowych szczególnie ważne jest zapewnienie odpowiednich parametrów dostarczanej wody. Za pomocą symulacji czasowo-przestrzennej określono wartości wskaźnika warunków poboru wody WWPW w całej analizowanej sieci oraz jego zmiany, w efekcie uszkodzenia wytypowanych odcinków rurociągów. Przedstawione metody faktoryzacji i symulacja czasowo-przestrzenna, do wyznaczenia wartości wskaźnika WWPW, wzajemnie się uzupełniają w ocenie niezawodności. Pozwalają zwiększyć wiarygodność oceny niezawodności i mogą być wykorzystywane w prowadzeniu racjonalnej eksploatacji sieci wodociągowych.

***Słowa kluczowe**: niezawodność układów sieciowych, sieć wodociągowa, algorytm faktoryzacji, symulacja czasowo-przestrzenna.*

1. Introduction

Reliability in a descriptive sense can be defined as the measure of trust that we have in a person or technical object that interests us. The development of reliability theory over the last decade or so has led to the elaboration of many methods of reliability assessment applied to solving a wide range of problems [1, 6, 7, 11]. The presented solutions pertain to both assessment of reliability and the risk of using water-pipe networks and methods of searching for optimal solutions for the implementation of economically justified redundancy in network structures [1, 7, 11, 14, 18]. Another significant aspect is the provision of supply of sufficient amounts of water, which must also fulfil qualitative requirements, to end users [6, 13]. Since the 1980s, the subject of water line and sewage system reliability has been undertaken in parallel by the Cracow University of Technology and the Warsaw University of Technology in Poland [15]. A series of interesting solutions have been found, which have been successfully implemented in practice for the control of water supply to users [2, 3].

This article presents the application of the factoring method for estimation of the reliability of water lines as a critical part of infrastructure, for the purpose of indicating locations in the network that, if damaged, pose a threat to supplying users with water. For such points in the network, the Water Supply Conditions Index (WSCI) was determined [3]. This index is significant in the assessment of water supply. Analysis of obtained results was also conducted to indicate segments that would significantly impact network reliability if damaged.

Analysis was conducted on a selected part of the water-pipe network being used under actual conditions.

2. Characteristics of a selected water-pipe network

The analyzed network consists of a transit system, water main, and distribution pipelines with diameters ranging from 100 [mm] – 400 [mm]. In this work, terminals and pipelines with diameters of less than 100 [mm] were omitted, as they do not have a significant impact on the functioning and reliability of the system as a whole.

Most pipelines form closed rings; however, the presence of branched endings justifies the classification of this network both as a ring and branched network. A large number of closed rings has a positive influence on the reliability of the operation of this network,

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl***

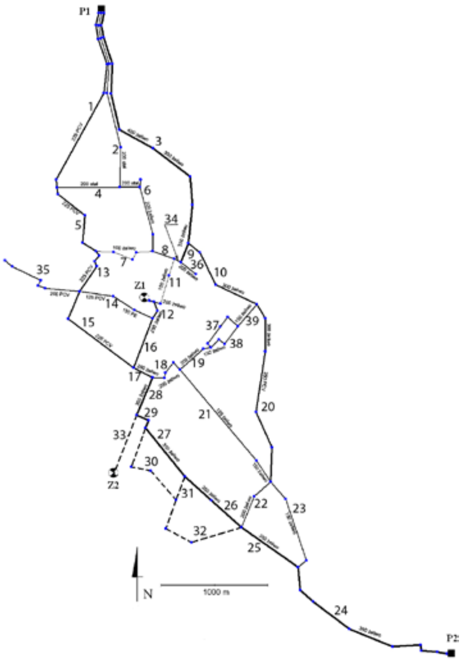

Fig. 1. Structure of water-pipe network connections

because in the case of failure of one pipeline water can be supplied to a consumer via a different route. The elements with the greatest influence on the functioning of the entire system are pumping plants, along with delivery lines and reservoirs. A supply source system comprising pumping plants is advantageous here, as their location on opposite sides of this extended network results in better functioning of the system, equalisation of pressure, and a high level of reliability.

