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Numerical investigations of the finite amplitude wave pressure distribution radiated by a circular transducer

Stanisław Dobrociński, Eugeniusz Kozaczka, Grażyna Grelowska
Naval Academy
ul.Śmidowicza 71, 81-919 Gdynia, Poland

ABSTRACT

The paper presents the results of numerical investigations of the circular transducer's nearfield. The investigations were carried out applying the numerical solution (FEM) of the nonlinear acoustics equation. It was assumed that the transducer generated harmonic vibrations. Different pressure distributions at the transducer's surface were taken to determine pressure distributions in its nearfield.

INTRODUCTION

It is well known that strong fluctuations in the pressure amplitude take place close to the transmitting transducer of finite dimensions. The numerical investigations of the nearfield of a circular transducer radiating a harmonic wave of an infinitesimal amplitude were presented in [17] whereas an analytical model applying a simplified parabolic equation was described in [14].

In case of the finite amplitude waves the situation is more complex. The wave equation of nonlinear acoustics which describes propagation of waves in a dissipative medium can be obtained basing on general equations of fluid mechanics. The equations suitable to study finite amplitude wave propagation are the equation of continuity, the force equation, the heat-exchange equation and the equation of state. The nonlinear equation of acoustics has been obtained assuming small relative changes of density and pressure [10, 16]. An analytical solution of this equation has not been found yet.

Presenting the functions describing the water movement according to assumptions of quasi-optical approximation [16] the equation of nonlinear acoustics can be transformed to a nonlinear parabolic equation called the KZK equation. It describes the changes in pressure in a beam considering the influence of nonlinear and dissipative features of the medium, as well as the diffraction of the acoustic beam.

Ingenito and Williams published one of the first results concerning the investigations of the nearfield of sources radiating finite amplitude [8]. The changes in pressure amplitude of the second harmonic in the nearfield were calculated solving the equation of nonlinear acoustics and neglecting the influence of dissipation. The first and the second harmonic of pressure in the field of a circular transducer radiating finite amplitude waves was determined by solving the KZK equation by the perturbation method [11]. The area of the validity of the solution was determined.

The KZK equation is currently a widely used tool in nonlinear acoustics. Aanonsen et. al. were able to solve numerically the equation in the nearfield of a plane and monochromatically excited sound source [1]. During the last decade several computer codes have been developed following the same line both for monochromatic and biharmonic sound sources of different shapes. A dialog system for PC has been elaborated for numerical simulation of physical phenomena connected with the nonlinear propagation and interaction of acoustic plane waves and a beam in ideal, dissipative and dispersive media [18]. The next model used to describe the nonlinear field propagation was based on the transformed beam equation (TBE) [7]. The TBE equation is derived from the KZK equation by changing the variables. Solutions of the KZK equations are still being modified [6]. Changes introduced to models depend on their application, i. e. investigations of pulse sources [4].

Szabo [13] proposed a new set of nonlinear wave equations for media with an attenuation described by a power law frequency dependence. For media with attenuation with quadratic dependence on frequency these new equations reduce to Burgers', the KZK and Westervelt's nonlinear equations.

There also exists a completely different approach to the problem of modelling the finite amplitude wave field. Vecchio and Lewin [15] widened the angular spectrum method developed by Stepanishen and Benjamin [12] for modeling the propagation of acoustic fields between two parallel planes. Their model considered the effects of the nonlinear distortion of a finite amplitude wave. On the other hand Christopher and Parker proposed a two step model of finite amplitude wave propagation [3]. A new discrete Hankel transform has been used with spatial transform techniques to propagate the field over a short distance with diffraction and attenuation. In the other substep, the temporal frequency domain solution to Burger's equation has been implemented to account for the nonlinear accretion and depletion of harmonics. The model has not been restricted by the usual parabolic wave approximation and the field's directionality has been explicitly accounted for at each point.

A piston transducer is often considered in calculations. The pressure distribution close to the surface of the transducer is usually assumed to be uniform. Otherwise it is described by the Gaussian function [2, 5] or a polynomial [2].

The paper will present the numerical results obtained applying a model based on the nonlinear acoustics equation. In the real conditions, often because of shielding, the pressure distribution close to the piston transducer's surface is not uniform. Therefore the investigations were carried out for a few pressure distributions close to the transducer's surface which approximately corresponded to the real pressure distributions. Fig.1. shows an example of the pressure distribution obtained experimentally in a plane placed 1 mm from the transducer's surface, which had the radius equal to 2.3 cm and radiated a wave of 1 MHz.

