



TRANSPORTATION PROBLEM BY MONALISHA'S APPROXIMATION METHOD FOR OPTIMAL SOLUTION (MAMOS)

Monalisha Pattnaik

Utkal University, Bhubaneswar, India

ABSTRACT. Background: This paper finds initial basic feasible solution and optimal solution to the transportation problem by using MAM's (Monalisha's Approximation Method).

Methods: Using the concept of comparison of the transportation problem by other methods of solution, the paper introduces a very effective method in terms of cost and time for solving these problems. This paper extends transportation problem by using different method of obtaining both initial basic feasible solution and optimal solution simultaneously other than existing methods.

Results and conclusions: It is presented a cost saving and less time consuming and accurate method for obtaining the best optimal solution of the transportation problem. With the problem assumptions, the optimal solution can still be theoretically solved using the existing methods. Finally, numerical examples and sensitivity analysis are presented to illustrate the effectiveness of the theoretical results, and to gain additional managerial insights.

Key words: Transportation Problem, Monalisha's Approximation Method for Optimal Solution (MAMOS), Comparative Analysis, Sensitive Analysis

INTRODUCTION

Over the last few years, more and more manufacturers had applied the optimization technique most frequently in linear programming to solve real-world problems and there it is important to introduce new tools in the approach that allow the model to fit into the real world as much as possible. Any linear programming model and transportation model representing real-world situations involves a lot of parameters whose values are assigned by experts' opinion, and in the conventional approach, they are required to fix an exact value to the aforementioned parameters. However, both experts and the decision maker frequently do not precisely know the value of those parameters. If exact values are suggested these are only statistical inference from past data and their stability is doubtful, so the

parameters of the problem are usually defined by the decision maker in a certain space. In the mean time typical method helps for obtaining the optimal solution of the transportation model then the post optimal solution gives the managerial implications for the given problem.

Two significant questions may be found in these kinds of problems: how to handle the relationship between the parameters, and how to find the optimal values for the objective function. The answer is related to the problem of numbers.

Lai and Hwang [1992] considered the situations where all parameters are in fuzzy number. (Lai and Huang, 1992) assume that the parameters have a triangular possibility distribution. Bazaraa, Jarvis and Sherali [1990] define linear programming problems with fuzzy numbers and simplex method is used for

finding the optimal solution of the fuzzy problem. Pattnaik [2012] presented several linear and nonlinear inventory models. Swarup, Gupta and Mohan [2006] explain the method to obtain sensitivity analysis or post optimality analysis of the different parameters in the linear programming problems.

In fact, in order to make transportation problem and linear programming more effective, the certainties that happen in the real world cannot be neglected. Those certainties are usually associated with per unit cost of the product, product supply, customer demand and so on.

The remainder of this paper is organized as follows. In Section 2, it is introduced transportation model. In Section 3, Monalisha's Approximation Method (MAM'S) Model is introduced and the steps are explained. In Section 4, applications are presented to illustrate the development of the model. Comparative analysis is developed in Section 5. The sensitivity analysis is carried out in Section 6 to observe the changes in the optimal solution. Finally Section 7 deals with the summary and the concluding remarks.

TRANSPORTATION MODEL

Minimize $z = \sum_{i=1}^m \sum_{j=1}^n x_{ij} c_{ij}$ subject to constraints:

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \dots, m \text{ (supply constraint)}$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \dots, n \text{ (demand constraint)}$$

$$\text{and } \forall x_{ij} \geq 0$$

MONALISHA'S APPROXIMATION METHOD (MAM'S) MODEL

STEPS

Step 1. Determine the cost table from the given problem.

- (i) Examine whether total demand equals total demand. If yes, go to step 2.
- (ii) If not, introduce a dummy row/column having all its cost elements as zero and supply/ demand as the (+ve) difference of supply and demand.

Step 2. Locate the smallest element in each row of the given cost matrix and then subtract the same from each element of that row.

Step 3. In the reduced matrix obtained in step 2, locate the smallest element of each column and then subtract the same from each element of that column.

Step 4. For each row of the transportation table identify the smallest and the next - to - smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.

Step 5. Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference correspond to i th row and let 0 be in the i th row. Allocate the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the (i, j) th cell and cross off either the i th row or the j th column in the usual manner.

Step 6. Recompute the column and row differences for the reduced transportation table and go to step 5. Repeat the procedure until all the rim requirements (the various origin capacities and destination requirements are listed in the right most outer column and the bottom outer row respectively) are satisfied.

