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FLOW-CAPTURE LOCATION MODEL WITH LINK CAPACITY CONSTRAINT OVER A MIXED TRAFFIC NETWORK

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Abstract

This paper constructs and settles a charging facility location problem with the link capacity constraint over a mixed traffic network. The reason for studying this problem is that link capacity constraint is mostly insufficient or missing in the studies of traditional user equilibrium models, thereby resulting in the ambiguous of the definition of road traffic network status. Adding capacity constraints to the road network is a compromise to enhance the reality of the traditional equilibrium model. In this paper, we provide a two-layer model for evaluating the efficiency of the charging facilities under the condition of considering the link capacity constraint. The upper level model in the proposed bi-level model is a nonlinear integer programming formulation, which aims to maximize the captured link flows of the battery electric vehicles. Moreover, the lower level model is a typical traffic equilibrium assignment model except that it contains the link capacity constraint and driving distance constraint of the electric vehicles over the mixed road network. Based on the Frank-Wolfe algorithm, a modified algorithm framework is adopted for solving the constructed problem, and finally, a numerical example is presented to verify the proposed model and solution algorithm.

Keywords: Traffic assignment problem, link capacity constraint, charging location, path distance constraint.

1 Introduction

With the rapid development of the automobile industry in recent years, the increasing production and usage of automobiles have led to increasingly serious pollution to our environment. Finding alternative fuels for gasoline vehicles is increasingly counted as one of the most effective strategies to reduce carbon dioxide and greenhouse gas emissions [1, 2]. Electric vehicle as the development trend of the future automobile has the advantages of environmentally sustainable development, however, it has not been widely used due to several bottlenecks. One of the key bottlenecks is that the amount of practical charging installations is still too small, which makes the drivers feel anxious that the battery power of the electric vehicle will drop to zero percent before arriving at their destinations (known as range anxiety [3]). Moreover, the current performance of electric vehicle batteries is not enough to satisfy all users, which is also an important reason for restricting the widespread use of electric vehicles. For example, electric vehicles are obviously not the best choice for drivers who like long-distance travel, because these drivers may need to charge or change batteries frequently during their long-distance travel. In a consequence of this, the battery performance (longer/shorter battery life) of electric vehicles directly affects the penetration of electric vehicles in the vehicle market. The effective way to solve these two bottlenecks is to improve battery performance while deploying more charging stations on a certain scale of the transportation network, which also demonstrates that high energy batteries and fast charging and discharging technology are the crucial challenges that electric vehicles must overcome to achieve widespread adoption.

Except for the above improvements in battery quality and the coverage of charging facilities, optimizing the location of charging facilities for electric vehicles is also a crucial step for the promotion of electric vehicles. A properly positioned charging device is conducive to improving the utilization rate and further saving the construction cost of public facilities. There exist many related studies on the charging facilities location problem of electric vehicles, such as [4, 5, 6, 7, 8, 9, 12, 13, 14]. Among these studies, Frade and Ribeiro et all in [4] took the capital of Portugal as an example to investigate the charging location problem of electric vehicles in employer-intensive areas. In a similar fashion in [5], a case study from Beijing is conducted to give a comprehensive analysis of three classic facility location models from two aspects of supply and demand, respectively. It should be noted that most of the studies mentioned above are mostly based on deterministic user equilibrium problems and it is generally believed that the effective way to improve the utilization of public charging facilities is to configure them on the most commonly used roads (flow-capturing location model). Research on the flow-capturing location models can be traced back to the 1990s, with one of the typical examples is [7]. After that, the research in [8, 9] extended the original model successively. Beyond these, with the widespread use of genetic algorithms in various fields [10, 11], there are also some studies that use genetic algorithms to study the charging position problem (see reference [12]). In addition to the above user equilibrium problems related to the location of charging facilities, there are also some studies on the optimal location of charging facilities for electric vehicles based on the random user equilibrium problem, such as [13, 14]. Along with other traces, the location model of electric vehicle charging device also needs to fully consider the vehicle type, range anxiety, driving distance, battery capacity, initial capacity, battery consumption rate, even external temperature, and so on [15, 16, 17, 18, 19].

