



## A simplified model of concrete for analysis of reinforced concrete elements

ANNA SZCZEŚNIAK, ADAM STOLARSKI

Military University of Technology, Faculty of Civil Engineering and Geodesy,  
00-908 Warsaw, 2 Kaliskiego Str., Poland,  
anna.szczesniak@wat.edu.pl, adam.stolarski@wat.edu.pl

**Abstract.** The elastic-plastic material model for concrete developed by considering the stress softening and degradation of the deformation modulus for the concrete was presented in the paper. A reduced plane stress state for the compression/tension range with shear was assumed. During the loading process, the model describes four phases of concrete behaviour during compression: achieving the elastic compressive concrete strength, perfectly plastic flow, material softening, and failure/crushing. The model describes three tension phases: achieving elastic tensile concrete strength, material softening, and failure/cracking. The failure phases were interpreted as achieving a stressless state in the material softening process. The proposed model is simplified and very effective to describe the most important properties of nonlinear behaviour of material. The model of concrete can be used for analysis of failure mechanism of reinforced concrete structural elements.

**Keywords:** model of concrete, reinforced concrete elements, geometrical nonlinearity, structural bent elements, eccentrically compressed structural elements

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### 1. Introduction

Description of the behaviour of structural materials, in particular concrete, is an integral part of the computational procedure for the effort analysis of reinforced concrete structural elements. Issues related to the modelling of concrete, which allows for description of the behaviour of the material corresponding to the experimental results, are taken into account in numerous publications. The simplest model that takes into account the static properties of concrete in terms of compression and

tension, is the double-surface model. This model was applied in the work of Feenstra and de Borst [6]. In this model, the concrete behaviour in terms of compression was described using the Drucker-Prager's criterion of plastic deformation, whereas the range of tension was defined on the basis of the Rankine's yield condition. Taken into account the phenomenon of the material stiffness degradation, caused by the strains in the loading and unloading processes, was the base to include the elastic deformation modulus degradation in the modelling of concrete. The research in this range was conducted among others by Pamin and de Borst [16], Hansen et al. [8], and also Cińcio and Wawrzynek [4], as well as Kmiecik and Kamiński [10], Kamal and Yazdani [9]. Nonstandard model describing inelastic, dynamic behaviour of concrete taking into account the material softening, was developed by Stolarski [20]. This model was used in nonlinear dynamic effort analysis of reinforced concrete beams, frames, arches, presented in work of Stolarski [19] and reinforced concrete deep beams presented in the work of Stolarski and Cichorski [21]. Reduced, static form of this model of concrete, has also been successfully used for analysis of the reinforced concrete structural elements statically loaded, the results of which are presented in the work of Cichorski and Stolarski [2, 21], Smarzewski [17], and Smarzewski and Stolarski [18]. Moreover, to the works that significantly influenced on the development of nonlinear modelling of concrete properties, can be included the publications of Wojewódzki et al. [25] Majewski [12], Mandera et al. [13], and Pamin [15]. The model of elastic-plastic material, taking in to account the non-local material softening and stiffness degradation, designed for modelling of concrete elements loaded cyclically, was proposed by Marzec and Tejchman [14]. The triaxial damage-plastic model for failure of concrete was presented by Grassl and Jirásek in [7]. This model was applied to the structural analysis of reinforced concrete columns. A plastic-damage constitutive model for plain concrete was presented by Cicekli et al. [3]. This material model was implemented to the advanced finite element program ABAQUS and the results of numerical simulations for uniaxial and biaxial tension and compression showed very good agreement with the experimental results. Krätzig and Pölling in [11] worked out the model of concrete that contains a minimum number of material parameters and which is effective to describe the behaviour of reinforced concrete structures under biaxial and uniaxial loading.

The aim of this work is to develop a simplified model of deformation of concrete allowing the description of the nonlinear properties of the material, with the assumption to use in the analysis of the failure mechanism of reinforced concrete elements, in particular of the rod elements subjected to the short-term static load. By assumption, the developed model is to reflect the basic features of the concrete behaviour in the subsequent phases of its effort and allows for its easy implementation in the numerical program. For this purpose, the elastic-perfectly plastic model with taking into account the material softening and degradation of deformation modulus, was developed. The proposed model of concrete allows for the description of the

elastic attaining the initial strength, perfectly plastic flow and material softening in compression and elastic attaining the initial strength and material softening in tension. The plane stress state reduced to the one-dimensional compression/tension with shear, was considered. The different loading and unloading cycles in the subsequent phases of the material effort, were also determined.

