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## FLOTATION KINETICS AND SEPARATION SELECTIVITY OF COAL SIZE FRACTIONS

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**Abstract:** Flotation recovery and kinetics for three size fractions of coal were investigated. Flotation of combustible matter recovery was approximated with the first order kinetic equation while flotation of the ash forming minerals with the second order equation. Next, the equations for each size fraction were combined and a formula was obtained which was used for approximation of the experimental results using the so-called Fuerstenau upgrading curve, which relates the recovery of combustible matter recovery and recovery of ash forming minerals, both in concentrate. The Fuerstenau upgrading plot showed that the best selectivity was obtained for the middle size fraction of 0.25–0.075 mm, while the flotation selectivity of larger 0.5–0.25 mm and smaller <0.075 mm particles was diminished. This finding agrees with many other investigations.

**Keywords:** coal, flotation rate, combustible matter recovery, ash, particle size

### Introduction

China is a country in which coal is the main energy source and in a very long period of time it will not change (Xu, 2003). The fine particle mineral processing technology has become one of the most important development directions in current field of mineral processing (Jameson, 2010; Albijanic et al., 2010; Bhattacharya and Dey, 2008). Flotation is the most widely used and effective method of separation of fine and very fine materials. The principle of flotation is based on different surface properties of mineral matter (Fan et al., 2010; William et al., 2010; Muganda et al., 2011). Very important aspect of flotation is its kinetics. The flotation rate is measured as the recovery change of the floating material in the flotation product per unit time and it is characterized by a rate constant and kinetics order (Vapur et al., 2010; Gui et al., 2011, Brozek and Mlynarczykowska, 2013). It is known that the coal particles of different sizes have different flotation rates. This has been confirmed with industrial data (Song et al., 2001).

Most flotation rate tests show that the fine coal particles can be described by the first order kinetic model and its rate constant changes with the increasing reagents dose (Chelgani et al., 2010; Abkhoshk et al., 2010; Ucurum and Bayat, 2007; Aktas et al., 2008; Brozek and Mlynarczykowska, 2007, 2013). The relationship between flotation rate constant and flotation recovery with particle size was found to be nonlinear (Abkhoshk et al., 2010). Particle size has also a great effect on the attachment/detachment of bubbles and particles (Ireland and Jameson, 2012).

It was shown recently by Drzymala and Luszczkiewicz (2011) as well as Bakalarz and Drzymala (2013) that having kinetics of flotation of the useful component of coal or ore and kinetics of the remaining components in the feed, it is possible directly produce the Fuerstenau upgrading plot. This is so because the Fuerstenau upgrading plot relates recovery of both components while the kinetic curves also relate both recoveries through the time of the process. The aim of this paper is to utilize this approach for coal by taking into account the kinetics of flotation of different size fractions present in ground coal.

## Experimental

### Analysis of coal sample

Table 1 provides the size composition of the coal sample. As the size becomes smaller, the ash content becomes greater. The ash content in the  $-0.045$  mm size fraction was 30.36%. It is more than 5.60 percent points greater than the average ash content. The yield of dominant 0.25–0.125 mm size fraction is 43.56%.

Table 1. Size analysis data of coal sample

Size fraction, mm	Yield,%	Ash,%	Oversize		Undersize	
			Yield,%	Ash,%	Yield,%	Ash,%
0.50–0.25	21.47	23.25	21.47	23.25	100.00	25.31
0.25–0.125	43.56	23.73	65.03	23.57	78.53	25.88
0.125–0.075	13.97	26.73	79.00	24.13	34.97	28.55
0.075–0.045	11.10	29.22	90.10	24.76	21.00	29.76
–0.045	9.90	30.36	100.00	25.31	9.90	30.36
Total	100.00	25.31	–	–	–	–

From Table 2, one can see that the dominating density fraction of the investigated coal is  $1.4\text{--}1.5$  g/cm<sup>3</sup>. Its yield is 37.93% with the ash content of 10.54%. The yield of the  $-1.5$  g/cm<sup>3</sup> density fraction is 57.03% with the ash content of 9.04%. Data for other fractions are given in Table 2. The data show that there is significant amount of ash forming matter in the coal fractions.