In this paper, we proposed a bi-level charging facilities location model to maximize the captured link flows of the battery electric vehicles. The purpose of upper level model is to maximize the covered electric vehicle flows under the assumption that the electric vehicle flows remain unchanged, and the lower layer invested a multiple vehicle/mode class of user equilibrium problem. The contributions of this paper are stated as follows. Firstly, the bi-level model proposed in this paper is based on a hybrid transportation network, which is different from the most existing research that only focuses on a single network [20]. It is more practical to consider a hybrid transport network, which allows users to travel in multiple types of vehicles with the increase of automobiles amount. Secondly, to the best of my knowledge, there are very few studies on the charging location problem of the electric vehicles that take both link capacity constraint and the driving distance constraint of the electric vehicles into consideration, for example, [13, 20, 21], etc. In a consequence of this enlightenment, we proposed a bi-level model that takes link capacity constraint and path distance constraint into account. Simulation results indicate that when the maximum driving distance of the electric vehicle is not considered, each driver will choose to use electric vehicles because of their low cost. Thirdly, the proposed algorithm for the studied two-layer model is different from the traditional way. The traditional way, for instance, [22], to tackle the lower model is to utilize the augmented Lagrange method to tackle the link capacity constraints, but the lower-layer model in this paper is converted to a linear programming problem through a series of transformations, and then settled efficaciously by virtue of Gurobi solver [25].

The remainder of this paper is organized as follows. Section 2 gives the studied problem formulation, problem analysis as well as optimality conditions of the lower level model. Section 3 then establishes the solution method of the proposed model and gives the concrete steps of the algorithm implementation, which is based on the Frank-Wolfe algorithm. Section 4 utilizes the Nguyen-Dupuis network to evaluate the presented algorithm. Finally, the conclusions and future suggestions are given in the Conclusion Section.

Notations:

$\mathcal{A}$: index set of links on the road network, $\mathcal{A} = \{a|1,...,A\}$

a: link index, $a \in \mathcal{A}$

A: total number of links on the road network

g: gasoline vehicles

e: electric vehicles

v_a: sum of traffic flow of gasoline vehicles and electric vehicles on link a, that is, $v_a = v_{a,g} + v_{a,e}$

$v_{a,g}$: traffic flow of gasoline vehicles on link a

$v_{a,e}$: traffic flow of electric vehicles on link a

v: column vector of all link flows: $v = (v_a)^{\mathrm{T}} \in \mathbb{R}^A$ with $a \in \mathcal{A}$

x: column vector of all location variables: $x = (x_a)^{\mathrm{T}} \in \mathbb{R}^A$ with $a \in \mathcal{A}$

x_a: binary variable ($x_a = 1$ indicates that a charging facility has been installed on link a, $x_a = 0$ means not)

p: number of charging facilities planned for installation on the road network, $p > 0$

ρ: value of time

v_a^{max}: maximum capacity of link a

t_a: travel time of link a

t_a^0: free-flow travel time of link a

β: function parameter

α: function parameter

c_g: operating cost per mile of gasoline vehicles

c_e: operating cost per mile of electric vehicles

$c_{a,g}$: generalized travel cost of gasoline vehicles of link a

$c_{a,e}$: generalized travel cost of electric vehicles of link a

$f_{k,g}^{rs}$: traffic flow of gasoline vehicles on path k for origin-destination (O-D) pair (r,s)

$f_{k,e}^{rs}$: traffic flow of electric vehicles on path k for O-D pair (r,s)

$\delta_{a,k}^{rs}$: a parameter to mark the subordination of routes and links ($\delta_{a,k}^{rs} = 1$, if link a belongs to path k, otherwise, $\delta_{a,k}^{rs} = 0$)

D: maximum range limit for electric vehicles

l_k^{rs}: length of path k for O-D pair (r,s), $l_k^{rs} = \sum_a \delta_{a,k}^{rs} d_a$

d_a: length of link a

q^{rs}: travel demand for O-D pair (r,s)

$\perp$: orthogonal sign for two vectors

$\otimes$: Kronecker product

I_2: two-dimensional identity matrix

1_m: m dimensional column vector with all the entries are equal to 1

2 Problem definition

2.1 Problem formulation

In this subsection, we propose a bi-level charging facility location model that takes into account the driving distance limit and link capacity constraint to maximize the total captured electric vehicle link flows in a mixed network. The infrastructure developer (system designer) and the drivers (system users) are specified as the upper and lower layers of this bi-level model, which can be solved by an iterative scheme to reach an equilibrium state. The upper level model is described as follows:

$$\begin{aligned} \max_{x} \quad & \sum_{a\in\mathcal{A}} v_{a,e}x_a \\ \text{s.t.} \quad & \sum_{a\in\mathcal{A}} x_a = p. \end{aligned} \tag{1}$$

