SOME MIXED MATRIX PROBLEMS OVER SEVERAL DISCRETE VALUATION RINGS

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Abstract. This article presents some results about several district valuation rings with a common skew field of fractions. They are obtained from the approximation theorem for discrete valuation rings. These results give the possibility to solve basic mixed matrix problems for such rings. We present the solution of some mixed flat matrix problems over several district valuation rings with common skew field of fractions.

Keywords: valuation rings, local invariant rings, discrete valuation rings, approximation theorem, mixed flat matrix problems

1. Introduction

In this paper we present the solution of some basic mixed flat matrix problems over several discrete valuation rings with a common skew field of fractions.

Let O be a discrete valuation ring with a skew field of fractions D. Recall that a right elementary row operation over D in an arbitrary matrix is one of the following:

(i) interchanging of two rows;
(ii) multiplying a row by a nonzero scalar (i.e. an element of D);
(iii) replacing a row by itself plus a scalar multiple of another row.

Analogously there can be introduced the left D-elementary column operations. In a similar fashion we can introduce right O-elementary row transformations and left O-elementary column transformations.

Let $\{O_i\}$ be a family of discrete valuation rings (not necessary commutative) with Jacobson radicals R_i ($i = 1, 2,\ldots, k$) and a common skew field of fractions D.

Assume that A is an indecomposable semiperfect ring with a decomposition of the identity into a sum of pairwise orthogonal idempotents

$$1 = f_1 + f_2 + \ldots + f_n \tag{1}$$

such that the two-sided Pierce decomposition of A has the following form:

$$A = \bigoplus_{i,j=1}^{n} f_i A f_j \tag{2}$$

where $f_i A f_i = H_{n_i}(O_i)$ for $i = 1,2,\ldots, k$; $f_j A f_j = D$ for $j = k+1, \ldots, n$; and $A_{ij} = f_i A f_j$ is an (A_{ii}, A_{jj})-bimodule, for $i, j = 1,2,\ldots, n$.

Consider the main flat matrix problem associated with a ring A given by (2) of the following form.

Main mixed flat matrix problem

Given a block matrix **T**:

$\mathbf{T}_{11}$	...	$\mathbf{T}_{1j}$	...	$\mathbf{T}_{1n}$
...	...	...	...	...
$\mathbf{T}_{i1}$	...	$\mathbf{T}_{ij}$	...	$\mathbf{T}_{in}$
...	...	...	...	...
$\mathbf{T}_{n1}$	...	$\mathbf{T}_{nj}$	...	$\mathbf{T}_{nn}$

partitioned into n horizontal and vertical strips so that a block $\mathbf{T}_{ij}$ has elements in $f_j A f_i$ and $\mathbf{T}_{ii}$ has elements in $f_i R f_i$ for $i,j = 1,..., n$.

Reduce the matrix **T** to a block-diagonal form with indecomposable blocks by means of admissible transformations $\mathbf{T} \rightarrow \mathbf{UTV}$, where $\mathbf{U} = (\mathbf{U}_{ij})$, $\mathbf{V} = (\mathbf{V}_{ij})$ are block unsingular matrices, and $\mathbf{U}_{ii}$, $\mathbf{V}_{ii}$ are square invertible matrices with entries in $f_i A f_i$ $(i = 1,2,..., n)$.

Definition 1.1. The vector of the form:

$$\mathbf{d} = \mathrm{d}(\mathbf{T}) = (d_1, d_2, ..., d_n; d^1, d^2, ..., d^n), \tag{3}$$

where d_i is the number of rows of the i-th horizontal strip of **T**, and d^i is the number of columns of the i-th vertical strip of **T** for $i = 1,2, ..., n$ is called the **dimension vector** of a stripped matrix **T**

$$\dim(\mathbf{T}) = \sum_{j=1}^{n} d_j + \sum_{i=1}^{n} d^i \tag{4}$$

Definition 1.2. We say that a mixed flat matrix problem with a matrix **T** is of **bounded representation type** if there is a constant C such that $\dim(\mathbf{T}) < C$ for all the indecomposable matrices **T**. Otherwise it is of the **unbounded representation type**.