## 2. Concrete model

An elastic-plastic material model for concrete that considers material softening and the degradation of deformation modulus was developed. A reduced plane stress state for the compression/tension range with shear ( $\sigma_{11}, \sigma_{22} = 0, \sigma_{12}$ ) was assumed.

During the loading process, the model describes four phases of concrete behaviour during compression: 1 achieving the elastic compressive concrete strength, 2 perfectly plastic flow in a limited strain range, 3 material softening, and 4 failure/

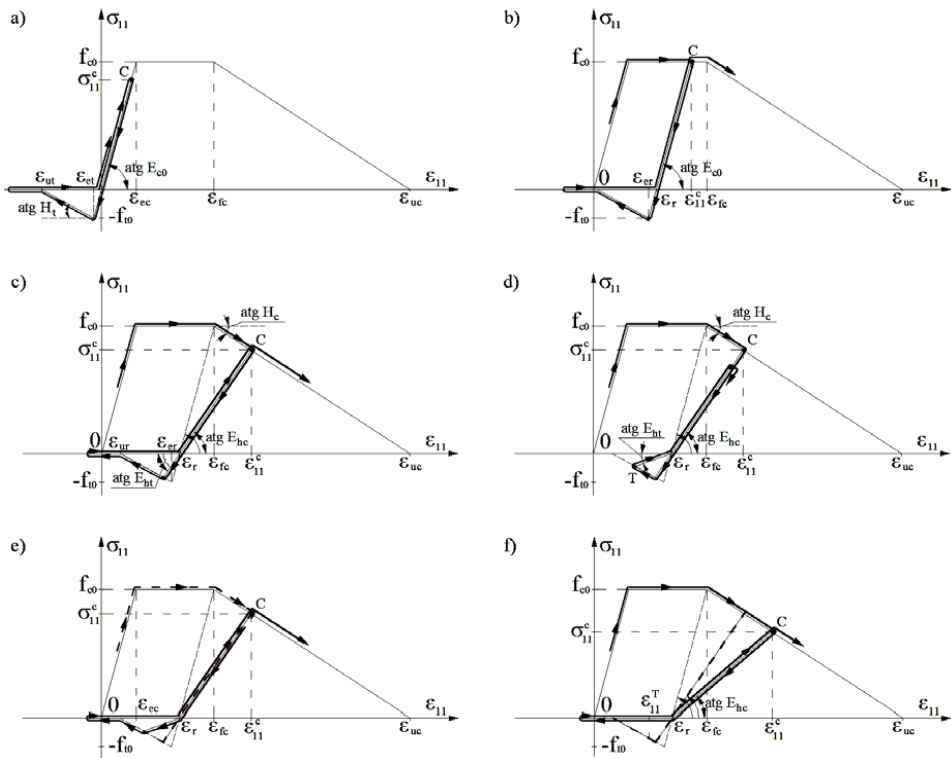


Fig. 1. Model of concrete in the plane  $\sigma_{11} - \epsilon_{11}$  and possible situations of loading and unloading cycles

crushing. The model describes three tension phases: 1 achieving elastic tensile concrete strength, 2 material softening, and 3 failure/cracking. The failure phases were interpreted as achieving a stressless state in the material softening process.

In figure 1, the possible cycles of loading and unloading processes in  $\sigma_{11} - \varepsilon_{11}$  plane are presented.

Figure 1 shows the possible situations of loading and unloading cycles for the proposed model of concrete: a) elastic loading and unloading in compression, loading to cracking in tension and reloading in compression after crack closing; b) elastic-perfectly plastic loading and unloading in compression, loading to cracking in tension and reloading in compression after cracks closing at attaining the permanent strains; c) elastic-perfectly plastic with material softening loading and unloading in compression, loading in the range of material softening to cracking in tension, and reloading in compression after cracks closing after attaining the permanent strains; d) elastic-perfectly plastic with material softening loading and unloading in compression, loading in the range of material softening and unloading in tension, and reloading in compression after attaining the permanent strains; e) elastic-perfectly plastic with material softening loading and unloading in compression, secondary loading in the range of material softening to cracking in tension, and reloading in compression after cracks closing at attaining the permanent strains; f) elastic-perfectly plastic with material softening loading and unloading in compression, reloading the previously cracked concrete in tension at stressless state, and reloading in compression after cracks closing at attaining the permanent strains.