In terms of the above model, the objective of the upper level is to maximize the covered electric vehicles flows. The goal of the model is to maximize the coverage of electric vehicle flows by deploying a certain number of charging facilities p. We mathematically translate this concept into the sum of link flows of electric vehicles only on links with charging facilities. More details can refer to the work in [23, 24]. For the lower level model, it is characterized as an electric vehicles drivers' route choice behavior problem with a generalized travel cost structure. Specifically, for any O-D pair (r,s), the lower level model can be constructed as follow:

$$\begin{aligned} \min_{v} \quad & \sum_{a\in\mathcal{A}} \left[\rho \int_0^{v_a} t_a(w)\,dw + (v_{a,g}c_g + v_{a,e}c_e)\,d_a\right] \\ \text{s.t.} \quad & \sum_k \left(f_{k,g}^{rs} + f_{k,e}^{rs}\right) = q^{rs}, \quad 0 \leqslant v_a \leqslant v_a^{max}, \\ & \left(D - l_k^{rs}\right) f_{k,e}^{rs} \geqslant 0, \quad f_{k,g}^{rs} \geqslant 0, \; f_{k,e}^{rs} \geqslant 0 \end{aligned} \tag{2}$$

where $v_{a,g} = \sum_{rs}\sum_k f_{k,g}^{rs}\delta_{a,k}^{rs}$ represents the traffic flow of gasoline vehicles on link a and $v_{a,e} = \sum_{rs}\sum_k f_{k,e}^{rs}\delta_{a,k}^{rs}$ corresponds to the traffic flow of electric vehicles on link a. The travel time for each link a is described as the following continuously differentiable, increasing function:

$$t_a(v_a) = t_a^0\left(1 + \alpha\left(\frac{v_a}{v_a^{max}}\right)^{\beta}\right). \tag{3}$$

To simplify, we define the feasible region of equality and inequality constraints of (2) as Ω which will be used in the following analysis. Intuitively, the objective function of the above optimization model (2) is an extension of the Beckmann transform, thus, it is easy to prove that the objective function of the optimization model (2) is convex and its feasible region Ω is convex. Therefore, this model has a unique solution.

Without loss of generality, the following assumptions are declared throughout the paper:

(i) It is assumed that the hybrid (mixed) traffic network in this paper refers to two categories of vehicles over the whole network, including electric vehicles and gasoline vehicles. Beyond that, it is also assumed that all users studied in this paper own both categories of vehicles.

(ii) The traffic demand for these two types of vehicles is fixed and equal, and elastic and stochastic demands is not taken into account.

(iii) The link flows of electric vehicles are identified as covered if there exists a charging station on this link.

(iv) The charging facility has enough electricity to supply the electric vehicles in demand, and the charging time is ignored in this study.

(v) Each electric vehicle is fully charged at the origin, which means that the maximum driving distance of the electric vehicle is not affected by the initial charge before the vehicle leaves.

(vi) The stochastic user equilibrium is not included in this study, which means we assumed that the drivers always choose the least cost path from a given origin to destination.

2.2 Problem analysis

In this subsection, we mainly analyse and transform the lower level model (2). It is noted that the low-level model is an extension of the traditional user equilibrium problem. Different from the traditional user equilibrium problem, there are two inequality constraints related to traffic and route respectively in problem (2). For the traditional user equilibrium problem, there was no effective algorithm until 1975 when Leblanc et al. used the Frank-Wolfe algorithm to solve it [26]. The Frank-Wolfe algorithm is effective for solving nonlinear programming problems with linear constraints. The basic idea of the Frank-Wolf algorithm is to use a linearized algorithm to approximate the nonlinear programming problem iteratively. However, since there are two inequality constraints in model (2) and these inequalities are not always guaranteed to be true in the process of solving the problem, the Frank-Wolf algorithm cannot be directly applied to model (2). Therefore, before formally proposing our algorithm, we need to modify the step of solving the feasible descent direction in the Frank-Wolfe algorithm to ensure that the inequality constraints in (2) are always satisfied.

