Myron CHERNETS*

A GENERALIZED METHOD FOR PREDICTING CONTACT STRENGTH, WEAR, AND THE LIFE OF INVOLUTE CONICAL SPUR AND HELICAL GEARS: PART 1. SPUR GEARS

UOGÓLNIONA METODA PROGNOZOWANIA WYTRZYMAŁOŚCI STYKOWEJ, ZUŻYCIA ORAZ TRWAŁOŚCI STOŻKOWEJ EWOLWENTOWEJ PRZEKŁADNI O ZĘBACH PROSTYCH ORAZ UKOŚNYCH: CZ. 1. PRZEKŁADNIA O ZĘBACH PROSTYCH

Key words: involute conical spur gear, tooth correction, contact and tribo-contact pressures, tooth wear, gear durability.

Abstract The paper presents the results of research undertaken to determine maximum contact pressures, wear, and the life of involute conical spur gear, taking into account gear height correction, tooth engagement, and wear-generated changes in the curvature of their involute profile. Moreover, we have established the following: (a) the initial contact pressures are higher in the internal section with double-single-double tooth engagement; (b) the highest values can be observed at the entry of single tooth engagement; (c) the maximal tooth wear of the wheels in the frontal section will be less than half of that in the internal section; (d) profile shift coefficients have an optimum at which the highest gear life is possible; and (e) gear life in the internal section will be less than half of that the frontal section. The calculations were made for a reduced cylindrical gear using a method developed by the authors. The effect of applied conditions of tooth engagement in the frontal and internal sections of a cylindrical gear ring is shown graphically. In addition, optimal correction coefficients ensuring the longest possible gear life are determined.

Słowa kluczowe: stożkowa ewolwentowa przekładnia o zębach prostych, korekcja technologiczna zazębienia, naciski kontaktowe oraz tribokontaktowe, zużycie zębów, trwałość przekładni.

Streszczenie W artykule predstawiono rezultaty oszacowania maksymalnych nacisków stykowych, zużycia oraz trwałości stożkowej przekładni o zębach prostych z uwzględnieniem korekcji technologicznej uzębienia, parzystości zazębienia zębów oraz zmiany krzywizny ich zarysów ewolwentowych wskutek zużycia. Ustalono, że: a) największe początkowe naciski kontaktowe są w przekroju wewnętrznym przy realizacji zazębienia dwu-jedno-dwuparowego; b) największe ich wartości powstają na wejściu w zazębienie jednoparowe; c) maksymalne zużycie zębów kół w przekroju czołowym będzie ponad dwukrotnie mniejsze niż w przekroju wewnętrznym; d) współczynniki korekcji profilu mają optimum, przy którym trwałość przekładni będzie najwyższa; e) trwałość przekładni w przekroju wewnętrznym będzie ponad dwukrotnie niższa niż w przekroju czołowym. Obliczenia przekładni stożkowej przeprowadzono jako zredukowanej przekładni walcowej przy użyciu metody opracowanej przez autora. Ustalone prawidłowości wpływu wymienionych warunków interakcji zębów w przekroju czołowym oraz wewnętrznym wieńca kół stożkowych przedstawiono w postaci graficznej. Ponadto określono optymalne współczynniki korekcji zapewniające maksymalnie możliwą trwałość przekładni.

INTRODUCTION

Conical involute gears are widely used in machine design. For a gear with no tooth profile correction, the maximum contact pressures generated during tooth engagement are determined in compliance with a relative ISO standard. However, no such methods have been developed for conical gears with corrected tooth profiles though they are quite widespread. Also, the methods for assessing wear and life of spur and helical gears reported in the literature **[L. 1–12]** can only be applied to gears with uncorrected profiles. But the engagement correction provides the increase of teeth

* Lublin University of Technology, Mechanical Engineering Faculty, Institute of Technological Systems of Information, Nadbystrzycka 36, 20-618 Lublin, Poland, tel.: (81) 538-42-76, e-mail: m.czerniec@pollub.pl.

bearing capacity, the lessening of their wear rate, and the increasing of transmission's durability as a result of teeth's form changes while cutting. The preliminary results of investigations conducted using methods which take into account gear tooth correction, changes in tooth profile curvature due to wear, and the number of engaged tooth pairs are reported only in the works **[L. 14–16]**. This study was undertaken for conical gears having teeth with involute profiles with respect to the effect of the above factors and tooth interaction conditions on variations in the maximum contact pressures and tribotechnical parameters that are completely new results for conic gears. The calculations for a conical gear were made in the same way as for a reduced cylindrical gear with frontal and internal modules of conical engagement made variable over a tooth length $m_{\min} \leq m \leq m_{\max}$ **[L. 13, 14]**. To solve the problem, we applied methods for assessing contact strength, wear, and the life of cylindrical spur and helical gears **[L. 15–20]**. This approach to the calculation of conical gears, which are the equivalent of cylindrical gears, completely conforms to ISO standards.

METHOD FOR SOLVING THE PROBLEM

To investigate kinetics of teeth wear in engagement, a mathematical model of sliding tribo-process was used, which is described with the system of linear differential equations **[L. 17].**

$$\frac{1}{v}\frac{dh_k}{dt} = \Phi_k^{-1}(\tau), \quad k = 1;2 \tag{1}$$

where
h linear wear of material of element in the tribosystem;
t is the time of wear;
ν is the sliding velocity;
$\Phi(\tau)$ function of wear resistance of the materials;
τ is the specific friction force;
k is the number of element in the tribosystem.