The solution of a mixed flat matrix problem in the case of one discrete valuation ring and its skew field of fractions was considered by A.G. Zavadskij and U.S. Revitskaya in [1]. Earlier similar problems were studied in [2-5].

We now describe briefly the contents of this paper. In Section 2 we present the main notions and results concerning valuations and structures of (noncommutative) discrete valuations rings (DVRs). Section 3 considers the approximation theorem for DVRs and presents some important corollaries which makes it possibile to solve basic mixed flat matrix problems. Using these results in Section 4 we give the solutions of some basic mixed flat matrix problems over several distinct discrete valuation rings and their common skew field of fractions. These matrix problems arise in connection with problems in the representation theory of rings.

We use the main notions and results from [6, 7].

All rings considered in this paper are assumed to be associative with $1 \neq 0$ and all modules are assumed to be unital.

2. Valuations and noncommutative DVRs

In this section we recall some main notions and results from the theory of valuations for noncommutative rings. First noncommutative valuation rings were considered and studied by O.F.G. Schilling in [8]. Below we give the main notions of a valuation and a valuation ring of a division ring. They are the generalizations of the concepts for commutative case which were first introduced and studied in 1932 by W. Krull [9].

Definition 2.1. [8] Let G be a totally ordered group (written additively) with order relation $\geq$. Add to G a special symbol ∞ such that $x + \infty = \infty + x = \infty$ for all $x \in G$. Let D be a division ring. A **valuation** on D is a surjective map $v: D \to G \cup \infty$ which satisfies the following:
1) $v(x) \leq \infty$
2) $v(x) = \infty$ if and only if $x = 0$
3) $v(xy) = v(x) + v(y)$
4) $v(x + y) \geq \min(v(x), v(y))$
for all $x, y \in D$.

Definition 2.2. [8] A subring R of a division ring D is called the **valuation ring** (=**non-commutative valuation ring**) of D if there is a totally ordered group G and a valuation $v: D \to G \cup \infty$ of D such that

$$R = \{x \in D : v(x) \geq 0\} \tag{5}$$

In his paper [8] O.F.G. Schilling introduced the important notions of invariant valuation rings and total valuations rings and systematically studied them.

Definition 2.3. [8] A subring R of a division ring D is called **invariant** of D^* if $dRd^{-1} = R$ for any $d \in D^*$. A subring R of a division ring D is called a **total valuation ring** if $d^{-1} \in R$ for any $d \notin R$.

The next theorem shows that the notions of valuation rings of division rings and total valuation rings are equivalent.

Theorem 2.4. [8] Let R be a subring of a division ring D. Then the FCAE:
1) R is a valuation ring of D with respect to some valuation v on R;
2) R is invariant total valuation ring of D.

Similar to the commutative case we can introduce the notion of discrete valuation rings for noncommutative case. In the literature it was defined by many different ways. Below we give the definition of DVR following to O.F.G. Schilling.

Definition 2.5. [8] A subring O of a division ring D is called the (noncommutative) **discrete valuation ring** (DVR for short) if there is a (discrete) valuation $v: D \to \mathbf{Z}$ of D such that

$$O = \{x \in D : v(x) \geq 0\}. \tag{6}$$

The basic properties of DVR's are presented in the following proposition.

Proposition 2.6. [10] Let O be a DVR of a division ring D with respect to a valuation v. Let t be a fixed element of O with $v(t) = 1$. Then
1) O is a local domain with the nonzero unique maximal ideal

$$M = \{x \in O : v(x) > 0\}$$

2) Any nonzero element $x \in O$ has the unique representation in the form

$$x = t^n u = w t^n$$

for some $u, w \in O^*$, and $n \in \mathbf{Z}$, $n \geq 0$.
   If D is a division ring of fractions of A then any element $y \in D^*$ has the form $y = t^n u = w t^n$ for some $u, w \in O^*$, and $n \in \mathbf{Z}$.
3) Any one-sided ideal I of O is a two-sided ideal and has the form $I = t^n O = O t^n$ for some $n \in \mathbf{Z}$, $n \geq 0$, i.e. O is a principal ideal ring. In particular, $M = tO = Ot$, and $I = M^n = t^n O = O t^n$.
4) $\bigcap_{i=0}^{\infty} M^i = 0$, where M is the unique maximal ideal of O.
5) O is a Noetherian uniserial ring.
6) O is a hereditary ring.