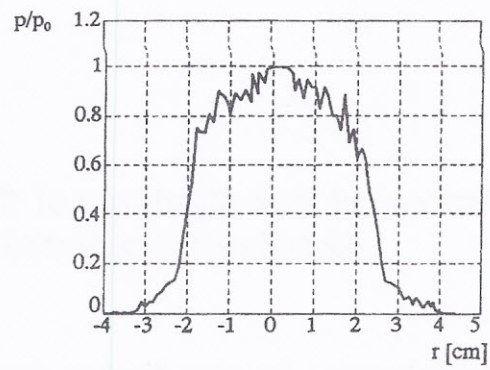

Fig.1. The pressure distribution at a distance of 1 mm from the surface of the examined transmitting transducer

FORMULATION OF THE PROBLEM

The numerical model of the finite amplitude wave propagation close to a piston source was worked out assuming that changes occurring in this field are described with the nonlinear acoustics equation [10, 16]:

$$\Delta p' - \frac{1}{c_o^2}\frac{\delta^2 p'}{\delta t^2} + \frac{b}{c_o^2 \rho_o}\frac{\delta}{\delta t}\Delta p' = -Q \qquad (1)$$

where:

$$Q = \frac{1}{c_o^4 \rho_o}\left(\frac{\delta p'}{\delta t}\right)^2 + \frac{B}{2A c_o^4 \rho_o}\frac{\delta^2 p'}{\delta t^2} + \frac{\rho_o}{2}\Delta v^2 + \rho_o \vec{v}\Delta\vec{v} \qquad (2)$$

$p' = p - p_s$ - acoustic pressure, c_o - the speed of sound, ρ_o - density of the undisturbed medium, b - attenuation factor, t - time, v - vibration velocity, B/A - the nonlinearity parameter.

This equation is solved using FEM in the coordinate system connected with the radiating transducer. A piston source is placed in such a way that the radiating surface is located in the plane xOy and the wave propagates in the z axis direction (the beam axis corresponds to the z axis). Assuming the axial symmetry of the source $r=\sqrt{x^2+y^2}$, we are looking for the solution in the z area (0,Z) and r∈<0,3a>, where a - the radius of the transmitting transducer. We are assuming the harmonic vibrations at the source's surface and the following distribution of the generated pressure:

$p(z=0,r,t)=-p(r)\sin\omega t,\ r\le a$

$p(z=0,r,t)=0,\qquad r>a.$

THE RESULTS OF NUMERICAL INVESTIGATIONS

Below there are presented the results of calculations of the transducer's nearfield distribution for two pressure distributions shown in the figure 2.

The pressure distribution in the area close to the transmitting transducer was determined for both boundary conditions. Next figures show the numerical investigation results which were carried out assuming that the pressure distribution close to the source is given by the curve 1. Fig. 3. shows the pressure distribution in the transverse section of the beam for a different distance from the source, whereas figure 4 shows the pressure distribution in a plane placed 33.1 mm from the source illustrated in three dimensions. Next figures show calculation results obtained assuming that the pressure distribution close to the source is given by the curve 2.

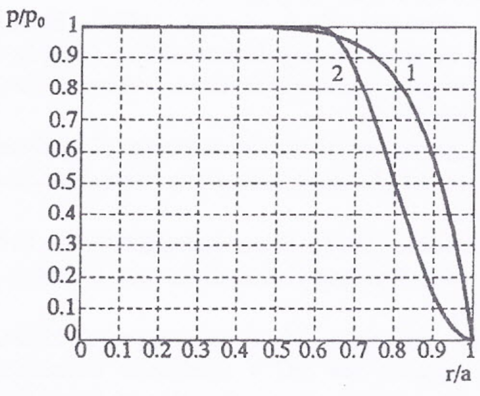

Fig. 2. The pressure distributions at a sound source assumed for the calculations