Experimental values of wear-resistance function are approximated by relation

$$\Phi_k(\tau) = C_k\left(\frac{\tau_S}{\tau}\right)^{m_k} \tag{2}$$

where
C_k, m_k are the indicators of resistance to wear of tribological pair materials;
τ_S shear strength of the material;
$\tau = fp$; $\tau_S = R_{0,2}/2$; $R_{0,2} = 0{,}7\,R_m$;
f is the sliding friction factor;
p is the maximum tribo-contact pressure;
$R_{0,2}$ conventional yield strength of the material in tension;
R_m the immediate tensile strength of material.

The wear-resistance functions $\Phi_i(\tau_i)$ of teeth materials are determined in the following way:

$$\Phi_i(\tau_i) = L / h_i\,,$$

where
h_i linear wear of material samples;
L friction length;
$i = 1, 2, 3, \ldots$ loading ratios.

Taking into account relation (2) after separation of variables and system integration (1) on condition that $\tau = fp = \text{const}$, the following will arise **[L. 17]**:

$$t_k = \frac{C_k}{v}\left(\frac{\tau_S}{\tau}\right)^{m_k} h_k \tag{3}$$

Then the function of the linear wear of teeth in arbitrary point j of working surface over a period of t_j of their interaction is as follows:

$$h_k = \frac{vt_k}{C_k}\left(\frac{\tau}{\tau_S}\right)^{m_k} \tag{4}$$

The linear wear of the gear teeth h'_{kjn}, at any point j of the profile in the tooth engagement time t'_{jh} is determined using the following formula **[L. 20]**:

$$h'_{kjn} = \frac{v_j t'_{jh}(fp_{jh\max})^{m_k}}{C_k(0.35R_m)^{m_k}} \tag{5}$$

where
$j = 0, 1, 2, 3\ldots, s$ are the contact points of the teeth profiles;
$j = 0$, $j = s$, are, respectively, the first and last point of tooth engagement;
$t'_{jh} = 2b_{jh}/v_0$ is the time of tooth wear at displacement along the profile of a j-th contact point over the contact area width $2b_{jh}$;
$v_j = v$ is the sliding velocity at a j-th point of the gear profile;
$p_{jh\max}$ is the maximum tribocontact pressure (at tooth wear) at a j-th contact point;
$v_0 = \omega_1 r_1 \sin\alpha_t$ is the velocity of contact point travel along the tooth profile;
ω_1 is the angular velocity of the pinion;
r_1 is the pitch radius of the pinion;
α_t is the pressure angle of engaged teeth.

Tooth wear causes an increase in the curvature radii of tooth profiles, which leads to a decrease in initial maximum contact pressures $p_{j\max}$ and contact area width $2b_j$ at every j-th point of contact. The values of $p_{jh\max}$ and $2b_{jh}$ are calculated in accordance with the modified Hertz equations, where the changeable in the result of wear composite radius of curvature ρ_{jh} is introduced:

$$p_{jh\max} = 0.564\sqrt{N'\theta/\rho_{jh}}$$
$$2b_{jh} = 2.256\sqrt{\theta N'\rho_{jh}} \tag{6}$$

where
$N' = N / l_{\min} w$; $N = 9550 P / r_1 n_1 \cos \pm_t$ is the engagement force;
P – is the power on the drive shaft (pinion);
l_{min} is the minimum length of the contact line;
w is the number of engaged tooth pairs;
$\theta = (1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2$;
E, ν are the Young modulus and Poisson's ratios of toothed gear materials, respectively;
n_1 is the number of revolutions of the drive shaft;
$\rho_{jh} = \frac{\rho_{1jh}\rho_{2jh}}{\rho_{1jh} + \rho_{2jh}}$ is the reduced radius of curvature of the gear profile subjected to changes due to wear in a normal section;
ρ_{1jh}, ρ_{jh} are the changeable radii of curvature of the pinion and gear teeth profiles, respectively.

In operation, due to the gear's wear, the initial curvature radii ρ_{1j}, ρ_{2j} **[L. 17, 18]** of the gear profiles and the reduced curvature radius ρ_j increase.

The reduced radius of curvature of the cylindrical gear involute profile is:

$$\rho_j = \frac{\rho_{1j}\rho_{2j}}{\rho_{1j} + \rho_{2j}} \tag{7}$$

The formulas for calculating the radii of curvature of the modified pinion and gear profiles of the cylindrical gear at j-th point of contact are **[L. 18]**:

$$\rho_{1j} = \frac{\rho_{t1j}}{\cos\beta_b}, \quad \rho_{2j} = \frac{\rho_{t2j}}{\cos\beta_b}$$

where

$$\beta_b = \arctan\left(\tan\beta\cos\alpha_t\right), \quad \alpha_t = \arctan\left(\frac{\tan\alpha}{\cos\beta}\right),$$

$$\rho_{t1j} = r_{b1}\tan\alpha_{t1j}, \ \rho_{t2j} = r_2\sqrt{\left(r_{2j}/r_2\right)^2 - \cos^2\alpha_t},$$

$$\alpha_{t1j} = \arctan\left(\tan\alpha_{t10} + j\Delta\varphi\right),$$

$$\alpha_{t1s} = \arctan\sqrt{\left(r_{1s}/r_1\right)^2 - \cos^2\alpha_t},$$

$$\alpha_{t2j} = \arccos\left[\left(r_2/r_{2j}\right)\cos\alpha_t\right],$$

$$r_{b1} = r_1\cos\alpha_t \quad r_1 = mz_1/2\cos\beta,$$

$$r_{b2} = r_2\cos\alpha_t \quad r_2 = mz_2/2\cos\beta,$$

$$\tan\alpha_{t10} = \left(1+u\right)\tan\alpha_t - \frac{u}{\cos\alpha_t}\sqrt{\left(r_{20}/r_2\right)^2 - \cos^2\alpha_t},$$

$$r_{a2} = r_2 + m \quad r_{20} = r_{a2} - r \quad r = 0.2m,$$

$$r_{2j} = \sqrt{a_W^2 + r_{1j}^2 - 2a_W r_{1j}\cos\left(\alpha_t - \alpha_{t1j}\right)},$$

$$r_{1j} = r_1\cos\alpha_t/\cos\alpha_{t1j} \quad a_W = \left(z_1 + z_2\right)m/2\cos\beta,$$