The network supplies water to approximately 40 000 residents with an average demand for water amounting to about 10 000 [m^3] daily. Users are located at heights fluctuating within the range of 283,5–320 [m] above sea level. The highest areas are in the downtown area, around the existing Z1 reservoir, and in the southwestern part of the city in the vicinity of the Z2 reservoir.

The north pumping plant (P1) is situated at a height of 293 [m] above sea level and operates in the pressure range of 4,2 [MPa] – 6,1 [MPa]. Water is supplied to the city through three transit pipelines running in parallel, with diameters of 200 [mm], 225 [mm] and 400 [mm]. In the case of failure of one of the pipelines, the two remaining pipelines take on the burden of water supply to the city. The south pumping plant (P2) is situated 6 metres higher, and so its pumping parameters are lower: 3,5 [MPa] – 5,1 [MPa]. From the P2 pumping plant, water is supplied to the city by only one transit pipeline. Thus, there is a danger that in the case of failure of this pipeline, pumping plant P2 will be put completely out of operation, which may cause insufficient pressure in the network and a decrease in comfortable water consumption for users. In such a situation, consumers situated at the greatest heights may be deprived of water from the network. The location of the Z1 reservoir in the downtown area is also advantageous because it additionally supplies areas with a greater population density and greater demand located farther from the pumping plant, and thus also stabilises pressure. Unfortunately, its ability to accumulate water in the event of failure is severely limited by its small capacity (550 [m^3]).

This network was built in the 1970s and is made mostly of cast iron, polyvinyl chloride, and several steel segments. Thus the main causes of failure are leaks in connections of cast iron pipelines and material defects in plastic pipelines.

3. Applied methods of reliability analysis of the water-pipe network

The connection structure of the water-pipe network shown in fig. 1 is complex from the perspective of reliability. The factoring algorithm was used to determine the reliability of the presented network. This is a method based on graph theory and graph reduction, making it possible to determine network reliability under the following basic assumptions [4, 9, 10, 16, 17]:

– the analyzed network represents an undirected graph:

$$G=(V, E) \tag{1}$$

where:

V – set of graph peaks – nodes in the network (connection of more than 2 pipelines or a recipient connection point): $V=(v_1, v_2,\dots,v_n)$,

E – set of graph edges – connections (pipelines) in the network: $E=(e_1, e_2,\dots,e_m)$,

– it is accepted that graph peaks, or nodes in the network, are always in a serviceable state,
– the probabilities of individual connections being damaged in the network are to be assumed or determined by using e.g. statistical methods and data on malfunctions gathered for the given network or for networks exploited under similar conditions,
– any incidents of damage are independent from one another.

Network reliability is defined as the probability of the existence of at least one serviceable connection between all nodes of set K in the network ($2\le|K|\le|V|$). This makes it possible to determine various measures of network reliability depending on the number of nodes contained in set K. When K=2, reliability is the probability of the existence of a connection between two nodes, e.g.: source – reception point or inlet – outlet. When K=V, reliability is the probability of the existence of a connection between all nodes of set V. In a water-pipe network, this can be interpreted as the probability that water supply is available for all nodes in set K, and thus, for all users.

Determination of a dependency making it possible to calculate reliability is done by way of the appropriate reduction of the network, represented by an undirected graph, according to the following notation [4, 8, 9, 17]:

$$R(G_K)=R_{ei}\cdot R(G_K|e_i\ serviceable)+(1-R_{ei})\cdot R(G_K|e_i\ unserviceable) \tag{2}$$

$$R(G_K)=R_{ei}\cdot R(G_{K'}*e_i)+(1-R_{ei})\cdot R(G_K-e_i) \tag{3}$$

where:

$G_{K'}*e_i=(V-u-v+w, E-e_i)$ – graph reduction when connection e_i is serviceable; peaks at the ends of connection e_i are subtracted from the set of peaks (V), a peak resulting from their connection is added in their place, and the reduced connection e_i is removed from connection set (E),

$$K'=\begin{cases} K & if\ u,v\notin K \\ K-u-v+w & if\ u\in K\ lub\ v\in K, \end{cases}$$

$G_K-e_i=(V, E-e_i)$ – graph reduction when connection e_i is unserviceable; the peak set (V) is unchanged and connection e_i is removed from the connection set.