Table 2. Density analysis of coal sample

Density, g/cm <sup>3</sup>	Yield,%	ash,%	Float accumulation		Sediment accumulation		(Separation density±0.1)	
			Yield,%	ash	Yield,%	ash	Density, g/cm <sup>3</sup>	Yield,%
-1.3	0.26	4.21	0.26	4.21	100.00	24.55	1.30	19.10
1.3-1.4	18.84	6.10	19.10	6.07	99.74	24.61	1.40	56.77
1.4-1.5	37.93	10.54	57.03	9.04	80.90	28.92	1.50	58.27
1.5-1.6	20.34	24.49	77.37	13.10	42.97	45.14	1.60	27.08
1.6-1.7	6.74	38.14	84.11	15.11	22.63	63.70	1.70	9.48
1.7-1.8	2.74	54.06	86.85	16.34	15.89	74.54	1.80	3.74
1.8-2.0	1.99	63.60	88.84	17.40	13.15	78.81	1.90	1.99
+2.0	11.16	81.52	100.00	24.55	11.16	81.52	-	-
Total	100.00	24.55	-	-	-	-	-	-

### Experimental procedure

Coal was mixed with distilled water and stirred for 120 s. Next collector was added and stirred for 60 s. The foaming agent contact time was 10 s, and then flotation was initiated. The clean coal collection intervals were 30, 30, 60, 60, and 120 s. The total flotation time was 5 min. The obtained six flotation (cleaned coal) products were labeled as concentrates J1, J2, J3, J4, J5 and tailings T. The reagents dosage and other operational parameters were: 320 g/Mg of collector (kerosene), 110 g/Mg of foaming agent (2-octyl alcohol), flotation feed pulp mass concentration was 90 g/dm<sup>3</sup>, air flow 0.37 m<sup>3</sup>/min, and the stirring speed during flotation was 1800 rpm.

### Results and discussion

The flotation kinetics of size fractions of the investigated coal (Table 3 and Fig. 1) can be expressed by the classical first order equation:

$$\varepsilon = \varepsilon_{\infty}(1 - e^{-k_1 t}) \quad (1)$$

where  $\varepsilon$  is the combustible matter recovery in concentrate,  $\varepsilon_{\infty}$  maximum combustible matter recovery in concentrate,  $t$  flotation time and  $k_1$  is the first order kinetics constant. The values of  $\varepsilon_{\infty}$  and  $k_1$  were determined with the Matlab software and they are presented in Table 4.

Table 3. Results of coal size fractions flotation

Products	Combustible matter recovery (%)			Ash content (%)		
	0.50–0.25	0.25–0.075	–0.075	0.50–0.250	0.25–0.075	–0.075
J1	79.64	67.87	62.23	10.58	11.05	10.78
J2	14.07	15.36	19.14	11.28	12.06	12.97
J3	1.82	8.60	10.25	13.99	12.48	15.20
J4	0.59	1.82	2.36	15.65	13.50	20.40
J5	0.74	1.93	1.14	44.30	25.26	28.38
T	3.14	4.42	4.88	84.88	83.47	80.75
Total	100.00	100.00	100.00	22.99	25.93	25.23

Figure 1 shows changes of the combustible matter recovery with time. The data were approximated with the first order kinetic equation (Eq. 1). It can be seen from Fig. 1 and Table 4 that the rate of flotation of the combustible matter increases with the particle size. This is so because the fine particles exhibit low collision efficiencies due to their low mass and inertial force while the coarse particles have a high degree of heterogeneity. The flotation rate of the 0.5–0.25 mm, 0.25–0.075 mm and –0.075 mm size fractions changes from fast to slow. This agrees with the results of other researchers (Brożek and Mlynarczykowska, 2013; Jameson, 2012; Polat et al., 1993; Gaudin et al., 1931).