Short Steps of MAM's Method to obtain IBFS and OS of Transportation Problem

- Row minimization/ row reduced matrix
- Column reduced matrix
- Evaluate penalty
- Highest penalty
- At zero do the allocation (corresponding)
- Draw the straight line
- Again draw the new table
- Continue the step from 1
- Until all are not adjusted
- Cost effective and consuming less time than VAM and other methods

NUMERICAL EXAMPLES

Example 1

	D1	D2	D3	D4	Supply (S)
O1	2	3	11	7	6
O2	1	0	6	1	1
O3	5	8	15	9	10
Demand (d)	7	5	3	2	17

Solve the transportation problem of the given data.

Solution

Using MAM'S Method this problem is solved.

Step 1. Determine the cost table from the given problem.

(i) Here total demand equals total supply, go to step 2.

	D1	D2	D3	D4	Supply
O1	2	3	11	7	6
O2	1	0	6	1	1
O3	5	8	15	9	10
Demand	7	5	3	2	17

Step 2. Locating the smallest element in each row of the given cost matrix and then subtracting the same from each element of that row.

	D1	D2	D3	D4	Supply
O1	0	1	9	5	6
O2	1	0	6	1	1
O3	0	3	10	4	10
Demand	7	5	3	2	17

Step 3. In the reduced matrix obtained in step 2, locating the smallest element of each column and then subtracting the same from each element of that column.

	D1	D2	D3	D4	Supply
O1	0	1	3	4	6
O2	1	0	0	0	1
O3	0	3	4	3	10
Demand	7	5	3	2	17

Step 4. For each row of the transportation table identifying the smallest and the next - to - smallest costs. Determining the difference between them for each row in the transportation table. Displaying them alongside the transportation table by enclosing them in parenthesis against the respective rows of the transportation table. Similarly computing the

differences for each column of the transportation table.

	D1	D2	D3	D4	Supply	Penalty
O1	0	1	3	4	6	(1)
O2	1	0	0	0	1	(0)
O3	0	3	4	3	10	(3)
Demand	7	5	3	2		
Penalty	(0)	(1)	(3)	(3)		

Step 5. Identifying the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference correspond to i th row and let 0 be in the i th row. Allocating the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the (i, j) th cell and cross off either the i th row or the j th column in the usual manner.

	D1	D2	D3	D4	Supply	Penalty
O1	0	0	5	0	1	(0)
O3	0	2	1	0	10	(0)
Demand	7	5	2	2		
Penalty	(0)	(2)	(1)	(1)		

Step 6. Recomputing the column and row differences for the reduced transportation table and go to step 5. Repeating the procedure until all the rim requirements (the various origin capacities and destination requirements are listed in the right most outer column and the bottom outer row respectively) are satisfied.

	D1	D3	D4	Supply	Penalty
O1	0	0	1	1	(0)
O3	0	1	0	10	(0)
Demand	7	2	2		
Penalty	(0)	(1)	(1)		

	D1	D3	D4	Supply	Penalty	
O3	0	7	1	1	10	(1)
	7	1	2			

Optimal Solution

	D1	D2	D3	D4	Supply
O1	2	3	5	1	7
O2	1	0	6	1	1
O3	5	8	15	1	9
Demand	7	5	3	2	17

Thus the optimal allocation is:

$$x_{12} = 5, x_{13} = 1, x_{14} = 5, x_{23} = 1, x_{33} = 1 \text{ and } x_{34} = 2.$$

The transportation cost according to the MAM's method is:

$$\begin{aligned} \text{Total cost} = TC &= (3 \times 5) + (11 \times 1) + (7 \times 5) \\ &+ (6 \times 1) + (15 \times 1) + (9 \times 2) \\ &= 100 \end{aligned}$$

Total minimum cost will be Rs. 100.

Example 2

	D1	D2	D3	D4	Supply (S)
O1	19	30	50	10	7
O2	70	30	40	60	9
O3	40	8	70	20	18
Demand (d)	5	8	7	14	34

Solve the transportation problem of the given data.

Solution

Using MAM's method the optimal solution is obtained.

Step 1. Determine the cost table from the given problem.

(i) Here total demand equals total supply, go to step 2.

	D1	D2	D3	D4	Supply
O1	19	30	50	10	7
O2	70	30	40	60	9
O3	40	8	70	20	18
Demand	5	8	7	14	34

Step 2. Locating the smallest element in each row of the given cost matrix and then subtracting the same from each element of that row.

	D1	D2	D3	D4	Supply
O1	9	20	40	0	7
O2	40	0	10	30	9
O3	32	0	62	12	18
Demand	5	8	7	14	

Step 3. In the reduced matrix obtained in step 2, locating the smallest element of each column and then subtracting the same from each element of that column.