Figure 2 demonstrates the method to reduce and obtain a dependency for calculation of the measure of network reliability sought. Depending on the number of nodes in set K, system reduction is carried out according to the method presented in figure 2. The method from fig. 2a is applied only when all nodes are included in set *K*. The method from fig. 2b, presented here for K=2, is analogous for all cases where *K*<*V*.

The application of the presented method enables the reduction and

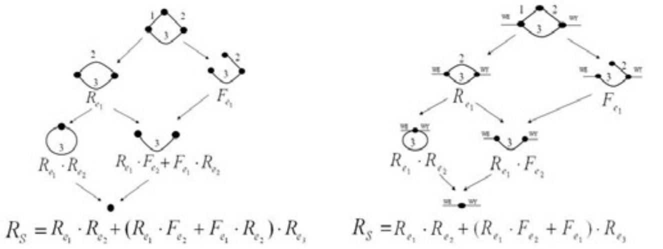

Fig. 2. Method of system reduction according to the factoring algorithm: a) K=V, b) K=2

determination of the reliability of a network system of any degree of complexity. However, the problem becomes significantly more complicated as the number of connections in the network increases, because the reduction procedure is more time-consuming and the form of the final dependency is extensive [4, 10, 16, 18]. Figure 3 shows a fragment of reduction for a simple system where, for 7 connections, the final dependency is already extensive [10]. For more complex systems, such as the water-pipe network analysed here, it is necessary to use a computer program.

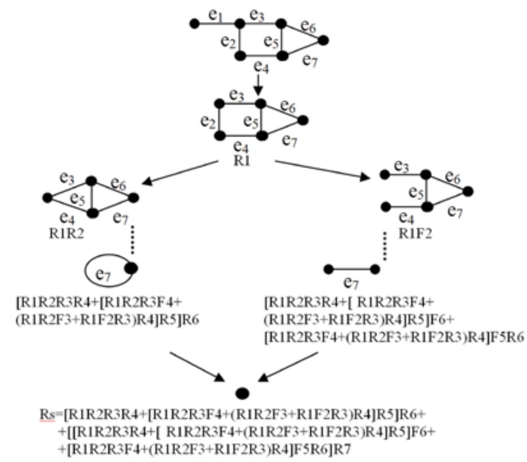

Fig. 3. Fragment of system reduction with seven connections for a case where K=V [9]

Besides providing water to recipients through undamaged pipelines, it is particularly important to ensure the required parameter values of supplied water in a water-pipe network. This concerns both hydraulic parameters and those of chemical composition. In this paper, aspects related to the chemical composition of water will not be taken into consideration. As for hydraulic parameters, it is most important to ensure the required pressure values for all consumers and their ability to receive necessary amounts of water at any time [2, 3, 13].

Because the factoring algorithm does not account for the above aspects in its analysis, the 'ISYDW' program, developed at the Cracow University of Technology, was used to calculate hydraulic parameters of water supply to consumers [2,3]. The computational model applied in this program accounts for the following, among other things:

- the structure of connections in the network,
- the diameters and lengths of all pipelines,
- flow resistance and resultant losses of pressure,
- altitudes of users in the network and the corresponding required pressures,
- variable outputs of network supply sources,
- capacities of reservoirs cooperating with the network,
- volume of water demand of consumers in the network.