As we mentioned above there were a few different definitions of DVRs. The following statement shows that all these definitions of DVRs are equivalent.

Proposition 2.7. [10] The following statements for a ring O are equivalent.
1) O is a DVR.
2) O is a local ring with nonzero maximal ideal M of the form $M = tO = Ot$, where $t \in O$ is a non-nilpotent element, and $\bigcap_{i=0}^{\infty} M^i = 0$.
3) O is a local principal ideal domain which is not a division ring.
4) O is a Noetherian local ring with nonzero maximal ideal which is two-sided and principal.
5) O is a right (left) Noetherian local ring with nonzero maximal ideal M of the form $M = tO = Ot$ with a non-nilpotent element $t \in O$.
6) O is a Noetherian non-Artinian uniserial ring.
7) O is a Noetherian valuation ring.

3. Approximation theorem for DVR

The approximation theorem is a basic tool for working with several valuation rings over a given skew field. In this section we consider the approximation theorem for noncommutative DVRs and obtain some corollaries from this theorem. In [11] H.H. Brungs and J. Gräter introduced the following general notion of a valuation on a skew field:

Definition 3.1. [11] Let D be a skew field and W_v a totally ordered set with largest element θ. A mapping $v: D \to W_v$ is called a **valuation** of D if for all $x,y,z \in D$ the following conditions hold:
(i) $v(x) = \theta$ if and only if $x = 0$;
(ii) $v(x + y) \geq \min\{ v(x), v(y)\}$;
(iii) $v(x) \geq v(y)$ implies $v(zx) \geq v(zy)$.

This valuation defines the subring

$$B_v = \{x \in D : v(x) \geq v(1)\} \tag{7}$$

which is a total ring in the sense that $x \notin B_v$ implies $x^{-1} \in B_v$ for each $x \in D$.

Note that if in the definition above the condition iii) is changed by
(iv) $v(xy) = v(x) + v(y)$
provided $W_v \backslash \theta = G$ is a totally ordered Abelian group (written additively) we obtain the definition of a valuation which was first proposed by Schilling in [8].

J. Gräter in [12] introduced the important notion of a locally invariant ring.

Definition 3.2. [12] A ring R is called **right locally invariant** if for each right ideal I of R

$$\sqrt{I} = \{x \in R \mid \exists n_x \in N, x^{n_x} \in I\} \tag{8}$$

is a completely prime ideal.

The following theorem gives the characterization of locally invariant valuation rings.

Theorem 3.3. [13, Theorem 3.1] A chain domain R is locally invariant if and only if any one of the following equivalent conditions is satisfied:

1) $\bigcap a^n R$ is a two-sided ideal for any $a \in R$.
2) $Ra^2 \subseteq aR$ for any $a \in R$.
3) $\sqrt{I} = \{x \in R \mid \exists n_x \in N, x^{n_x} \in I\}$ is a completely prime ideal for every right ideal I in R.
4) Between any two prime ideals of R there is another two-sided ideal of R.
5) $xP(x) = P(x)x$ for any $x \in R$ and $P(x)$ the minimal completely prime ideal of R containing x.

From this theorem it immediately follows the following result:

Corollary 3.4. Any discrete valuation ring O of a division ring D (in sense of Definition 2.5) is locally invariant.

In [12] J. Gräter proved the approximation theorem for total valuation rings of a skew field when these rings are locally invariant. Here we give this theorem following to [13]. We need some definitions which were introduced in this paper.

Let B_1, B_2 be valuation rings of a skew field D. Then there exists a minimal valuation ring $B_{1,2}$ of D containing both B_1 and B_2. The right ideal I_1 of B_1 and the right ideal I_2 of B_2 are compatible if $I_1 B_{1,2} = I_2 B_{1,2}$. If in addition, $a_1, a_2 \in D$ then a_1, a_2, I_1, I_2 are called **compatible** if $a_1 - a_2 \in I_1 B_{1,2} = I_2 B_{1,2}$.