Figure 2 shows an idealization of the concrete model in the plane  $\sigma_{12} - \varepsilon_{12}$ .

Figure 3 shows the idealized limit curve for a plane, with reduced stress state ( $\sigma_{11}, \sigma_{22} = 0, \sigma_{12}$ ). This defines the yield criterion for the assumed material model.

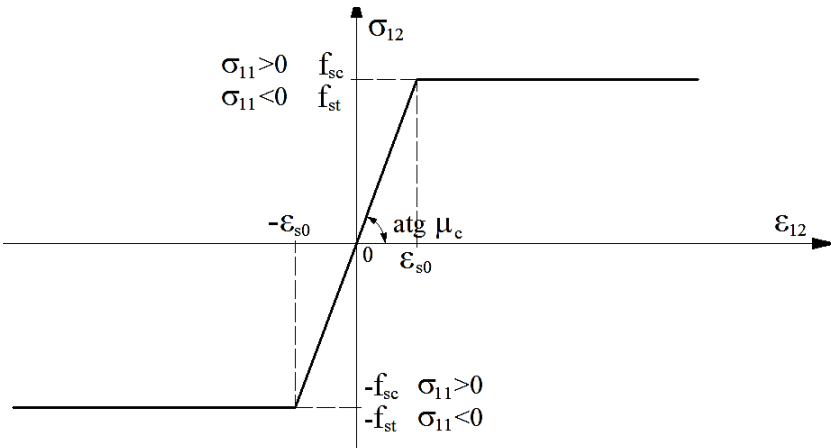


Fig. 2. Model of concrete in the plane  $\sigma_{12} - \varepsilon_{12}$ .

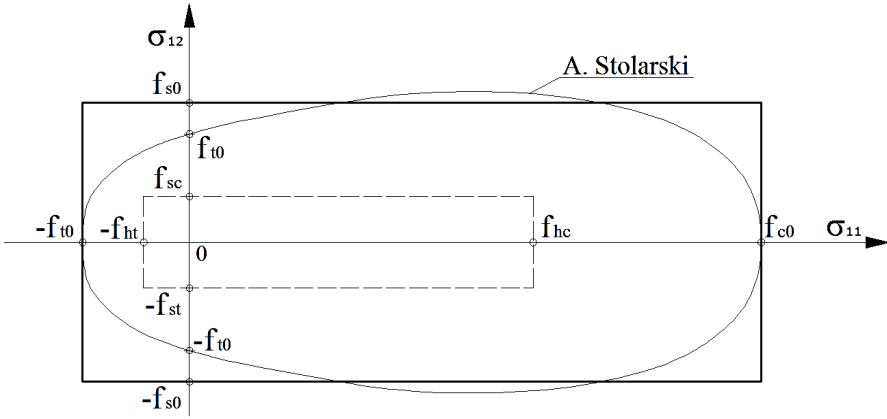


Fig. 3. Limit curve for the plane stress state

The concrete model describes the incremental equations for stresses while considering the limitations that are a result of the yield condition. That is,

$$\left\{ \begin{array}{l} \bar{\sigma}_{11}^n = \sigma_{11}^{n-1} + E_c \Delta \varepsilon_{11}^n \quad \sigma_{11}^n = \begin{cases} \bar{\sigma}_{11}^n & \text{for } -f_t^n < \bar{\sigma}_{11}^n \leq f_c^n \\ -f_t^n & \text{for } \bar{\sigma}_{11}^n < -f_t^n \\ f_c^n & \text{for } \bar{\sigma}_{11}^n > f_c^n \end{cases} \\ \bar{\sigma}_{12}^n = \sigma_{12}^{n-1} + 2\mu_c \Delta \varepsilon_{12}^n \quad \sigma_{12}^n = \begin{cases} \bar{\sigma}_{12}^n & \text{for } |\sigma_{12}^n| \leq f_s^n \\ f_s^n & \text{for } |\sigma_{12}^n| > f_s^n \end{cases} \end{array} \right. \quad (1)$$

where  $n$  is the instantaneous step of the stress-strain state;