The program carries out a simulation of network operation, and the execution of calculations makes it possible to observe changes of hydraulic parameters at individual points in the network over a selected time interval, the method in which the network functions, and changes in these parameters in the event of failure of pipelines, reservoirs, or network supply sources. The determined values for hydraulic parameters in the network are then compared to the required values. A convenient method of assessing the degree to which required parameter values are fulfilled is to observe the Water Supply Conditions Index (WSCI). This index defines the degree to which the capability to supply water to consumers is reduced. It is defined as the quotient of the volume of water consumption and the volume of demand. Full user comfort is defined as a WSCI of 1; when this value falls below 1, users do not receive water with the appropriate parameters (pressure in the network is lower than the required pressure, and the user cannot receive the required amount of water). For an individual node in the network, this value is determined according to the following dependency [2, 3]:

$$WSCI = \frac{ZU}{ZP} \leq 1 \tag{4}$$

where:
ZU – value of water consumption,
ZP – value of water demand, which is known.

The *ZU* value is determined according to the dependency:

$$ZU = ZP \cdot \left(\frac{h}{h_{min}} \right)^{0,5} \tag{5}$$

where:
h – pressure in the node at a given moment in time,
h_{min} – minimum pressure in a given node providing full comfort of water consumption.

Depending on pressure value h in the node, the following situations are possible:

- if $h = h_{min}$ then $ZU=ZP$ and $WSCI=1$,
- if $h = 0$ then $ZU=0$ and $WSCI=0$,
- if $0<h< h_{min}$ then $0<ZU<ZP$ and $0<WSCI<1$.

The value of the index for the entire network is the quotient of the sum of consumption values for all nodes and the sum of the demand values for all nodes at a given moment in time.

4. Results of calculations

Due to the labour demand related to reduction of the graph representing the network analyzed in this paper, reliability calculations were carried out using a computer programme developed at the AGH Technical University [12].

The nodes in the network consist of all branches, connection points of the network with supply sources and user connection points. For the presented water-pipe network, all 28 nodes were included in set *K*. In this case, a state of network serviceability is every situation in which water transfer to every node in the network is possible, and calculated reliability is the probability of such a situation.

Network reliability, determined as the probability of the existence of a connection between all nodes, with changes dependent on the probability of connections remaining in a serviceable state, is presented on fig. 4.

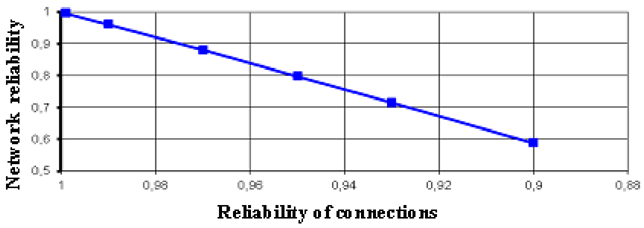

Fig. 4. Network reliability as a dependency of the probability of serviceability of its connections

The main goal of analysis is to determine the impact of damage to individual connections in the network on its reliability. To simplify the calculations presented further on in the paper, only one of the values from figure 4 was taken as a point of reference. This is a case in which the value of the probability of connections remaining in a serviceable state is expressed as $R_p = 0{,}99$, according to literature data elaborated on the basis of many years of study of water-pipe networks [5,15]. Network reliability, determined as the probability of existence of a connection between all nodes, is then equal to $R_s = 0{,}95997$.

The determined values of network reliability in the event of damage to a given connection, as well as the impact of connection damage on network reliability expressed as a percentage, are shown in figures 5 and 6. The zero value of reliability in fig. 5 signifies that damage to a given connection causes a state of network unserviceability in terms of the accepted assumptions. There is no way water can be transferred to the node at the end of this connection. In the case where a supply source is located at the end of the damaged connection, it cannot supply water to the network. These connections (24, 33, 35, 36) can be seen as critical from the perspective of the reliability of the analyzed network.