Theorem 3.5. [13] (Approximation Theorem).
Let $B_1, ..., B_n$ be valuation rings of a skew field D and assume that B_i is locally invariant for $i = 1,..., n-1$. Let $a_i \in D$ for $i = 1,..., n$ and let I_i be a right ideal of B_i such that a_i, a_j, I_i, I_j are compatible for every $i, j \in \{1,..., n\}$. Then there exists an $x \in D$ with

$$x - a_i \in I_i \tag{9}$$

for $i = 1, ..., n$.

This theorem holds for valuation rings in finite dimensional division algebras.

Let B_1, B_2 be discrete valuation rings of D. Since any discrete valuation ring O of D is a maximal valuation ring in D, then $B_{1,2} = D$. Therefore $I_1 B_{1,2} = I_2 B_{1,2}$ for

each ideal I_1 of B_1 and each ideal I_2 of B_2, and so a_1, a_2, I_1, I_2 are compatible for any elements a_1, $a_2 \in D$ and any ideal I_1 of B_1, any ideal I_2 of B_2.

Taking into account that any discrete valuation ring is locally invariant we immediately obtain the following corollary from Theorem 3.5, which is an approximation theorem for discrete valuation rings.

Corollary 3.6. (Approximation Theorem for DVRs) Let O_1, ... , O_n be distinct discrete valuation rings of a skew field of fractions D. Then for any elements $a_i \in D$ and ideals I_i of O_i for $i = 1, ..., n$ there is an element $x \in D$ such that

$$x - a_i \in I_i \qquad (10)$$

for $i = 1, ..., n$.

Proposition 3.7. Let O_1, O_2 be distinct discrete valuation rings in a skew field of fractions D. Then
1. $D = O_1 + O_2$.
2. There exist elements $\pi_1 \in M_1 \backslash M_1^2$ and $\pi_1 \in O_2^*$. Analogously there exist elements $\pi_2 \in M_2 \backslash M_2^2$ and $\pi_2 \in O_1^*$.
3. $D^* = O_1^* \cdot O_2^*$.

Proof.
1. Put $I_1 = O_1$, $I_2 = O_2$, $a_1 = d \in D$, $a_2 = 0$ in Corollary 3.6. Then there exist elements $x \in D$ that $x - d \in O_1$ and $x \in O_2$, i.e. $x - d = a$, $x = b$. So $d = b - a$, where $b \in O_2$, $a \in O_1$.
2. Put $I_1 = M_1^2$, $I_2 = M_2$, $a_1 \in M_1 \backslash M_1^2$, $a_2 = 1$ in Corollary 3.6.
3. This follows immediately from Theorem 2.6 (3).

Proposition 3.8. Let O_1, O_2 be distinct discrete valuation rings in a skew field of fractions D and $A = O_1 \cap O_2$. Then there exists an element $x \in D$ such that $x^n \notin A$ and $x^{-n} \notin A$ for any $n \in \mathbf{N}$.

Proof. Suppose that A is a total valuation ring of D. Then by [14, Corollary 1.3] subrings of D containing A must be totally ordered by inclusion. Since $O_1 \not\subset O_2$ and $O_2 \not\subset O_1$ we have a contradiction.

Therefore A is not a total valuation ring and so by definition there exists and element $x \in D$ such that $x \notin A$ and $x^{-1} \notin A$.

Suppose there exists $n \in \mathbf{N}$ such that $x^n \in A$ or $x^{-n} \in A$. Since $x^n \in A$, $x^n \in O_1$ and $x^n \in O_2$. Therefore $v_1(x^n) = nv_1(x) \geq 0$, which implies that $v_1(x) \geq 0$, that is $x \in O_1$. Analogously, $x \in O_2$, that is, $x \in A$. A contradiction! In the same way we can show that $x^{-n} \notin A$ for any $n \in \mathbf{N}$.

4. Some mixed matrix problems

In this Section we give the solution of two mixed matrix problems. Consider the first one.