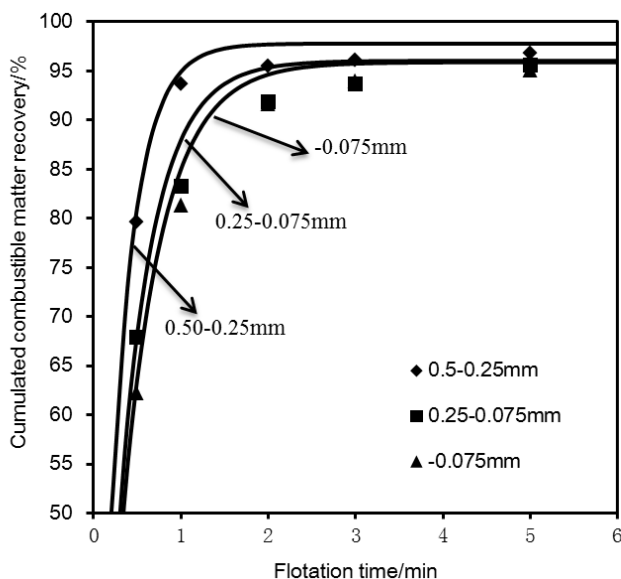


Fig. 1. Relationship between cumulated combustible matter recovery for different size fractions of investigated coal and flotation time

In the case of the ash forming minerals flotation, the second order kinetic equation was used

$$\epsilon_a = \frac{\epsilon_{a\infty}^2 k_2 t}{1 + \epsilon_{a\infty} k_2 t} \tag{2}$$

where  $\epsilon_a$  is the ash matter recovery in the concentrate,  $\epsilon_{a\infty}$  maximum ash matter recovery in concentrate,  $t$  flotation time and  $k_2$  is the second order kinetics constant. The values of  $\epsilon_{a\infty}$  and  $k_2$  were determined with the Matlab software and they are presented in Table 4.

Table 4. Kinetics of flotation

Kinetic equation	Size fraction/mm	$\epsilon_a$ or $\epsilon_{a\infty}$	$k_1$ or $k_2$	Correlation coefficient
First order (combustible)	0.50–0.25	97.8	3.52	0.998
	0.25–0.075	96.0	2.47	0.988
	–0.075	95.9	2.17	0.998
Second order (ash)	0.50–0.25	41.7	0.16	0.972
	0.25–0.075	38.2	0.09	0.997
	–0.075	43.5	0.05	0.996

Figure 2 shows the change of ash recovery to concentrate and its approximation with the second order kinetic equation presented in Eq. 2. It can be seen from Fig. 2 and

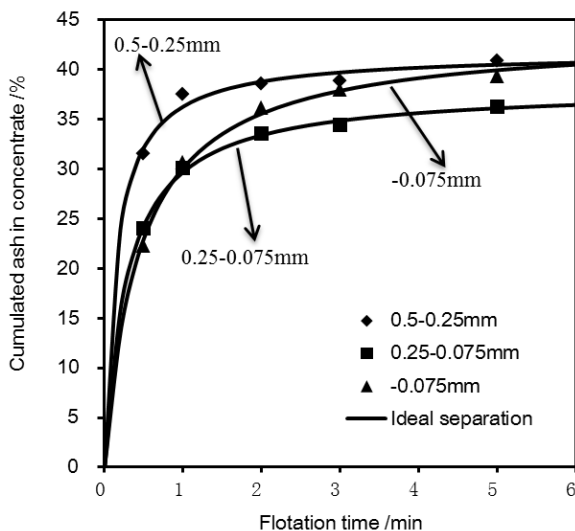


Fig. 2. Relationship between cumulated ash in concentrate for different size fractions and flotation time

Table 4 that the rate of flotation of ash is different from that of combustible matter because the finest fraction floats better than the 0.25–0.075 mm size fraction and worse than the 0.5–0.25 mm size fraction. This can partly be explained by a combined effect of collision and attachment/detachment sub-processes, dominant for small and large sizes, respectively. The cleaned coal –0.075 mm particles has the most obvious change and the highest cumulated ash content due to a high content of ultrafine ash forming matter. This agrees with the results of other researchers (Brozek and Mlynarczykowska, 2013; Rahman et al., 2012; Vapur et al., 2010; Polat and Chander, 2000; Al Taweel et al., 1986).

As presented recently by Drzymala and Luszczkiewicz (2011) as well as Bakalarz and Drzymala (2013) the flotation kinetics, that is relations between recoveries and time can be combined providing the Fuerstenau upgrading curves (Drzymala and Ahmed, 2005; Drzymala, 2006) relating recovery of the combustible mater and recovery of ash in the concentrate. The Fuerstenau upgrading curves for the investigated size fractions are given in Fig. 3. The experimental points were approximated with the equation

$$\varepsilon = \varepsilon_{\infty} \left( 1 - e^{\frac{-k_1 \varepsilon_a}{\varepsilon_{a\infty} k_2 (\varepsilon_{a\infty} - \varepsilon_a)}} \right) \tag{3}$$

resulting from Eqs 1 and 2 after removing time as the parameter. The obtained equations for each size fraction of the investigated coal are given in Table 5.