In what follows, we first give the following definitions to pave the way for the transformation of model (2), and then point out that the descent direction of the Frank-Wolfe algorithm can be solved by finding the gradient of optimization model (2), i.e. (6). Specifically, we abbreviate the objective function in (2) as

$$S(f^{rs}) = \sum_{a\in\mathcal{A}}\left[\rho\int_0^{v_a} t_a(w)\,dw + (v_{a,g}c_g + v_{a,e}c_e)\,d_a\right].$$

For a given network, we numbered the different O-D pairs over the whole network from 1 to M, and numbered the corresponding feasible paths of each O-D pair as N_1 and N_M. Thus, M and N respectively stand for the numbers of O-D pairs and the possible paths for the specified O-D pair. Based on these, redefine the decision variable of model (2) as the following stacked vector:

$$f^{rs_i} = \left[f^{rs_i}_{l,g}, f^{rs_i}_{l,e}\right]_{l=1}^{N_i} \in \mathbb{R}^{1\times 2N_i}$$

where $i \in \{1,...,M\}$ represents the ith O-D pair, $N_1 + N_2 + ... + N_M = N$. The superscript rs_i of f^{rs_i} represents the ith O-D pair (r,s). Hence, f^{rs_i} consists of the path flow of gasoline vehicles and electric vehicles for ith O-D pair (r,s), in consequence, we utilize f^{rs} to represent an integrated vector consisting of the corresponding path flow of two categories of vehicle types for different O-D pairs on different paths in a certain order. More specifically, let f^{rs} be a vector that consists of N components, that is,

$$f^{rs} = [f^{rs_i}, ..., f^{rs_M}]^T \in \mathbb{R}^{2N}.$$

Based on the Frank-Wolfe algorithm for user equilibrium problem, the approximate linear objective function of model (2) can be obtained through the first-order Taylor expansion for the objective function of problem (2) in the nth iteration at $f^{rs}(n)$, which is described as:

$$\begin{aligned} \min_{f^{rs}} \quad & [\nabla S(f^{rs}(n))]^{\mathrm{T}} \cdot f^{rs} \\ \text{s.t.} \quad & f^{rs} \in \Omega \end{aligned} \tag{4}$$

where, as mentioned above, Ω stands for the shorthand of the constraint set (feasible region) presented in model (2), and $\nabla S(f^{rs}(n))$ represents the gradient of the path flow f^{rs} of objective function $S(f^{rs})$ at iteration n:

$$\nabla S(f^{rs}(n)) = \begin{bmatrix} \left[\sum_{a\in\mathcal{A}} \delta^{rs_i}_{a,1} c_{a,g}(n), \sum_a \delta^{rs_i}_{a,1} c_{a,e}(n)\right]^{\mathrm{T}} \\ \vdots \\ \left[\sum_{a\in\mathcal{A}} \delta^{rs_M}_{a,N} c_{a,g}(n), \sum_a \delta^{rs_M}_{a,N} c_{a,e}(n)\right]^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{2N}.$$

where $c_{a,g} = \rho t_a(v_a) + c_g d_a$ and $c_{a,e} = \rho t_a(v_a) + c_e d_a$ are the generalized travel cost with respect to gasoline and electric vehicles for any link $a \in \mathcal{A}$, respectively. Intuitively, the structure of $[\nabla S(f^{rs}(n))] \in \mathbb{R}^{2N}$ is complex according to the specific form of $\nabla S(f^{rs}(n))$ described above. To simplify, Define the following stacked column vector

$$C(n) = \begin{bmatrix} c_{a,g}(n) \\ c_{a,e}(n) \end{bmatrix}_{a=1}^{A} \in \mathbb{R}^{2A}$$

as a sum of energy cost and time cost of each link over the road network. According to the specific forms of $\nabla S(f^{rs}(n))$ and $C(n)$ presented above, we can give its equivalent but simplified form as follows:

$$\nabla S(f^{rs}(n)) = \delta^{rs}_{I_2} C(n) \in \mathbb{R}^{2N} \tag{5}$$

where $\delta^{rs}_{I_2} = \delta^{rs} \otimes I_2 \in \mathbb{R}^{2N\times 2A}$. Given that the total number of links in the road network is A, the total number of possible links is N, then the path-link incidence matrix for the studied road network is assumed to be $\delta^{rs} \in \mathbb{R}^{N\times A}$. For every possible path in matrix δ^{rs}, the corresponding element is set as 1 if the path contains the corresponding link, otherwise 0. In addition, for a given link a, $C(n)$ is a constant vector in the nth iteration, which can be abbreviated to a constant vector $C \in \mathbb{R}^{2A}$.