Matrix problem I

Let O_1, O_2 be a district valuation rings of the division ring D. Given a matrix $\mathbf{T}$ with entries in D it is possible the following admissible transformations:
1) left O_1 - elementary transformations of rows of the matrix $\mathbf{T}$.
2) right O_2 - elementary transformations of columns of the matrix $\mathbf{T}$.
Find all indecomposable matrices.

Lemma 4.1. The matrix problem I can be reduced by admissible transformations to the following form:

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} \tag{11}$$

Proof.
1-st step.

Let M_i be a maximal ideal of a ring O_i. By Proposition 3.7 there exists an element $\pi_1 \in M_1 \backslash M_1^2$ and $\pi_1 \in O_2^*$. Analogously there exists an element $\pi_2 \in M_2 \backslash M_2^2$ and $\pi_2 \in O_1^*$.

So that any element $x \in D$ can be written in the different forms $x = \pi_1^n \varepsilon = \eta \pi_1^n$ and $x = \pi_2^m \mu = \xi \pi_2^m$, where $\varepsilon, \eta \in O_1^*$ and $\mu, \xi \in O_1$. Then $v_1(x) = n$ and $v_2(x) = m$.

Pick out in the matrix $\mathbf{T}$ an element $x = \pi_2^m \mu \in D$ such that $v_2(x)$ is minimal among all elements of $\mathbf{T}$ and put it in the place (1,1). Multiplying the first column of this matrix on $\mu^{-1} \in O_2^*$ we obtain that $t_{11} = \pi_2^m$. By this element we can make all elements in the first row to be zero using the transformations on the columns. So we obtain the equivalent form of the matrix:

$$\mathbf{T} = \begin{bmatrix} \pi_2^n & \mathbf{O} \\ * & \mathbf{T}_1 \end{bmatrix} \tag{12}$$

Since $\pi_2^n \in O_1^*$, multiplying the first row by $\pi_2^{-n} \in O_1^*$, we can reduce the matrix $\mathbf{T}$ to the following equivalent form:

$$\mathbf{T} = \begin{bmatrix} 1 & \mathbf{O} \\ * & \mathbf{T}_1 \end{bmatrix} \tag{13}$$

Applying this procedure to the matrix $\mathbf{T}_1$ we obtain the following equivalent form of the matrix $\mathbf{T}$:

$$\mathbf{T} = \begin{bmatrix} \mathbf{T}_2 & \mathbf{O} \\ * & \mathbf{O} \end{bmatrix}, \text{ where } \mathbf{T}_2 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ * & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ * & * & * & 1 \end{bmatrix}. \tag{14}$$

Since $O_1 + O_2 = D$, then for any nonzero element $x = t_{ij} \in \mathbf{T}_2$ with $i > j$ there exist elements $a \in O_1$ and $b \in O_2$ such that $-x = a + b$. So by transformations on the i-th row and the j-th column of $\mathbf{T}_2$ we can make this element to be 0. So we reduce the matrix $\mathbf{T}$ to the following equivalent form:

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{T}_3 & \mathbf{O} \end{bmatrix} \tag{15}$$

where $\mathbf{I}$ is the identical matrix.

2-nd step.

Let elements of $\mathbf{T}_3$ have form t_{ij} with $i \geq k$. We pick out in the matrix $\mathbf{T}_3$ an element $y \in D$ with minimal $v_1(y)$ for all elements in $\mathbf{T}_3$ and put it in the place $(k,1)$. Let $y = \pi_1^n$. If $n \geq 0$ then by the element $t_{11} = 1$ we can make all elements zero below the element y in the 1-st column. So we obtain the indecomposable direct summand of $\mathbf{T}$ of the form (1). So we can assume $v_1(y) = n < 0$.