β is helix angle of the teeth,
α = 20ºC is the pressure angle of engaged teeth,
r_1, r_2 are the radii of pitch circles of the pinion and gear, respectively,
r is the radius of the gear tooth fillet,
$\Delta\varphi$ is the applied angle of rotation of the teeth of the pinion from the point of initial contact (point 0) to point 1, and so on,
u is the gear ratio,
m is the engagement modulus,
z_1, z_2 are the numbers of teeth in the wheel pinion;
r_{b1}, r_{b2} are the radii of base circles of the pinion and gear, respectively;
r_{a1}, r_{a2} are the radii of addendum circles of the pinion and gear, respectively;
r is the radius of the gear tooth fillet;
a_W centre distance of gear pair;
α_{t10} is the angle of the first point on the contact line.

The minimum length of the line of contact is **[L. 18]**:

$$l_{\min} = \frac{b_W\varepsilon_\alpha}{\cos\beta_b}\left[1 - \frac{\left(1-n_\alpha\right)\left(1-n_\beta\right)}{\varepsilon_\alpha\varepsilon_\beta}\right]$$

at

$$n_\alpha + n_\beta \rangle 1 \quad l_{\min} = \frac{b_W\varepsilon_\alpha}{\cos\beta_b}\left[1 - \frac{n_\alpha n_\beta}{\varepsilon_\alpha\varepsilon_\beta}\right] \tag{8}$$

at

$$n_\alpha + n_\beta \le 1$$

where
b_W is the width of the pinion;
ε_α, ε_β are the coefficients describing the top and step-by-step overlaps of the gear,
n_α, n_β are the fractional parts of coefficients ε_α, ε_β,

$$\varepsilon_\alpha = \frac{t_1 + t_2}{t_z}, \quad \varepsilon_\beta = \frac{b_W\sin\beta}{\pi m}, \quad \varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta,$$

$$t_1 = \frac{e_1}{\omega_1 r_{b1}}, \quad t_2 = \frac{e_2}{\omega_1 r_{b1}}, \quad t_z = \frac{2\pi}{z_1\omega_1},$$

$$e_1 = \sqrt{r_{1s}^2 - r_{b1}^2} - r_1\sin\alpha_t, \quad e_2 = \sqrt{r_{20}^2 - r_{b2}^2} - r_2\sin\alpha_t,$$

$$r_{1s} = r_{a1} - r, \ r_{a1} = r_1 + m, \quad r_{a1} = r_1 + (1 + x_1)m.$$

In case of technological teeth correction, $r_{a1} = r_1 + (1 + x_1)\,m$, x_1, x_2 – teeth correction coefficients.

The work **[L. 17, 18]** presents a method that takes into account wear-generated changes in the initial radii of gear curvature in every revolution of the gear as follows:

$$\rho_{kjh} = \rho_{kj} + D_{jk}\sum^{n} K_{kjn}^{-1} \tag{9}$$

where
$n = n_k$ = 1, 2, 3, ... is the number of revolution of the gear;
k is the numeration of gears (1 – pinion, 2 – gear);
$D_{jk} = K_{kj}^2$ are the nondimensional constants at every j-th contact point depending on wear.

The wear-generated changes in gear profiles during every tooth interaction is

$$K_{kj} = 8h'_{kj} / l_{kj}^2 \tag{10}$$

where
h'_{kj} in the linear wear of gear teeth at any j-th point of the profile;
l_{kj} is the length of a gear chord which substitutes the involute between points $j – 1$, $j + 1$.

To make the computation time shorter, we developed a block-based method for solving this problem. With this method, changes in profile curvature radii, reduced curvature radii, and maximum contact pressures are determined following a selected number of revolutions (blocks of interactions), and not for every gear revolution (tooth engagement) as was done previously. In a block, computations are made under constant conditions of tooth engagement based on linear cumulative changes in given parameters. In a successive block of computations, the cumulative changes are taken into account after (11) and (12), and then the computations are continued using new data. The changeable curvature radii ρ_{kjh} are determined as follows:

$$\rho_{kjh} = \rho_{kj} + \tilde{E}_k \sum_{B_1}^{B_{\max}} D_{kjB} K_{kjB}^{-1} \tag{11}$$

where
a the size of a block can be selected as: $B = 1$ revolution pinion (accurate solution), $B = n_1$ revolutions per one minute, $B = n_1$ revolutions per one hour, $B = n_1$ pinion revolutions per 10 hours, and so on; B_1 and $B_{\max}$ are the first and last computational block, respectively;
B_1 and $B_{\max}$ are the first and last computational block, respectively;
$D_{kjB} = K_{kjB}^2$ is a constant which remains unchanged in one block but changes in every other block;
$\tilde{E}_k$ is a nondimensional constant dependent on the maximum acceptable tooth wear h_{k*};
$\tilde{E}_1 = 3(h_{1*} + h_{2*}),\ \tilde{E}_2 \approx (97...98)(h_{1*} + h_{2*}),$
$\tilde{E}_1 + \tilde{E}_2 = 100(h_{1*} + h_{2*}).$