Remark 1 *The reason for expanding the matrix δ^{rs} in equation (5) is that there may be two categories of vehicles driving on the same link instead of just one type of vehicle, therefore the path and link relation matrix should be extended to the matrix $\delta^{rs}_{I_2}$.*

Based on the above analysis, for any feasible path k corresponding to O-D pair (r,s), the problem

(4) can be transformed into the following equivalent problem:

$$\begin{aligned} \min_{f^{rs}} \quad & C^{\mathrm{T}}\left([\delta_{I_2}^{rs}]^{\mathrm{T}} f^{rs}\right) \\ \text{s.t.} \quad & \xi f^{rs} = Q^{rs},\ \ \zeta^{\mathrm{T}}\left[\delta_{I_2}^{rs}\right]^{\mathrm{T}} f^{rs} \le V_a, \\ & \left(D - l_k^{rs}\right) f_{k,e}^{rs} \geqslant 0,\ f_{k,g}^{rs} \geqslant 0,\ f_{k,e}^{rs} \geqslant 0, \end{aligned} \tag{6}$$

where ζ and ξ are both superposition matrices, $\zeta = I_A \otimes 1_2^{\mathrm{T}}$,

$$\xi = \begin{bmatrix} 1_{2N_1}^{\mathrm{T}} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1_{2N_M}^{\mathrm{T}} \end{bmatrix} \in \mathbb{R}^{M \times 2N},$$

$V_a = [v_i^{\max}]_{i=1}^{A} \in \mathbb{R}^A$ and $Q^{rs} = \left[q^i\right]_{i=1}^{M} \in \mathbb{R}^M$. It is noted that problem (6) is a linear programming problem with the linear constraints.

2.3 Optimality conditions

The main purpose of this subsection is to prove that the optimality conditions of (2) described in subsection 2.1 are equivalent to the extended Wardriop's equilibrium principles. It can be stated that the drivers' choice of driving vehicle type and route choice of drivers under equilibrium state is ultimately the result of driving distance limit and travel cost function.

For each O-D pair, we firstly give the Lagrangian function of the lower level model (2) as

$$\begin{aligned} & L\left(f^{rs}, \gamma_k^{rs}, \lambda_k^{rs} \psi_a\right) \\ = & \sum_{rs}\sum_{k} \gamma_k^{rs}\left[q^{rs} - \sum_{k}\left(f_{k,g}^{rs} + f_{k,e}^{rs}\right)\right] \\ & + S\left(f^{rs}\right) + \psi_a \sum_{a \in \mathcal{A}}\left(v_a - v_a^{max}\right) \\ & + \sum_{rs}\sum_{k} \lambda_k^{rs}\left(l_k^{rs} - D\right) f_{k,e}^{rs} \end{aligned} \tag{7}$$

where λ_k^{rs} , ψ_a and γ_k^{rs} are respectively the corresponding Lagrange multipliers associated with inequality constraints $(D - l_k^{rs}) f_{k,e}^{rs} \geqslant 0$, $v_a \leqslant v_a^{max}$, and equality constraint $\sum_k \left(f_{k,g}^{rs} + f_{k,e}^{rs}\right) = q^{rs}$. Consequently, for any path k and link a, the optimal conditions for the above Lagrangian function are listed as follows:

$$0 \leqslant \lambda_k^{rs*} \perp \left(l_k^{rs} - D\right) f_{k,e}^{rs*} \leqslant 0, \tag{8a}$$

$$0 \leqslant \gamma_k^{rs*} \perp \left(v_a - v_a^{max}\right) \leqslant 0, \tag{8b}$$

$$q^{rs} - \sum_k \left(f_{k,g}^{rs*} + f_{k,e}^{rs*}\right) = 0, \tag{8c}$$

$$0 \leqslant f_{k,g}^{rs*} \perp \left(\sum_a \left(c_{a,g} + \psi_a\right) \delta_{a,k}^{rs} - \gamma_k^{rs*}\right) > 0, \tag{8d}$$

$$0 \leqslant f_{k,e}^{rs*} \perp \left(\begin{array}{c} \sum_a \left(c_{a,e} + \psi_a\right) \delta_{a,k}^{rs} \\ + \lambda_k^{rs*}\left(l_k^{rs} - D\right) - \gamma_k^{rs*} \end{array}\right) > 0. \tag{8e}$$