$\varepsilon_{11}^n$ ,  $\varepsilon_{12}^n$ ,  $\Delta \varepsilon_{11}^n = \varepsilon_{11}^n - \varepsilon_{11}^{n-1}$ , and  $\Delta \varepsilon_{12}^n = \varepsilon_{12}^n - \varepsilon_{12}^{n-1}$  are the known strains and strain increments;

$f_{c0}$ ,  $f_{t0}$ , and  $f_{s0}$  are the initial compressive, tensile, and shear strengths;

$E_{c0}$  is the initial modulus of deformation,  $\nu_{c0}$  is the initial Poisson's ratio,

$\varepsilon_{c0} = \frac{f_{c0}}{E_{c0}}$  and  $\varepsilon_{t0} = \frac{f_{t0}}{E_{c0}}$  are the elastic limit strains in compression and tension;

$\varepsilon_{fc}$  and  $\varepsilon_{uc}$  are the strain limits for the perfectly plastic flow and the material softening range in compression;

and  $\varepsilon_{ut}$  is the strain limit for the material softening range in tension.

The stress states in the loading/unloading processes are determined using the degradation law with different deformation moduli for compression and tension, and the shear deformation modulus in the form:

$$E_c = \langle s_\sigma \rangle^{n-1} E_{cc} + (1 - \langle s_\sigma \rangle^{n-1}) E_{ct} \text{ and } \mu_c = \frac{E_c}{2(1 + \nu_{c0})}, \quad (2)$$

where  $\langle s_\sigma \rangle^{n-1} = (|s_\sigma| + s_\sigma) / 2$  is the Macaulay bracket function;  
 $s_\sigma = \text{sign}(\sigma_{11}^{n-1})$  is the sign of the normal stress value in the previous time step;

$E_{cc} = s_{fc} E_{c0} + (1 - s_{fc}) E_{hc}$  is the deformation modulus in compression (where  $\sigma_{11}^{n-1} > 0$ );

$E_{ct} = s_{ft} E_{c0} + (1 - s_{ft}) [\min(E_{ht}, E_{hc})]$  is the deformation modulus in tension (where  $\sigma_{11}^{n-1} < 0$ );

$E_{hc} = s_{uc} \frac{\sigma_{11}^C}{\varepsilon_{11}^C - \varepsilon_r}$  is the deformation modulus in the material softening range during compression, at the instant that unloading state  $C(\varepsilon_{11}^C, \sigma_{11}^C)$  is achieved, Figs. 1a-f;

and  $E_{ht} = s_{ut} \frac{\sigma_{11}^T}{\varepsilon_{11}^T - \varepsilon_r}$  is the deformation modulus in the material softening range during tension, at the instant that unloading state  $T(\varepsilon_{11}^T, \sigma_{11}^T)$  is achieved, Fig. 1d.

The following group of indicators is introduced:

$$s_{fc} = \begin{cases} 1 & \text{for } \varepsilon_{11}^n \leq \varepsilon_{fc} \\ 0 & \text{for } \varepsilon_{11}^n > \varepsilon_{fc} \end{cases} \text{ is an indicator of the material softening range in compression,}$$

$$s_{ft} = \begin{cases} 1 & \text{for } \varepsilon_{11}^n \geq \varepsilon_{er} \\ 0 & \text{for } \varepsilon_{11}^n < \varepsilon_{er} \end{cases}$$

is an indicator of the material softening range in tension,

$$s_{uc} = \begin{cases} 1 & \text{for } \varepsilon_{11}^n \leq \varepsilon_{uc} \\ 0 & \text{for } \varepsilon_{11}^n > \varepsilon_{uc} \end{cases}$$

is an indicator of a stressless state in compression,

$$s_{ut} = \begin{cases} 1 & \text{for } \varepsilon_{11}^n \geq \varepsilon_{ur} \\ 0 & \text{for } \varepsilon_{11}^n < \varepsilon_{ur} \end{cases}$$

is an indicator of a stressless state in tension, and

$$\varepsilon_r = \begin{cases} 0 & \text{for } -\varepsilon_{ut} \leq \varepsilon_{11}^T < 0 \\ 0 & \text{for } 0 \leq \varepsilon_{11}^C < \varepsilon_{c0} \\ \varepsilon_{11}^C - \varepsilon_{c0} & \text{for } \varepsilon_{c0} \leq \varepsilon_{11}^C \leq \varepsilon_{fc} \\ \varepsilon_{fc} - \varepsilon_{c0} & \text{for } \varepsilon_{fc} < \varepsilon_{11}^C \leq \varepsilon_{uc} \end{cases}$$

is a permanent strain. Here  $\varepsilon_{er} = \varepsilon_r - \varepsilon_{i0}$  and  $\varepsilon_{ur} = \varepsilon_r - \varepsilon_{ut}$  are the limits on the strains that describe the stress-strain relationship during tension.