Table 5. Upgrading equations used for approximation of data points of the Fuerstenau upgrading curve for each size fraction

Size fraction/mm	$\varepsilon - t$	$\varepsilon_a - t$	The derived $\varepsilon - \varepsilon_a$
kinetic equation	$\varepsilon = \varepsilon_{\infty} (1 - e^{-k_1 t})$	$\varepsilon_a = \frac{\varepsilon_{a\infty}^2 k_2 t}{1 + \varepsilon_{a\infty} k_2 t}$	
0.50–0.25	$\varepsilon_{\infty} = 97.79, k_1 = 3.52$	$\varepsilon_{a\infty} = 41.72, k_2 = 0.16$	$\varepsilon = \varepsilon_{\infty} \left( 1 - e^{\frac{-k_1 \varepsilon_a}{\varepsilon_{a\infty} k_2 (\varepsilon_{a\infty} - \varepsilon_a)}} \right)$
0.25–0.075	$\varepsilon_{\infty} = 96.04, k_1 = 2.47$	$\varepsilon_{a\infty} = 38.20, k_2 = 0.09$	
–0.075	$\varepsilon_{\infty} = 95.89, k_1 = 2.17$	$\varepsilon_{a\infty} = 43.53, k_2 = 0.05$	

Figure 3 indicates that the selectivity of flotation is the best for the middle size fraction while both smaller and larger size fractions have diminished selectivity. This observation agrees well with numerous flotation data. For each size fraction, the recovery of combustible matter in concentrate is greater than the recovery of ash in concentrate.

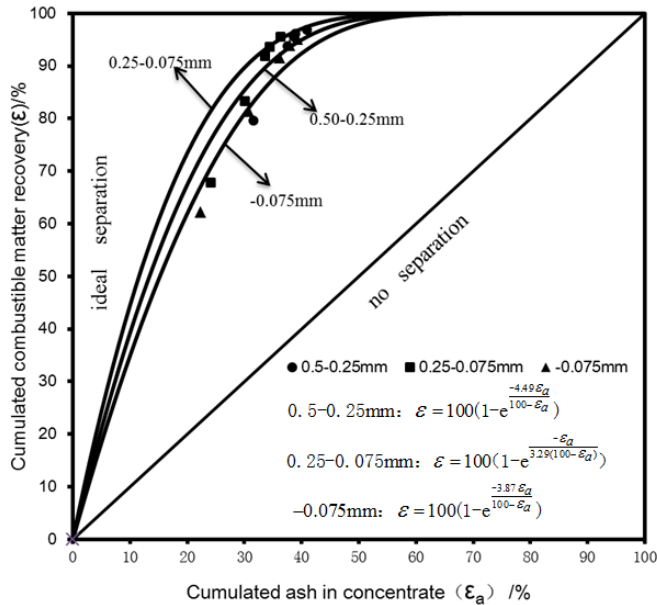


Fig. 3. The Fuerstenau upgrading curve showing flotation results for different size fractions of investigated coal

### Conclusions

According to the tests conducted for coal narrow size fractions, the flotation rate of the 0.5–0.25 mm, 0.25–0.075 mm and –0.075 mm size fractions changes from fast to slow. As the flotation proceeds, the cumulated combustible matter recovery gradually increases and reaches a plateau level at about 95%. The combustible matter recovery in concentrate can be approximated with the first order kinetic equation while the ash matter recovery in the concentrate can be approximated with the second order kinetic equation.

Combination of kinetic equations for combustible and ash matter provides an

equation ( $\epsilon = \epsilon_{\infty} (1 - e^{\frac{-k_1 \epsilon_a}{\epsilon_{a\infty} k_2 (\epsilon_{a\infty} - \epsilon_a)})}$ ) which can be used for approximation of the data points for each flotation size fraction in the recovery-recovery Fuerstenau upgrading curve. The Fuerstenau upgrading curves show that the best selectivity of separation occurs for the middle size fraction of coal.

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