where the variable with superscript $*$ represents the optimal solution corresponding to the variable. The Wardrop principle states that the generalized travel cost of the selected paths is identical and less than or equal to the generalized travel cost of the non-selected paths for any O-D pair over the traffic network. Suppose $\sum_{a \in \mathcal{A}} \left(c_{a,g} + \psi_a\right) \delta_{a,k}^{rs} = c_{k,g}^{rs}$ as the generalized travel cost of the gasoline vehicles on path k for O-D pairs (r,s), and denote γ^{rs*} existed in the above optimal conditions as the lower limit of generalized travel cost for O-D pairs (r,s). According to the formulas (8d), all drivers will choose the gasoline vehicles for O-D pairs (r,s) when the generalized travel cost of the gasoline vehicles on path k for O-D pairs (r,s) is equal to the corresponding minimum generalized travel cost γ_k^{rs*}. Otherwise, no drivers would choose the gasoline vehicles if the generalized travel cost of the gasoline vehicles on path k for O-D pairs (r,s) is higher than the corresponding minimum generalized travel cost γ_k^{rs*}. Similarly to the definition of the generalized travel cost of the gasoline vehicles $c_{k,g}^{rs}$, denote the generalized travel cost of the electric vehicles as a sum of two components: $c_{k,e}^{rs} = \sum_a \left(c_{a,e} + \psi_a\right) \delta_{a,k}^{rs} + \lambda_k^{rs*}\left(l_k^{rs} - D\right)$, where $\sum_a \left(c_{a,e} + \psi_a\right) \delta_{a,k}^{rs}$ represents the the path travel cost and $\lambda_k^{rs*}\left(l_k^{rs} - D\right)$ is the out-of-range cost when the path length exceeds the maximum distance limit of electric vehicles. Thus, as for formulas (8e), if there exist the electric vehicle flows on this path, the distance of this path is shorter than the driving distance limit and the generalized travel cost equals to the corresponding lower limit of generalized travel cost. Therefore, (8a) and (8e) collaboratively describe the situation that the distance of some but not all paths is shorter than the maximum travel distance of the electric vehicles for O-D pair (r,s). For this situation, some drivers choose to use electric vehicles under the condition that the generalized travel cost $c_{k,e}^{rs}$ of the gasoline vehicles on path k for O-D pairs (r,s) equal to the corresponding minimum generalized travel cost ξ^{rs*}, other will choose the gasoline vehicles if $c_{k,g}^{rs} = \gamma^{rs^*}$ holds.

3 Solution method

In this section, we will present the designed algorithm to tackle the proposed bi-level problem, in which the upper level model involves bilinear functions while the lower level model is a user equilibrium assignment problem considering the capacity constraint for each link over the whole network. In the initial state, we assume that there are no charging facilities in the transportation network. The initial flow state of electric and gasoline vehicles is obtained by solving the traffic equilibrium problem under various constraints. Perform the upper model to get a maximum of the covered electric vehicle flows of the charging device location. Record the charging facility location and compare it with the optimal charging device location obtained next iteration. If the position of charging facilities does not change, the current recorded charging position is taken as the optimal solution and the problem is terminated. Otherwise, the lower problem needs to be solved again to find a better solution. The detailed steps of the feasible solution iteration algorithm are listed as follows.

Step 1. Let the iteration counter τ of the upper level problem be 1. Initializing the charging facility, that is, suppose there is no charging facility in the whole mixed network and slack the driving distance limit of electric vehicles, eventually, we can obtain the initial link flow pattern.

Step 2. Set $\tau = \tau + 1$. Sort all links in ascending order of their BEV flows over the whole network and find the top p of them. Locate the charging facilities in the middle of p links.

Step 3. Perform user equilibrium assignment in the mixed network. The specific implementation procedure goes as follows:

Step 3.1 (Initialization). For each O-D pair (r,s), a group of link flows are generated randomly, which meet the constraints listed in (2). This yields link flows $v_{a,g}(1)$ and $v_{a,e}(1)$. Let the loop counter be $n = 1$.

Step 3.2 (Update). Update the link cost in terms of $v_{a,g}(1)$ and $v_{a,e}(1)$.

Step 3.3 (Direction finding). Based on the link cost deduced from Step 3.2, the feasible descending direction is determined by the linear programming problem described in subsection 2.2 Problem analysis of Section 2 and the resulting link flows yield auxiliary flows $y_{a,g}(1)$ and $y_{a,e}(1)$ for any link a.