The wear-induced change in the curvature of a gear tooth profile for every single block of interaction is:

$$K_{kjB} = 8\sum^{B} h'_{kjn} / l_{kj}^2 \tag{12}$$

The chord length l_{kj} is calculated from:

$$l_{kj} = 2\rho_{kjh} \sin \mu_{kjh} = \text{const} \tag{13}$$

where
$\varepsilon_{kjh} = S_{kj} / \rho_{kjh}$ is the angle between the points j and $j+1$;
$S_{kj} = \left| \frac{mz_k}{4} \left(\frac{1}{\cos^2\alpha_{kj}} - \frac{1}{\cos^2\alpha_{k,j+1}} \right) \cos \pm \right|$ is the length of the involute between the points j, $j + 1$;
α_j, α_{j+1} are the angles of tooth engage of selected involute points j, $j + 1$ **[L. 16]**.

Hence, following every interaction or a block of interaction, the parameters h_{1j}, h_{2j}, ρ_{1jh}, ρ_{2jh}, ρ_{jh}, $p_{jh\max}$, $2b_{jh}$, t'_{jh} will change.

The angles of transition from a double tooth engagement ($\Delta\varphi_{1F_2}$) to a single tooth engagement and, again, to a double tooth engagement ($\Delta\varphi_{1F_1}$) in a cylindrical gear with profile correction are determined in the following way **[L. 19]**:

$$\Delta\varphi_{1F_2} = \varphi_{10} - \varphi_{1F_2},\ \Delta\varphi_{1F_1} = \varphi_{10} + \varphi_{1F_1} \tag{14}$$

where
$\varphi_{1F_2} = \tan\alpha_{F_2} - \tan\alpha_t,\ \varphi_{1F_1} = \tan\alpha_{F_1} - \tan\alpha_t$
$\phi_{10} = \tan\alpha_{10} - \tan\alpha_w$

$$\tan\alpha_{F_2} = \frac{r_1 \sin\alpha_t - (p_b - e_1) + 0.5 n_\beta p_b}{r_1 \cos\alpha},$$
$$\tan\alpha_{F_1} = \frac{r_1 \sin\alpha_t - (p_b - e_2) - 0.5 n_\beta p_b}{r_1 \cos\alpha} \tag{15}$$

In spur gear e_β, $n_\beta = 0$, and $l_{\min} = b_W$.

The angle $\Delta\varphi_{1E}$ describing the moment of teeth exit of engagement is:

$$\Delta\varphi_{1E} = \varphi_{10} + \varphi_{1E} \tag{16}$$

where
$\varphi_{1E} = \tan\alpha_E - \tan\alpha_t,\ \alpha_E = \arccos(r_{b1} / r_{1s})$
$p_b = \pi m \cos\alpha_t / \cos\beta$ is the pitch of teeth;
$z_1 = z_{1K} / \cos\delta_1,\ z_2 = z_{2K} / \cos\delta_2.$

For triple-double-triple tooth engagement:

$$\tan\alpha_{F_2} = \frac{r_1 \sin\alpha_t - (p_b - e_1) + 0.5 \tilde{n}_\beta p_b}{r_1 \cos\alpha},$$
$$\tan\alpha_{F_1} = \frac{r_1 \sin\alpha_t - (p_b - e_2) - 0.5 \tilde{n}_\beta p_b}{r_1 \cos\alpha}; \tag{17}$$
$$\tilde{n}_\beta = \begin{Bmatrix} n_\beta \text{ at } n_\alpha + n_\beta \rangle 1, \\ 1 - n_\beta \text{ at } n_\alpha + n_\beta \le 1 \end{Bmatrix}$$

The sliding velocity of engaged teeth is calculated as **[L. 17]**:

$$v_j = \omega_1 r_1 (\tan\alpha_{t1j} - \tan\alpha_{t2j}) \tag{18}$$

where
α_{t1j}, α_{t2j}, in compliance with **[L. 18]**.

For the applied number of pinion revolutions n_{1s} and gear revolutions n_{2s}, and the corresponding number of interaction blocks, the total tooth wear h_{1jn} and h_{2jn} at the j-th points of contact are calculated as **[L. 17]**:

$$h_{1jn} = \sum_{1}^{n_{1s}} h_{1jB} \qquad h_{2jn} = \sum_{1}^{n_{2s}} h_{2jB} \tag{19}$$

where
$n_{2s} = n_{1s} / u$; $h_{kjB} = \sum h'_{kj}$ is the tooth wear in every block of interaction.

The service life $t_{B\min}$ of gear operation for the number of gear revolutions n_{1s} or n_{2s}, at which the permissible teeth wear is reached, is determined by the following **[L. 17]**:

$$t_{B\min} = n_{1s} / 60n_1 = n_{2s} / 60n_2 \tag{20}$$

NUMERICAL SOLUTION FOR THE CONICAL SPUR GEAR PROBLEM

Fig. 1. Involute conical spur gear
Rys. 1. Przekładnia stożkowa ewolwentowa o zębach prostych

The input data included the following: z_{1K} = 20; u_1 = 3; n_1 = 750 rpm; P = 20 kW; b_W = 50 mm; β = 0; m_{max} = 5 mm – a normal module of tooth engagement in the frontal section of a gear at β = 0; m_{min} = 3.391 mm – a normal module of tooth engagement in the internal section of a gear at β = 0; $\Delta\phi$ = 4°; h_{k*} = 0,5 mm – maximum acceptable wear of gear teeth; B = 900000 = 1200 n_1 revolutions. We applied boundary lubrication with a sliding friction factor set to f = 0.05. The applied profile shift coefficients were: $x_1 = - x_2$ = 0; 0,1; 0,2; 0,3; 0,4.