During compression ( $\sigma_{11} \geq 0$ ), when the unloading occurs in the elastic phase or perfectly plastic flow phase, the strain at the beginning of unloading process  $\varepsilon_{11}^C$  does not exceed the limit on the strain value  $\varepsilon_{fc}$ . Then, the stress state is defined by the initial deformation modulus  $E_c = E_{c0}$ , Figs. 1a, b. However, during the unloading in the material softening range  $\varepsilon_{fc} < \varepsilon_{11}^C \leq \varepsilon_{uc}$ , the deformation modulus is  $E_c = E_{hc}$ , Figs. 1d, c.

The method used to model the unloading process in compression corresponds to the work of Bąk and Stolarski [1].

During tension ( $\sigma_{11} < 0$ ), if the unloading occurs in the elastic phase, the strain characterizing the beginning of unloading process  $\varepsilon_{11}^T$  does not exceed the limit on the elastic strain value  $\varepsilon_{er}$ . Then, the stress state is defined by the initial deformation modulus  $E_c = E_{c0}$ . However, if the unloading is during the material softening range [where-(2)-21], the deformation modulus is  $E_c = E_{ht}$ , Fig. 1d.

During tension ( $\sigma_{11} < 0$ ), if the unloading occurs in the first cycle ( $m = 1$ ), from the material softening phase in compression, the stress state is determined by the same deformation modulus as during compression, i.e.,  $E_{ht}^{(m=1)} = E_{hc}$ , Figs. 1d, c. But, in subsequent unloading cycles ( $m > 1$ ), the deformation modulus in tension ( $E_{ht}$ ) has the lowest value that was achieved in the previous cycles, i.e.,  $E_{ht} = E_{ht}^{(m)} = \min\{E_{ht}^{(1)}, \dots, E_{ht}^{(m-1)}\}$ , Fig. 1e.

The limit curve (Fig. 3) describes the relationships

$$\begin{cases} F_{1,k}(\sigma_{11}) = (\sigma_{11}^n - f_c^n) \langle s_\sigma \rangle^n + (\sigma_{11}^n + f_t^n) (1 - \langle s_\sigma \rangle^n) = 0 \\ F_2(\sigma_{12}) = |\sigma_{12}^n| - f_s^n = 0, \end{cases} \quad (3)$$

where  $f_c^n = s_{fc} f_{c0} + (1 - s_{fc}) f_{hc}^n$  is the current compressive strength;  
 $f_{hc}^n = s_{uc} [f_{c0} - H_c (\varepsilon_{11}^n - \varepsilon_{c0})]$  is the current compressive strength in the material softening range;

$H_c = \frac{f_{c0}}{\varepsilon_{uc} - \varepsilon_{fc}}$  is the material softening modulus in compression;

$f_t^n = s_{ft} f_{t0} + (1 - s_{ft}) f_{ht}^n$  is the current tensile strength;

$f_{ht}^n = s_{ut} [f_{t0} - H_t (\varepsilon_{11}^n - \varepsilon_{er})]$  is the current tensile strength in the material softening range;

$H_t = \frac{f_{t0}}{\varepsilon_{ur} - \varepsilon_{er}}$  is the material softening modulus in tension;

$f_s^n = \langle s_\sigma \rangle^n f_{sc} + (1 - \langle s_\sigma \rangle^n) f_{st}$  is the current shear strength;

$f_{sc} = \kappa_c s_\varepsilon f_{s0}$  is the current shear strength in compression;

$f_{st} = \kappa_c s_{ut} s_\varepsilon f_{s0}$  is the current shear strength in tension;

$s_\varepsilon = \text{sign}(\Delta \varepsilon_{12}^n)$  is the sign of the shear strain increment;

$\langle s_\sigma \rangle = (|s_\sigma| + s_\sigma) / 2$  is the Macaulay bracket function for the sign of the normal stress value in the current time step  $s_\sigma = \text{sign}(\sigma_{11}^n)$ ;

and  $\kappa_c = \frac{f_c^n}{f_{c0}}$  is the softening parameter in compression.