	D1	D2	D3	D4	Supply
O1	0	20	30	0	7
O2	31	0	0	30	9
O3	23	0	52	12	18
Demand	5	8	7	14	

Step 4. For each row of the transportation table identifying the smallest and the next - to - smallest costs. Determining the difference between them for each row. Displaying them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly computing the differences for each column.

	D1	D2	D3	D4	Supply	Penalty	
O1	0	20	30	0	7	(0)	
O2	31	0	0	7	30	9	(0)
O3	23	0	52	12	18	(12)	
Demand	5	8	7	14			
Penalty	(23)	(0)	(30)	(12)			

Step 5. Identifying the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference correspond to ith row and let 0 be in the ith row. Allocating the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the (i, j)th cell and cross off either the ith row or the jth column in the usual manner.

	D1	D2	D4	Supply	Penalty	
O1	0	20	0	7	(0)	
O2	31	0	2	30	2	(30)
O3	23	0	12	18	(12)	
Demand	5	8	14			
Penalty	(23)	(0)	(12)			

Step 6. Recomputing the column and row differences for the reduced transportation table and go to step 5. Repeating the procedure until all the rim requirements (the various origin capacities and destination requirements are listed in the right most outer column and the bottom outer row respectively) are satisfied.

	D1	D2	D4	Supply	Penalty	
O1	0	5	20	0	7	(0)
O3	23	0	12	18	(12)	
Demand	5	6	14			
Penalty	(23)	(20)	(12)			

	D2	D4	Supply	Penalty		
O1	20	0	2	2	(20)	
O3	0	6	12	12	18	(12)
Demand	6	14				
Penalty	(20)	(12)				

Optimal Solution

	D1	D2	D3	D4	Supply		
O1	19	5	30	50	10	2	7
O2	70	30	2	40	7	60	9
O3	40	8	6	70	20	12	18
Demand	5	8	7	14			34

Thus the optimal allocation is:

$$x_{11} = 5, x_{14} = 2, x_{22} = 2, x_{23} = 7, x_{32} = 6, x_{34} = 12.$$

The transportation cost according to the MAM's method is:

Total cost

$$TC = (19 \times 5) + (10 \times 2) + (30 \times 2) + (40 \times 7) + (8 \times 6) + (20 \times 12) = 743$$

Total minimum cost will be Rs. 743.

COMPARATIVE ANALYSIS

Table 1 explains the comparative analysis of the minimum total transportation cost of MAMOS with VAM and MODI methods. MAMOS method of obtaining the minimum total transportation cost is the best method in comparison to the other methods as the total cost is optimum with exercising less steps.

Table 1. Comparative Analysis of MAMOS with VAM and MODI
 Tabela 1. Analiza porównawcza MAMOS z VAM i MODI

Examples	Minimum	Method							% change of VAM and MAM	% Change of MODI and MAM
		Row minima	Column minima	Matrix minima	North West Corner Rule (NWCR)	Vogel's Approximation	MODI	Monalisha's Approximation method (MAM) for IBFS and OS		
1	Total Cost (Rs.)	112	116	112	116	102	100	100	2	0
2	Total Cost (Rs.)	1110	779	814	1015	779	743	743	36	0

Table 2. Sensitivity Analysis of Optimal Total Transportation Cost with respect to the Parameters such as Supply and Demand (Example 1)

Tabela 2. Analiza wrażliwości optymalnego kosztu transportu z uwzględnieniem paramaterów dostaw i popytu (Przykład 1)

Variable	Value	Iteration	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}	TC	% Change in TC
S_1	5	6	0	5	0	0	0	0	1	0	6	0	2	2	99	1
d_1	6		0	4	0	0	0	0	1	0	5	1	2	2	99	1
S_1	4	6	0	3	0	0	0	0	1	0	4	2	2	2	99	1
d_1	5		0	3	0	0	0	0	1	0	4	2	2	2	99	1
S_1	3	6	0	5	3	0	0	0	0	1	9	0	0	1	103	3
d_1	4		0	5	3	0	0	0	0	1	9	0	0	1	103	3
S_2	2	6	0	6	0	0	0	0	2	0	7	0	1	2	98	2
d_2	6		0	6	0	0	0	0	2	0	7	0	1	2	98	2
S_2	2	7	0	5	1	0	0	0	2	0	7	0	1	2	106	6
d_3	4		0	5	1	0	0	0	2	0	7	0	1	2	106	6
S_3	9	7	0	5	1	0	0	0	1	0	6	0	1	2	95	5
d_1	6		0	5	1	0	0	0	1	0	6	0	1	2	95	5
S_3	8	6	0	3	3	0	0	0	0	1	7	0	0	1	87	13
d_2	3		0	3	3	0	0	0	0	1	7	0	0	1	87	13
S_3	8	6	0	5	1	0	0	0	0	1	7	0	0	1	71	29
d_3	1		0	5	1	0	0	0	0	1	7	0	0	1	71	29
S_3	9	7	0	5	1	0	0	0	1	0	7	0	1	1	91	9
d_4	1		0	5	1	0	0	0	1	0	7	0	1	1	91	9