Step 3.4 (Line search). Apply any interval reduction line search method to find the optimal value of θ by solving

$$\min_{0<\theta<1} \left\{ \begin{array}{c} \sum_{a\in\mathcal{A}} [\rho \int_0^{v_a(n)+\theta(y_a(n)-v_a(n))} t_a(w)\,dw \\ +d_a\left[(v_{a,g}(n)+\theta(y_{a,g}(n)-v_{a,g}(n)))c_g \right. \\ \left. +(v_{a,e}(n)+\theta(y_{a,e}(n)-v_{a,e}(n)))c_e\right]] \end{array} \right\}$$

Step 4 (Move). Obtain the new flow pattern of the mixed network in terms of the following rules:

$$v_{a,e}(n+1) = v_{a,e}(n) + \theta\left(y_{a,e}(n) - v_{a,e}(n)\right), \ \forall a,$$

$$v_{a,g}(n+1) = v_{a,g}(n) + \theta\left(y_{a,g}(n) - v_{a,g}(n)\right), \ \forall a.$$

Step 5 (Convergence test). Let

$$\bar{v}_a(n) = \frac{1}{m}\left(v_a(n) + v_a(n-1) + \ldots + v_a(n-m+1)\right)$$

hold, and the values of m and n are identical and both are positive integers. If the convergence criterion

$$\frac{\sqrt{\sum_{a\in\mathcal{A}}\left(\bar{v}_a(n+1) - \bar{v}_a(n)\right)^2}}{\sum_{a\in\mathcal{A}}\bar{v}_a(n)} \leqslant \rho$$

is not satisfied, set $n = n+1$ and go to **Step 3.2**; otherwise, stop and $v_a(n+1)$ and $v_{a,e}(n+1)$ are equilibrium link flows of total vehicle flow and electric vehicle flow for each link a, respectively.

Step 6. Perform the **Step 2** to renew the present charging device location. Identify whether the position of the latter charging device location changes with that of the former one, jump to **Step 3** if the judgment is positive, otherwise, stop and keep a record of the current locations.

4 Numerical experiment

In this section, a numerical example is presented to look into the link-flow patterns of gasoline and electric vehicles given different driving distance limits and link capacity constraints. The road network used in this section is the Nguyen-Dupuis network in [13], which is shown as Figure 1. Intuitively, there exist 4 O-D pairs, 13 nodes, and 19 links in this road network.

Table 1. The demand-dependent input parameters of Nguyen-Dupuis network.

O-D pair	O-D demand	O-D pair	O-D demand
$(1,2)$	200	$(1,3)$	300
$(4,2)$	300	$(4,3)$	200

The four colored nodes in Figure 1 represent two origins (nodes 1 and 2) and two destinations (nodes 3 and 4) of the directed Nguyen-Dupius network, which indicates that there exist four O-D pairs $(1,2)$, $(1,3)$, $(4,2)$ and $(4,3)$. In addition, the directional arrows in Figure 1 denote the direction of vehicle flow between two nodes, and the number marked on each link is its serial number. The traffic demand of Nguyen-Dupuis for each O-D pair over the whole network is presented in Table 1.

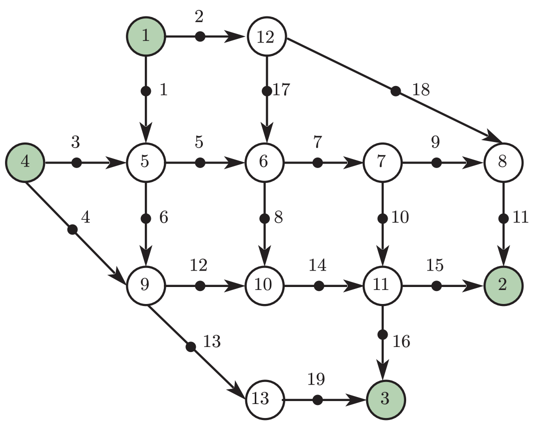

Figure 1. Nguyen-Dupius network

Table 2 shows the corresponding route information for different O-D pairs over the Nguyen-Dupuis network, while the longest and shortest paths are marked in bold. In this experiment, the relevant parameters are set as follows. We assume that only five charging facilities are allowed to be installed in the whole Nguyen-Dupuis network to charge the electric vehicle, and only one charging pile can be installed in each link, which obviously results in $p = 5$. Following the existing studies that also utilized BPR function [27], set the function coefficients α and β in travel time function (3) on link a as 0.15 and 4, respectively. The value of time $\rho = 4$, $c_g = 0.16$ and $c_e = 0.04$ are the component parameters of the generalized travel cost of gasoline and electric vehicle on link a separately.

Table 2. The routing information of Nguyen-Dupuis network.