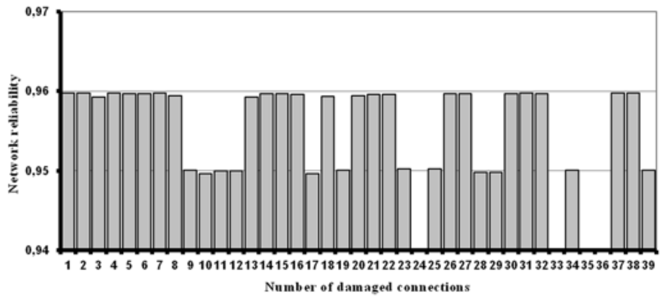

Fig. 5. Network reliability in the event of damaged connections

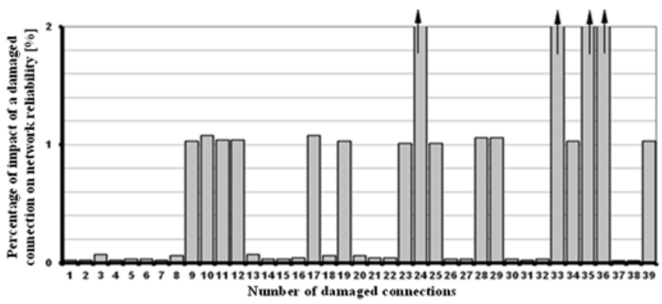

Fig. 6. Percentage of impact of a damaged connection on network reliability

It can also be seen that if certain connections in the ring structures are damaged, this will not cause a state of network unserviceability; however, such damage has a decidedly larger influence on the reliability of the network as a whole. Such critical connections are: 9, 10, 11, 12, 17, 19, 23, 25, 28, 29, 34, 39. Thus, it turns out that from the perspective of maintaining the cohesion of the network structure and the capability of supplying water to all nodes in the network, the serviceability of these connections is also of great significance.

Next, assuming damaged connections in the network, a simulation of network functioning was carried out in the ISYDYW program and WSCI index values were determined. Each time-space simulation was carried out for a full day. Based on information from the use of actual networks, it was assumed that the time to repair a damaged connection would not exceed 1 day [2,3].

When reservoirs and pumping plants are operating correctly, the WSCI index for the entire network is equal to 1. Temporary disengagement of a reservoir does not cause a reduction in comfortable water consumption, whereas total planned closing of a reservoir at peak hours causes a reduction of the WSCI value to 0,9 for several nodes. This means that the small number of consumers supplied from these nodes and residing at the highest altitudes may be affected by insufficient pressure. If reservoir closing is unplanned, then the entire western part of the city is subject to slight inconvenience (WSCI = 0,9), and in the vicinity of several nodes, this inconvenience is significantly greater (at peak hours WSCI = 0,3 – 0,4).

Failure of a 200 [mm] or 225 [mm] transit pipeline (connection 1, 2) from pumping plant P1 does not have a significant impact on WSCI. However, failure of the 400 [mm] pipeline (connection 3) at peak hours causes a drop in WSCI value in several nodes on the west side of the city to 0,9, and, at the most disadvantageous point, to 0,7. However, the effects of disconnection of this pipeline can be minimised by increasing the output of pumping plant P2.

In the case of total shutdown of pumping plant P1 at the most disadvantageous time of day, the WSCI index in the eastern part of the network reaches a value of approximately 0,85; in the western part 0,15 – 0,55.

The most difficult situation occurs when pumping plant P2 is shut down (damage to connection 24). Pressure drops throughout the city, and in the eastern part, the greatest values of the WSCI index are in the range of 0,8 – 0,9 and fall to 0,5 at peak hours. However, in the western part of the city, WSCI fluctuates in the range of 0,15 – 0,55. This means that the water supply needs of only 50% of consumers are met. The course of changes of the mean WSCI index determined for the entire network during a given day is shown in figure 7. It is clearly visible that in the hours from 3 pm to 11 pm, water supply conditions will deteriorate significantly.