The gears were ascribed the following material properties: the pinion was made of 38HMJA steel after nitriding at a depth ranging from 0.4 mm to 0.5 mm described by 58 HRC, R_m= 1040 MPa, C_1 = 3.5×10^6, m_1 = 2; the gear was made of bulk hardened 40H steel with 53 HRC, R_m = 981 MPa, C_2 = 0.17×10^6, m_2 = 2.5; E = 2.1×10^5 MPa, μ = 0.3.

The frontal and internal section of the gear ring is characterized by a double-single-double tooth engagement. The results are illustrated in the figures below.

I. Frontal section, $m_n = m_{max}$ = 5 mm

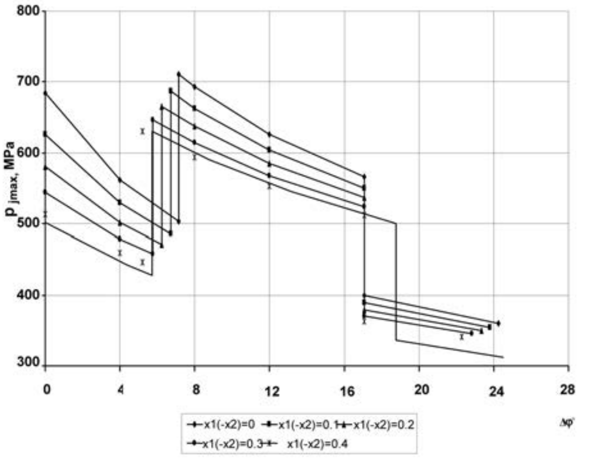

Fig. 2. Changes in p_{jmax} during tooth interaction
Rys. 2. Zmiana p_{jmax} w trakcie interakcji zębów

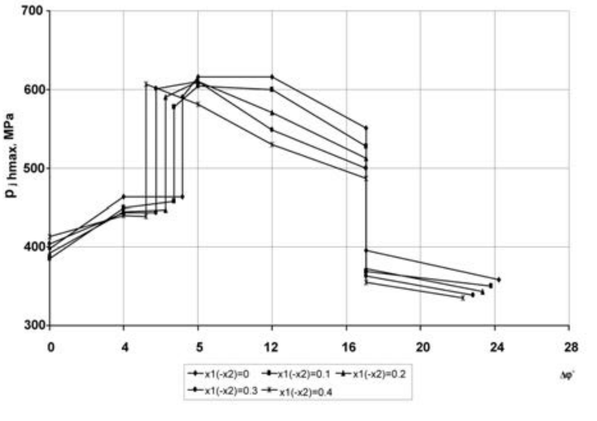

Fig. 3. Tribocontact pressures p_{jhmax}
Rys. 3. Naciski tribokontaktowe p_{jhmax}

The left- and right-hand sides of the figures show double tooth engagement, while single tooth engagement can be observed in the centre of the figures. An increase in the profile shift coefficients leads to decreasing p_{jmax}, and this decrease is particularly significant on the left (**Fig. 2**). **Fig. 3** illustrates variations in the tribocontact pressures p_{jhmax} due to tooth wear to the highest acceptable values h_* at one point of the gear profile. This value is remarkably high in the entire left-hand zone of the double tooth engagement and quite visible at the beginning of the double tooth engagement zone.

Fig. 4 shows the diagrams of linear wear of gear profiles in the engagement zone.

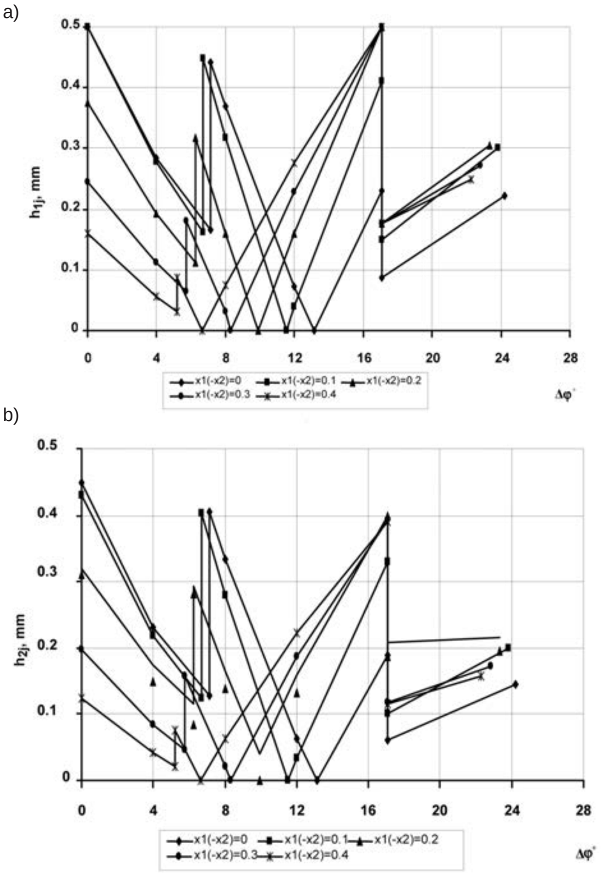

Fig. 4. Linear wear of corrected teeth: a) pinion, b) gear
Rys. 4. Zużycie liniowe zębów z korekcją technologiczną: a) zębnik, b) koło zębate

Depending on a value of $x_1 = -x_2$, the maximum acceptable wear of cylindrical gear teeth occurs at the entry of the left-hand zone of double tooth engagement ($x_1 = -x_2 = 0$; 0.1) and at the exit of single tooth engagement ($x_1 = -x_2 > 0.1$). Wheel teeth wear less than pinion teeth.