SENSITIVITY ANALYSIS

Table 2 represents the sensitivity analysis of the optimum total transportation cost with respect to the parameters such as demand and supply of Example 1. TC is less sensitive to the parameters (S_1, d_1) and (S_2, d_2) . TC is moderately sensitive to the parameters (S_2, d_3) and (S_3, d_1) and TC is more sensitive to the parameters (S_3, d_2) , (S_3, d_1) and (S_3, d_4) .

CONCLUSIONS

The main contribution of this paper is to deriving the optimal transportation cost by using Monalisha's Approximation Method with less steps in comparison to the VAM and MODI methods. Based on the optimal solution it allows taking a decision interactively with the decision maker in decision space. The decision maker also has additional information about the availability of the violation of requirement factor and availability factor in the constraints, and about the compatibility of the cost of the solution with his wishes for the values of the objective function which extend the classical transportation models with no sensitivity analysis in the past. By using LINGO 13.0 version software sensitivity analysis is evaluated for obtaining the managerial implications in decision space.

These analysis of the results are established which present a number of insights into the economic behavior of the firms, and can serve as the basis for empirical study in the future. Thus, there are possible extensions to improve our model. The decision maker can intervene in all the steps of the decision process which makes our approach very useful to be applied in a lot of real-world problems where the information is uncertain with nonrandom, like environmental management, marketing, production area.

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METODA MONALISHY OPTYMALIZACJI PROBLEMU TRANSPORTOWEGO (MAMOS)

STRESZCZENIE. Wstęp: W pracy zostało przedstawione rozwiązanie problemu transportowego przy zastosowaniu MAM (metody przybliżeń Monalishy).

Metody: Poprzez porównanie rozwiązania problemu transportowego z innymi możliwymi rozwiązaniami, została zaprezentowana metoda efektywnie uwzględniająca takie czynniki jak koszt i czas. Metoda ta rozwiązywania problemu transportowego stosuje inne podejście dla uzyskania rozwiązania bazowego, jaki i optymalnego w porównaniu do innych istniejących metod.

Wyniki i wnioski: Metoda ta minimalizuje koszty i czas realizacji dla uzyskaniu optymalnego rozwiązania problemu transportowego. Rozwiązanie te może być teoretycznie osiągnięte przy zastosowaniu innych metod pod warunkiem pewnych założeń. Zaprezentowane przykłady liczbowe oraz analiza wrażliwości przybliża efektywność teoretycznych rezultatów i możliwość praktycznego ich zastosowania.

Słowa kluczowe: problem transportowy, optymalizacyjna metoda przybliżeń Monalishy, analiza porównawcza, analiza wrażliwości

DIE MONALISHA-METHODE FÜR DIE OPTIMIERUNG DES TRANSPORTPROBLEMS (MAMOS)

ZUSAMMENFASSUNG. Einleitung: In der vorliegenden Arbeit wurde eine Lösung des Transportproblems unter Anwendung der MAM (der Monalisha-Approximationsmethode) dargestellt.

Methoden: Mithilfe des Vergleiches einer Lösung des bestehenden Transportproblems mit anderen möglichen Problemlösungen wurde eine Methode, die die Faktoren Kosten und Zeit effektiv berücksichtigt, projiziert. Die betreffende Methode für die Lösung von Transportproblemen nimmt eine andere Vorgehensweise für die Erzielung sowohl einer grundlegenden, als auch einer optimalen Lösung im Vergleich mit anderen bestehenden Methoden, in Anspruch.

Ergebnisse und Fazit: Die betreffende Methode reduziert weitgehend die Kosten und Ausführungszeit zwecks der Erzielung einer optimalen Lösung des gegebenen Transportproblems. Die Lösung kann theoretisch erzielt werden bei Anwendung von anderen Methoden und bestimmten Annahmen. Die dargestellten Zahlenbeispielen und die Empfindlichkeitsanalyse projizieren die Effizienz der theoretischen Resultate und die Möglichkeit deren praktischer Anwendung.

Codewörter: Transportproblem, Monalisha-Approximationsmethode, Vergleichsanalyse, Empfindlichkeitsanalyse.

Monalisha Pattnaik
PG. Dept. of Business Administration
Utkal University, Bhubaneswar
751004, **India**
e-mail: monalisha_1977@yahoo.com