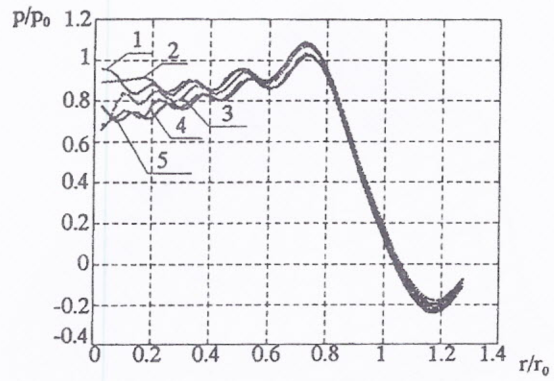

Fig. 3. The pressure distribution in the transverse section of the beam for the boundary conditions described by the curve 1 in the Fig. 2. at a varying distance from the transducer: 1-28.9 mm; 2-30.3 mm; 3-31.7 mm; 4-33.1 mm; 5-34.5 mm

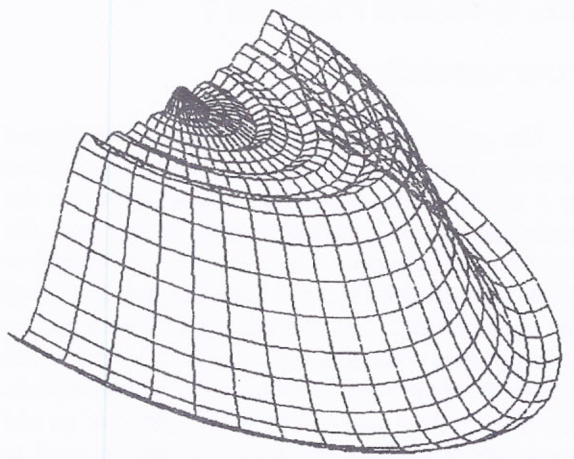

Fig. 4. The pressure distribution in the transverse section of the beam at a distance of 33.1 mm from the source for the boundary conditions given by the curve 1 in the Fig. 2

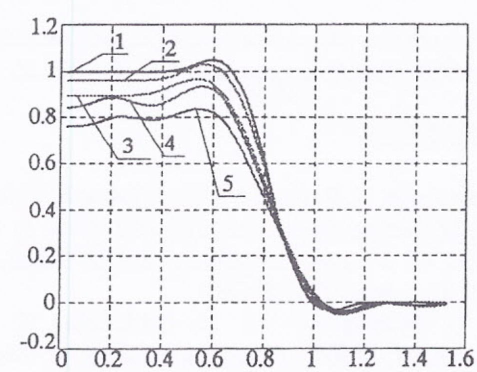

Fig. 5. The pressure distribution in the transverse section of the beam for the boundary conditions described by the curve 2 in the Fig. 2. at a varying distance from the transducer: 1-13.0 mm; 2-19.9 mm; 3-26.8 mm; 4-32.4 mm; 5-39.4 mm

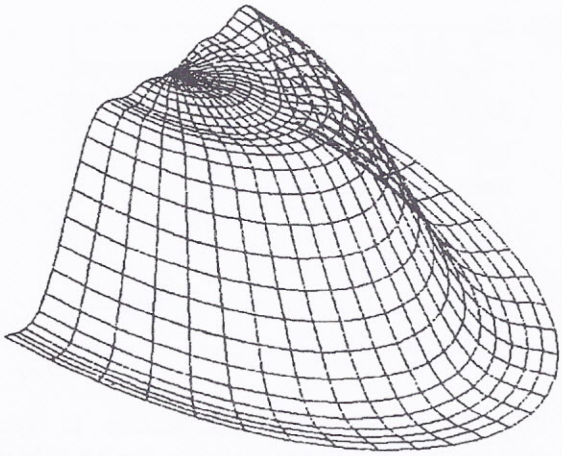

Fig. 6. The pressure distribution in the transverse section of the beam at a distance of 32.4 mm from the source for the boundary conditions given by the curve 2 in the Fig. 2

CONCLUSIONS

The paper presented the results of numerical investigations of the pressure distribution radiated by a circular transducer. The model used for the investigations was worked out basing on the equation of nonlinear acoustics in the complete form. This model does not have the restrictions of the area in which it can be used. The investigations of the pressure distribution were carried out in the nearfield for different pressure distributions close to the source. The model has no restrictions on the boundary conditions. Therefore it can be used to foresee the field distribution of real sources of finite amplitude waves.

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