Fig. 5 illustrates the effect of the minimal life of gears (gear tooth profile points where h_{2*} is attained) on the profile shift coefficients $x_1 = -x_2$.

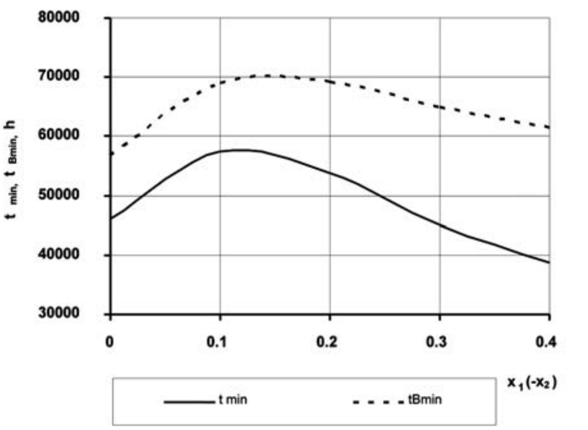

Fig. 5. Gear life: $t_{\min}$ when $p_{j\max}$ = const, $t_{B\min}$ when $p_{jh\max}$ = var
Rys. 5. Trwałość przekładni: $t_{\min}$ gdy $p_{j\max}$ const, $t_{B\min}$ gdy $p_{jh\max}$

Accordingly, the gear life $t_{B\min}$ takes into account changes in $p_{j\max}$ due to tooth wear, while $t_{\min}$ is described by the assumption that $p_{j\max}$ remains constant. For a selected range of change in the profile shift coefficients $0 \leq x_1 = -x_2 \leq 0{,}4$, the optimal gear life is at $x_1 = -x_2 \approx 0.13$. In this case, the tooth wear is similar at three characteristic points of the profile, i.e. at the entry of the left-hand zone of double tooth engagement, the entry of single tooth engagement, and the exit thereof. The gear life $t_{B\min}$ will be higher than $t_{\min}$ by 1.22 times.

II. Internal section, $m_{\min}$ = 3.391 mm

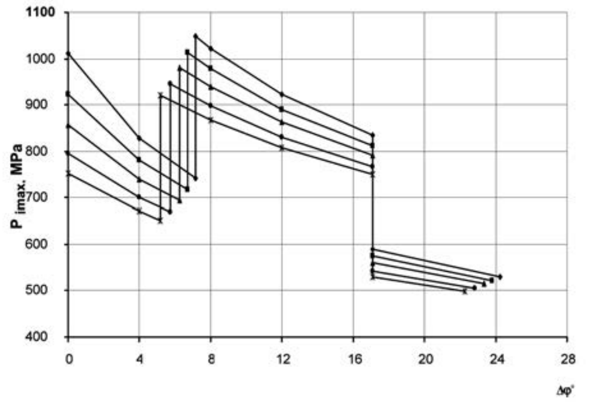

Fig. 6. Changes in $p_{j\max}$ during tooth interaction
Rys. 6. Zmiana $p_{j\max}$ w trakcie interakcji zębów

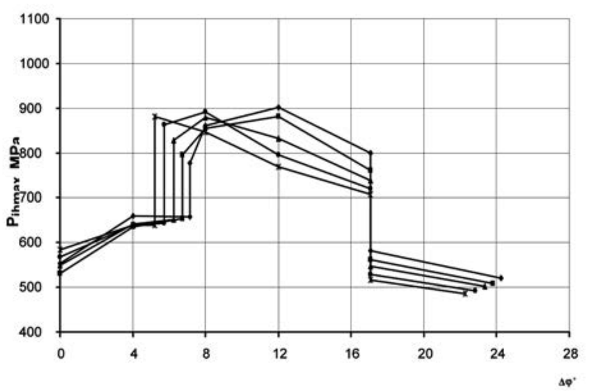

Fig. 7. Tribo-contact pressures $p_{jh\max}$
Rys.7. Naciski tribokontaktowe $p_{jh\max}$

As for this section (**Fig. 6**), $p_{j\max}$ is approx. 1.48 times higher than that in the frontal section. The change in $p_{j\max}$ (**Fig. 7**) is similar to the one given above (**Fig. 3**), i.e. the reasonable decrease of $p_{j\max}$ in the left zone of double-teeth engagement is observed.

In this case, the wear of gear teeth (**Fig. 8**) is almost the same as that shown in **Fig. 4a**. Here, the wear of gear teeth (**Fig. 8b**) is much more rapid than it is the case with the frontal section (**Fig. 4b**), and also the maximum acceptable wear h_{1*} = 0.5 mm is attained at $x_1 = -x_2 =$ = 0; 0.1.