This concrete model can approximate a cracking mechanism (i.e., the formation, expansion, and closing of cracks) or a crushing mechanism, in both monotonic deformation and cyclic alternating deformation processes. The mechanism that causes concrete to crack or crush is a result of the assumed material softening law. It describes a gradual loss in the material's capacity until achieving a failure state.

If the cracking state is achieved during the tension process ( $\sigma_{11} < 0$ ), it does not mean that the material is irreversibly destroyed. A cracking state does not reduce the compressive strength of the concrete, and it is possible that the deformation will be reconstructed if compression closes the cracks ([where-(3)-14]), Figs. 1a, b, c, e. Previously cracked concrete loses its ability to carry tensile stresses, Fig. 1f.

During compression, destruction occurs after reaching the strain limit  $\varepsilon_{uc}$ , and is equivalent to crushing the concrete. Crushing concrete is an irreversible process, and irrevocably destroys concrete's capability to carry stresses.

The yield criterion used in our concrete model is consistent with experimental results for the reduced plane stress state. This fact was confirmed by comparing the proposed model with that described by Stolarski [20], which was calibrated with experimental results (see Fig. 3).

Figure 4 shows the proposed model of concrete in comparison with the relation presented in Eurocode 2 [26]. A function describing the relationship between the strains and stresses for short-term axial load is described in [26] by the formula:

$$\sigma_{11} = f_{c0} \frac{k\eta - \eta^2}{1 + (k-2)\eta}, \quad (4)$$

where  $\eta = \frac{\varepsilon_{11}}{\varepsilon_{c1}}$  and  $k = 1.05 E_{c0} \frac{\varepsilon_{c1}}{f_c}$  are the coefficients;



$\epsilon_{c1} = 0.7 f_{c0}^{0.31} = \epsilon_{fc}$  is the strain corresponding to the stress  $f_c$ , in the strain range  $\epsilon_{c1} \leq 0.85$ , while

$$\epsilon_{cu1} = \begin{cases} 3,5\text{‰} & \text{for } f_{c0} < 50 \text{ MPa} \\ 2.8 + 27[0.01(98 - f_{c0})]^4 & \text{for } f_{c0} \geq 50 \text{ MPa} \end{cases} \text{ — is the nominal limit strain.}$$

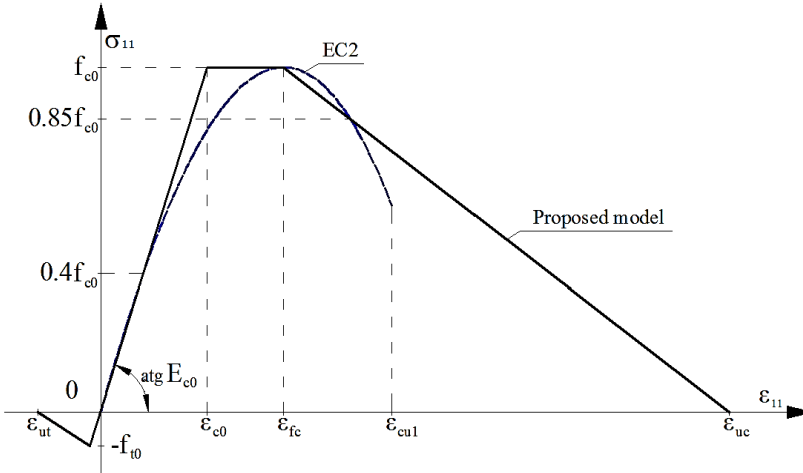


Fig. 4. The proposed concrete model in relation to EC2 [26]

Relation (4) is adapted to description of the deformability of concrete, for which the compressive strength does not exceed 90 MPa.

Table 1 contains the strain-stress parameters of the proposed model adapted to the recommendations of the EC2.

Limit strains for the range of tension are determined according to the formula:

$$\epsilon_{uc} = \alpha_{ct} \cdot \epsilon_{uc}, \tag{5}$$

where  $\alpha_{ct} = \frac{f_{t0}}{f_{c0}}$  dimensionless coefficient expressing the relationship between the tensile and compressive strength of concrete.