Therefore $y^{-1} \in O_1$. By the row transformations on $\mathbf{T}_3$ we can make all elements zero below the element y in the 1-st column. Since $y^{-1} \in O_1$ we can add the k-th row of $\mathbf{T}$ to the first row of it. Then we obtain in the reducing matrix the element $a_{11} = 0$ and $v_1(a_{1j}) > 0$ for all $j > 1$ since $v_1(y^{-1} t_{kj}) \geq 0$ owing $v_1(y)$ is minimal. Then using the elements $a_{ii} = 1$ for $i = 2, ..., k-1$ we reduce our matrix to the following form:

$$\mathbf{T} = \begin{bmatrix} \mathbf{X} & \mathbf{O} \\ \mathbf{Y} & \mathbf{O} \end{bmatrix}, \text{ where } \mathbf{T} = \begin{bmatrix} \mathbf{X} & \mathbf{O} \\ \mathbf{Y} & \mathbf{O} \end{bmatrix} \text{ and } \mathbf{Y} = \begin{bmatrix} y & * \\ \mathbf{O} & * \end{bmatrix}. \tag{16}$$

Since element $y \in D$ can be represented in the form $y = ab$, where $a \in O_1^*$, $b \in O_2^*$, then multiplying the 1-st column by b^{-1} and the k-th row by a^{-1} we can make the element t_{k1} to be 1, i.e. we obtain:

$$\mathbf{T} = \begin{bmatrix} \mathbf{X} & \mathbf{O} \\ \mathbf{S} & \mathbf{O} \end{bmatrix}, \text{ where } \mathbf{S} = \begin{bmatrix} 1 & * \\ \mathbf{O} & \mathbf{M}_3 \end{bmatrix}. \tag{17}$$

Now all elements in the k-th row we can make 1, since $O_1 + O_2 = D$. So we obtain the decomposition of matrix $\mathbf{T}$:

$$\mathbf{T} = \begin{bmatrix} 1 & \mathbf{O} \\ \mathbf{O} & \mathbf{A} \end{bmatrix}, \text{ where } \mathbf{A} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{M}_3 & \mathbf{O} \end{bmatrix}. \tag{18}$$

Since the dimension of the matrix $\mathbf{M}_3$ is smaller than the previous matrix, by mathematical induction we obtain that the matrix $\mathbf{T}$ can be reduced to the form (11), as required.

Matrix problem II

Let O_1, O_2 be a distinct discrete valuation rings of the common division ring of fractions D. Given a matrix $\mathbf{T}$ with entries in D with the following admissible transformations:

1) left $O_1 \cap O_2$ - elementary transformations of rows of the matrix $\mathbf{T}$.
2) right D - elementary transformations of columns of the matrix $\mathbf{T}$.

Find all indecomposable matrices.

Lemma 4.2. The matrix problem II is of unbounded representation type.

Proof.
By proposition 3.8 there exists an element $x \in D$ such that $x^n \notin A$ and $x^{-n} \notin A$ for any $n \in \mathbf{N}$, where $A = O_1 \cap O_2$.
Then the matrix of the following form:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ x & x^2 & \cdots & x^n \end{bmatrix} \tag{19}$$

is indecomposable under admissible transformations 1) and 2) of the matrix problem II for any $n \in \mathbf{N}$.

Matrix problem III

Let O_1, O_2 be distinct discrete valuation rings of the division ring D. Given a matrix $\mathbf{T}$ with entries in D partitioned into two vertical strips:

$$\mathbf{T} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \end{bmatrix} \tag{20}$$

with the following admissible transformations:
1) O_1 - elementary transformations of rows of the matrix $\mathbf{T}$.
2) O_2 - elementary transformations of columns of the matrix $\mathbf{A}_1$.
3) D - elementary transformations of columns of the matrix $\mathbf{A}_2$.

Find all indecomposable matrices.

Lemma 4.3. The matrix problem III is of the unbounded representation type.

Proof. By admissible transformations we can reduce our matrix problem to the form:

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & \mathbf{O} & \mathbf{O} & \mathbf{B} \\ \mathbf{O} & \mathbf{O} & \mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} \end{bmatrix} \tag{21}$$

where there are possible the following admissible transformations:
1) left $O_1 \cap O_2$ - elementary transformations of rows of the matrix **B**;
2) right D - elementary transformations of columns of the matrix **B**.

Therefore we get the matrix problem II which is unbounded representation type by Lemma 4.2.

Conclusions

In this article we consider the approximation theorem for discrete valuation rings and obtain some important corollaries from this theorem, which gives the possibility to solve mixed flat matrix problems over several district valuation rings with common skew field of fractions. We give the solution of some such mixed flat matrix problems.

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