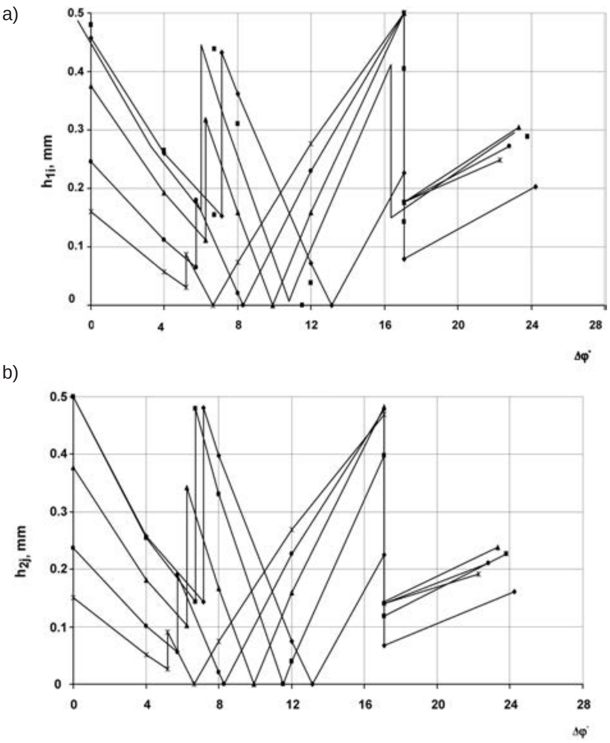

Fig. 8. Linear wear of corrected teeth: a) pinion, b) gear

Rys. 8. Zużycie liniowe zębów z korekcją technologiczną: a) zębnik, b) koło zębate

The minimal gear life for the change $x_1 = -x_2$ is shown in **Fig. 9**.

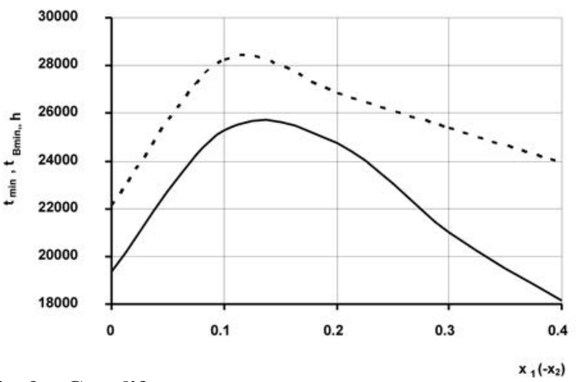

Fig. 9. Gear life

Rys. 9. Trwałość przekładni

Moreover, we can observe optimal values $x_1 = -x_2 \approx 0.13$. The results demonstrate that the gear life in the internal section of a gear (**Fig. 9**) is up to 2.27 times lower than that in the frontal section of a gear (**Fig. 5**). As a result, for this gear life, we calculated $p_{jh\max}$. $p_{jh\max}$, h_{1j}, h_{2j} (**Figs. 10, 11**).

III. Frontal section, t_{min} set identical as in internal section (Fig. 10)

In this case, the change in $p_{j\max}$ is less significant than on **Fig. 3**, both at the entry of the left-hand zone of double tooth engagement and at the entry of single tooth engagement.

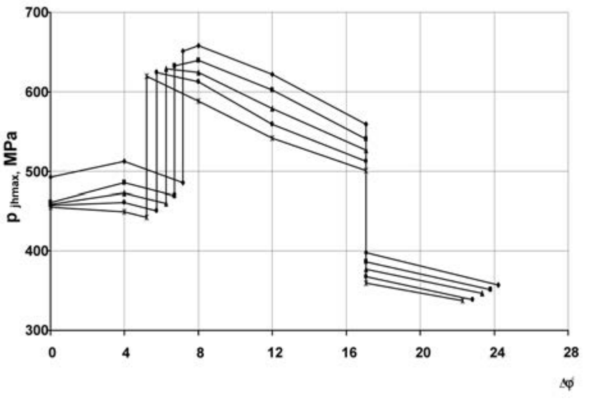

Fig. 10. Real transformation pressures $p_{j\max}$ in frontal section

Rys. 10. Rzeczywista zmiana nacisków $p_{j\max}$ w przekroju czołowym

The real teeth wear in shown in **Fig. 11**.

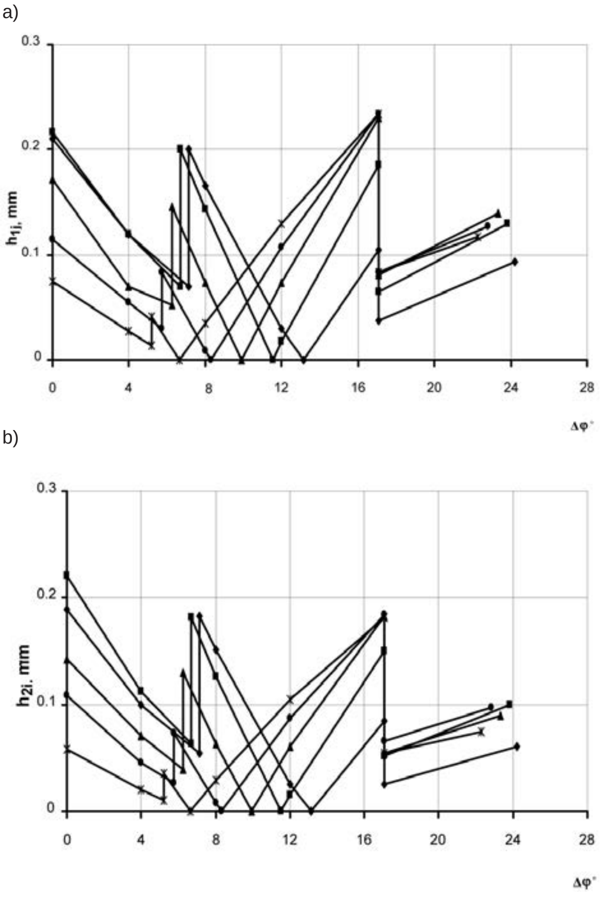

Fig. 11. Linear wear of gear teeth in frontal section: a) pinion, b) gear

Rys. 11. Zużycie liniowe zębów kół w przekroju czołowym: a) zębnik, b) koło zębate

The maximum acceptable wear of the pinion does not exceed 0.234 mm, while that of the gear is not higher than 0.22 mm.