TABLE 1

Proposed concrete model parameters related to the EC2 [26]

$f_{c0}$	$E_c$	$\epsilon_{c0}$	$\epsilon_{fc}$	$\epsilon_{uc}$	$\alpha_t$
MPa	GPa	‰			
< 50	23 ÷ 34	0.85 ÷ 1.48	1.7 ÷ 2.4	7.7 ÷ 7.4	0.08 ÷ 0.11
55 ÷ 90	35 ÷ 43	1.55 ÷ 2.1	2.42 ÷ 2.82	7.3 ÷ 6.66	0.05 ÷ 0.07

To consider the specificity of high-strength concrete, the relationships developed by Collins et al. [5], was included in modelling the material behaviour.

The stress function for concrete takes the form

$$\sigma_{11} = k_3 f'_c \frac{\varepsilon_{11}}{\varepsilon'_c} \frac{n}{n-1 + \left(\frac{\varepsilon_{11}}{\varepsilon'_c}\right)^{nk}} \quad (6)$$

where  $k_3 = 0.6 + \left(\frac{10}{f'_c}\right) \leq 0.85$  is the reducing factor for the concrete strength;

$n = 0.8 + \frac{f'_c}{17}$  is a factor that depends on the concrete strength;

$$k = \begin{cases} 0.67 + \frac{f'_c}{62} & \text{if } \frac{\varepsilon_{11}}{\varepsilon'_c} > 1 \\ 1.0 & \text{if } \frac{\varepsilon_{11}}{\varepsilon'_c} \leq 1 \end{cases} \quad \text{is a factor that depends on the strength}$$

and strain state of the concrete, and  $\varepsilon'_c = \frac{f'_c}{E_c} \frac{n}{n-1} = \varepsilon_{fc}$  is a strain that corresponds to the concrete's compressive strength.

The deformation modulus of concrete was determined using the formulae from the work [5] i.e.,

$$E_c = 3320\sqrt{f'_c} + 6900. \quad (7)$$

The proposed model of concrete behaviour was adjusted in accordance with the criterion of the concrete strength given by Collins et al., as shown in Fig. 5.

The proposed model has been adapted to model features presented in EC2 [26] and can be modified in the range of limit strains for the perfectly plastic flow phase and for the material softening phase. In the proposed model, the slope angle of the softening branch is determined by the material softening modulus  $H_c$  defined by the limit strains' values. In practice, the softening branch is the straight line passing through two specific points. The first one indicates the end of the perfectly plastic flow phase ( $\varepsilon_{11} = \varepsilon_{fc}$ ;  $\sigma_{11} = f_{c0}$ ), and the second one corresponds to the stress value  $\sigma_{11} = 0.85 f_{c0}$  in the softening phase. However, it is possible to modify the range of strains for the perfectly plastic flow phase, which causes changing the position of the first point, while the position of the second point in the plane  $\varepsilon_{11} - \sigma_{11}$  is rather unchangeable. Increasing the limit strain value  $\varepsilon_{fc}$  will extend the perfectly plastic flow phase and simultaneously will reduce the length of the material softening phase, by reducing the limit strain value  $\varepsilon_{uc}$ . This modification will influence in the more rapid course of the material failure process which will occur at a lower strain

value. In an analogous manner, the proposed concrete model has been adapted to the features of the model proposed by Collins et al [5].

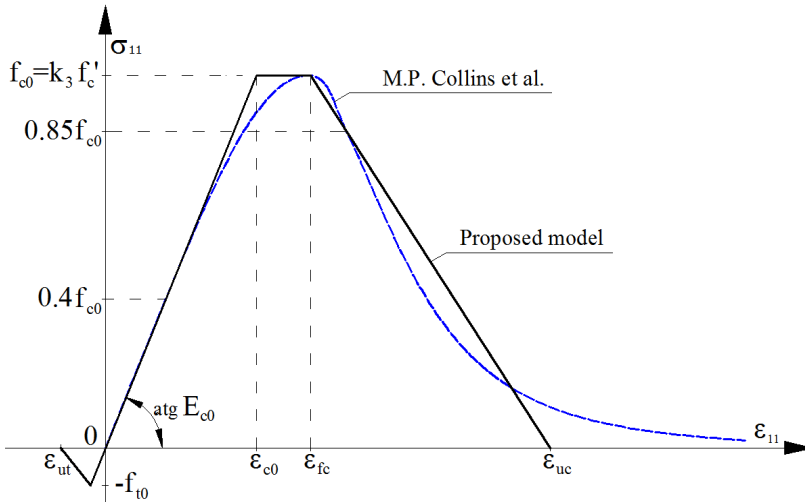


Fig. 5. Modification of the concrete model proposed by Collins et al. [5]

In turn, the stress and strain parameters of the proposed model taking into account the modifications in accordance with the criterion of Collins et al. [5], are shown in Table 2.