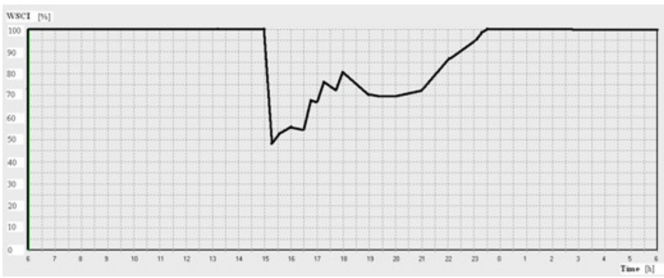

Fig. 7. Change in mean WSCI if connection 24 is damaged

Based on the calculations, it can be stated that the most significant connections in the analyzed network are designated by the numbers: 17, 24, 25, 28, 33. Damage to these connections significantly aggravates water supply conditions at certain points in the network and in large areas of the network.

A list of the most important connections indicated by reliability analysis and time-space simulation has been presented in table 1. Connections of the greatest significance have been marked with the "**X**" symbol.

The results obtained in both methods show a certain convergence. It can, however, be seen that not all connections indicated in the reliability analysis are significant from the perspective of time-space

Table 1. Most important connections in the network indicated by both computing methods

Damaged connection no.	Influence on structural reliability	Influence on water supply conditions to recipients (WSCI)	Minimum mean WSCI value during the day calculated for the entire network	Minimum WSCI value during the day for the worst point in the network
9	x		≅1	0,98
10	x		≅1	0,98
11	x		≅1	0,98
12	x		≅1	0,96
17	x	**X**	0,97	0,37
19	x		≅1	0,98
23	x		≅1	0,98
24	X	X	0,48	**0,13**
25	x	**X**	0,86	**0,13**
28	x	**X**	0,91	**0,13**
29	x		≅1	0,98
33	X	X	0,56	**0,13**
34	x		≅1	0,98
35	**X**		≅1	0,98
36	**X**		≅1	0,98
39	x		≅1	0,98

simulation and have a weaker influence on the determined value of the WSCI index.

Differences mainly result from different approaches to the problem in the two methods. Reliability analysis determines whether water can be transferred to all points in the network with connections being damaged within a certain probability. The most important connections are determined in terms of the fulfilment of this condition.

In time-space analysis, certain quantities not taken into consideration in reliability analysis were accounted for. These include both the diameters and lengths of individual pipelines as well as loss of flow and variable outputs of supply sources. The obtained results make it possible to determine whether, and to what degree, required hydraulic parameters of water will be supplied to consumers.

It can thus be said that the methods are complementary, and in the final assessment of network reliability, connections indicated as significant in both methods should be given special consideration. The critical connections indicated in both methods (17, 24, 25, 28, 33, 35, 36) are the most important from the perspective of structural reliabiltiy as well as the ability to ensure required water supply conditions for consumers in the network. This constitutes a direct guideline for undertaking necessary inspection and technical activities in order to ensure their correct functioning.

In the case under consideration, the following can be recommended:
- impose strict inspection of: construction work, and particularly of performance of excavations and other work involving heavy machinery performed in the area of critical pipeline segments,
- in the case of water main pipelines (24, 25), the construction of a backup should be considered (as in the case of pumping plant P1), particularly if further development of the existing network is planned and in light of the fact that segment 24 is the only connection between pumping plant P2 and the network.

5. Conclusions

The presented factoring and time-space methods used to determine the values of the WSCI index are mutually complementary in the assessment of reliability and can be used to conduct rational usage of water-pipe networks.

Running a time-space simulation in order to determine WSCI index values and to assess structural reliability of the water-pipe network leads to the identification of critical pipeline segments, which, if damaged, could cause the greatest inconvenience to water users. Ensuring the appropriate supervision and technical inspection for these parts of the network is a significant element of usage strategy, because water supply systems are one of the most critical parts of the infrastructure, of particular significance to the functioning of the country and its citizens. Accordingly, ensuring their safe and reliable operation is a particularly important matter.

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