CONCLUSIONS

The results have demonstrated the following:

1. With double-single-double tooth engagement, the initial contact pressures p_{jmax} will be higher in the internal section of the gear by 1.475 times than in the frontal section due to a decrease in the module.
2. The highest values of p_{jmax} can be observed at the entry of single tooth engagement in both sections.
3. We determined optimal profile shift coefficients $x_1 = -x_2 = 0.13$ that will produce the highest possible gear life.
4. The gears have the lowest acceptable life in their internal section.
5. The real values of maximum wear would be 46.8% in frontal section of pinion teeth and 44% for wheel teeth from the permissible $h_{k*} = 0.5$ mm.

DISCUSSION

The investigation results of conical involute gear presented in this paper according to the author's method most fully represent the real conditions of teeth interaction, which consider such geometrical factors as correction and parity of teeth engagement, as well as obvious exploitation factor – change of teeth involute profile in the result of their wear. The solutions of tribo-contact problems for conical gears, performed by other researchers, are absent in the literature. The existing investigation results of cylindrical gears do not consider the influence factors, which are mentioned above. In the previous authors' publications which concern spur conical gears, the total consideration of the influence of these factors on contact pressures, wear, and service life also has not been considered.

REFERENCES

1. Drozdov Yu.: To the development of calculation methods on friction wear and modeling. Wear resistance. Science, Moscow, 1975, pp. 120–135.
2. Pronikov A.: Reliability of machines. Mashinostroenie, Moscow 1978.
3. Grib V.: Solution of tribotechnical tasks with numerous methods. Science, Moscow 1982.
4. Brauer J., Andersson S.: Simulation of wear in gears with flank interference – a mixed FE and analytical approach. Wear, 2003, № 254, pp. 1216–1232.
5. Flodin A., Andersson S.: Simulation of mild wear in spur gears. Wear, 1997, № 207 (1–2), pp. 16–23.
6. Flodin A., Andersson S.: Wear simulation of spur gears. Tribotest J, 1999, № 5 (3), pp. 225–250.
7. Flodin A., Andersson S.: Simulation of mild wear in helical gears. Wear, 2000, № 241 (2), pp. 123–128.
8. Flodin A., Andersson S.: A simplified model for wear prediction in helical gears. Wear, 2001, № 249 (3–4), pp. 285–292.
9. Kahraman A., Bajpai P., Anderson N.E.: Influence of tooth profile deviations on helical gear wear. J. Mech. Des., 2005, Vol. 127, Issue 4, pp. 656–663.
10. Kolivand M., Kahraman A.: An ease-off based method for loaded tooth contact analysis of hypoid gears having local and global surface deviations. J. Mech. Des., 2010, Vol. 132, Issue 7.
11. Pasta A., Mariotti Virzi G.: Finite element method analysis of a spur gear with a corrected profile. J. Strain Analysis, 2007, Vol. 42, pp. 281–292.
12. Brandão José A, Martinsa Ramiro, Seabra Jorge H.O., Castro Manuel J.D.: Calculation of gear tooth flank surface wear during an FZG micropitting test. Wear, 2014, Volume 311, Issues 1–2, pp. 31–39.
13. Chernets M.V., Bereza V.V.: Method for the evaluation of wear and durability of involute conical straight-bevel gears. Materials Science, 2009, № 4, pp. 110–116.
14. Chernets M.V.: Influence of straight bevel gear teeth hooking conditions on their contact strength. Problems of Tribology, 2013, № 3, pp. 109–113.
15. Chernets M.V., Chernets Yu.M.: Investigation of tooth engagement conditions in cylindrical involute gear on contact strength, wear and durability. Part 3. Changeable conditions of interaction in uncorrected engagement. Problems of Tribology, 2014, № 4, pp. 49–53.
16. Chernets M.V., Chernets Yu.M.: Investigations of tooth engagement conditions in cylindrical involute gear on contact strength, wear and durability. Part 4. Changeable conditions of interaction in corrected engagement. Problems of Tribology, 2015, № 1, pp. 69–76.
17. Chernets M.V., Jarema R.Ya.: Generalized value method of teeth correction influence on resource, wear and contact durability of cylindrical involute gears. Materials Science, 2011, № 4, pp. 56–58.
18. Chernets M.V., Yarema R.Ya., Chernets Yu.M.: A method for the evaluation of the influence of correction and wear of the teeth of a cylindrical gear on its durability and strength. Part 1. Service live and wear. Materials Science, 2012, № 3, pp. 289–300.
19. Chernets M.V., Chernets Yu.M.: Evaluation of the Strength, Wear, and Durability of a Corrected Cylindrical Involute Gearing, with Due Regard for the Tooth Engagament Conditions. Journal of Friction and Wear, 2016, № 1, pp. 71–77.
20. Chernets M., Chernets Yu.: The simulation of influence of engagement conditions and technological teeth correction on contact strength, wear and durability of cylindrical spur gear of electric locomotive. Proc. IMechE Part J: J. Engineering Tribology, 2017, Vol. 231, № 1, pp. 57–62.