TABLE 2  
Parameters of the proposed model taking into account the modifications in accordance with [5]

$f_{c0}$	$E_{c0}$	$\epsilon_{c0}$	$\epsilon_{fc}$	$\epsilon_{uc}$	$\alpha_t$
MPa	GPa	‰			
50 ÷ 90	24.0 ÷ 29.5	1.47 ÷ 1.65	2.25 ÷ 2.8	4.2 ÷ 4.9	0.07-0.09

### 3. Conclusions

The paper presents the theoretical model of deformation of concrete that allows for the description of the most important non-linear properties of the material, such as softening in compression and in tension, and degradation of deformation modulus during the loading and re-loading processes. The main advantages of the model are the following:

- description of the elastic reaching of the initial strength, perfectly plastic flow and material softening in compression as well as of the elastic reaching of the initial strength and material softening in tension;

- description of the failure processes, i.e., cracking and crushing, modelled as the stressless states achieved in the material softening process, at tension and compression, respectively;
- description of material behaviour in the unloading and re-loading process, taking into account the degradation of deformation modulus, which is especially important in post-critical effort analysis of the eccentrically compressed reinforced concrete elements;
- the possibility to adapt the model to the specific behaviour of concrete of different strengths, including high-strength concrete, through the appropriate selection of model parameters;
- low degree of complexity, which allows easy use in the computational program and possible modifications.

The following simplifications could be seen as deficiencies of the proposed model:

- the direct, single phase, and linear transition from the elastic to the plastic flow phase in compression, which causes a slight over-stiffening of the computational model of the structural element. As the result, the reduction of the displacements is observed in the elastic effort phase of the compressed structural element. The influence of this phenomenon decreases with the reduction of the impact of compression on the effort of the reinforced concrete structural element, and for the bending its influence is negligible;
- rectangular shape of the limit curve in the shear and normal stresses plane, which, however, does not substantially affect the description of the effort of reinforced concrete structural elements especially subjected to compression;
- inability to describe the behaviour of unreinforced concrete elements.

The proposed model is very effective in the effort analysis of bending and eccentrically compressed reinforced concrete structural elements. This analysis requires to take into account both, the material softening for compression and tension, and degradation of the deformation modulus due to appearance of the local unloadings.

The model was the basis of effort analysis of the bending and eccentrically compressed reinforced concrete elements, the results of which were presented in the works of Szcześniak and Stolarski [22, 23, 24].

The model can be subjected to further modifications in the range of description of the plastic hardening for compression, as well as in the description of the nonlinear nature of the relationship between shear and normal stresses. The including of the dynamic strength criterion into the material model, will allow the description of the dynamic properties of concrete in the effort analysis of reinforced concrete elements under dynamic loading.

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## A. SZCZEŚNIAK, A. STOLARSKI

### Uproszczony model betonu do analizy elementów żelbetowych

**Streszczenie.** W pracy przedstawiono model betonu jako materiału sprężysto-plastycznego z uwzględnieniem osłabienia i degradacji modułu odkształcenia. Przyjęto założenie zredukowanego, płaskiego stanu naprężenia dla ściskania/rozciągania i ścinania. Model betonu pozwala na opis sprężystego osiągnięcia początkowej wytrzymałości, idealnego płynięcia plastycznego i osłabienia materiałowego przy ściskaniu oraz sprężystego osiągnięcia wytrzymałości i osłabienia materiałowego przy rozciąganiu. Procesy zniszczenia, tj. zarysowania i zmiażdżenia, modelowane są jako stany beznaprężeniowe osiągnięte w procesie osłabienia materiałowego, odpowiednio przy rozciąganiu i ściskaniu. Proponowany model odkształcenia betonu umożliwia efektywny opis najistotniejszych właściwości nieliniowego zachowania materiału i może być stosowany do analizy mechanizmu zniszczenia prętowych, żelbetowych elementów konstrukcyjnych.

**Słowa kluczowe:** model betonu, elementy żelbetowe, nieliniowość geometryczna, zginane elementy konstrukcyjne, mimośrodowo ściskane elementy konstrukcyjne

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