O-D pair	Route number	Route composition	Route length
	1	1-5-6-7-8-2	**29**
	2	1-5-6-7-11-2	33
	3	1-5-6-10-11-2	38
$(1,2)$	4	1-5-9-10-11-2	41
	5	1-12-6-7-8-2	35
	6	1-12-6-7-11-2	39
	7	1-12-6-10-11-2	**44**
	8	1-12-8-2	32
	9	1-5-6-7-11-3	32
	10	1-5-6-10-11-3	37
$(1,3)$	11	1-5-9-10-11-3	40
	12	1-5-9-13-3	36
	13	1-12-6-7-11-3	38
	14	1-12-6-10-11-3	43
	15	4-5-6-7-8-2	31
	16	4-5-6-7-11-2	35
$(4,2)$	17	4-5-6-10-11-2	40
	18	4-5-9-10-11-2	43
	19	4-9-10-11-2	37
	20	4-5-6-7-11-3	34
	21	4-5-6-10-11-3	39
$(4,3)$	22	4-5-9-10-11-3	42
	23	4-5-9-13-3	38
	24	4-9-10-11-3	36
	25	4-9-13-3	32

Assume the maximum range limit for electric vehicles is $D = 37$. Table 3 contains three types of information, namely, link length, link capacity, and equilibrium link flow. It should be noted that we have given two kinds of experimental results on the total equilibrium flow on each link: one contains link-free capacity constraints and the other contains link-capacity constraints, whose corresponding results are described by the inclusion and exclusion columns in Table 3.

Table 3. The total link flow information under whether or not the link capacity constraint is included

Link Number	Link Length	Link Capacity	Link flow	
			Inclusion	Exclusion
1-5	7	300	300	300
1-12	9	200	300	200
4-5	9	350	200	0
4-9	12	300	300	**500**
5-6	3	200	200	0
5-9	9	350	300	300
6-7	5	200	200	0
6-10	13	300	0	0
7-8	5	200	100	0
7-11	9	300	100	0
8-2	9	300	300	200
9-10	10	350	300	300
9-13	9	300	300	**500**
10-11	6	300	300	300
11-2	9	300	200	300
11-3	8	300	200	0
12-6	7	300	0	0
12-8	14	200	200	200
13-3	11	300	300	**500**

From Table 3, it can be easily found that the link flow sometimes exceeds the maximum capacity of the corresponding link if the link capacity constraint over the network is not considered (for example, link numbers 4-9, 9-13 and 13-3), which is inconsistent with the premise of this study that there is no blocking of the entire road network. In contrast, if the link capacity constraint is always considered in the process of solving the bi-level model, the obtained link flow for each link will not exceed the capacity of the corresponding link.

Moreover, we also conducted the sensitivity analysis on the parameters of driving distance limit D for electric vehicles. The reason why we analyse the sensitivity of the distance limit of electric vehicles is that the length of each link over the whole network will affect the route selection behavior of electric vehicle drivers. If the length of a link exceeds the maximum driving distance for an electric vehicle, it is clear that the route chosen by the drivers will not include that route. The results of the experiment also confirmed this fact. The details for partial path flow of electric and gasoline vehicles are shown in Figure 2 and Figure 3, respectively. What needs to be explained is that only part of the path flow information is given, and the path information without traffic flow is omitted.

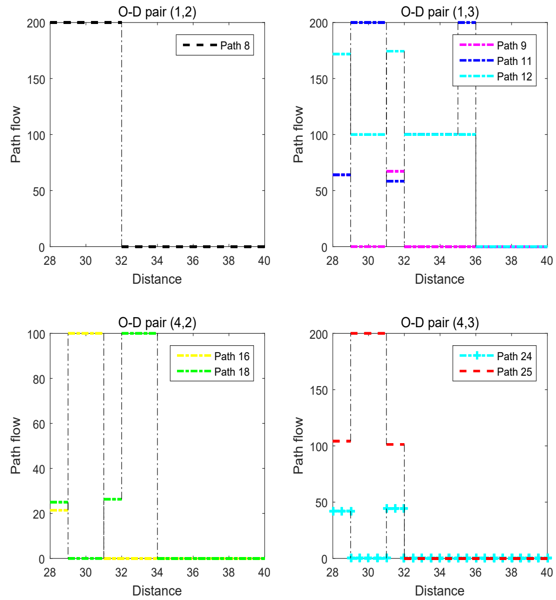

Figure 2. The path flow of electric vehicles for different O-D pairs

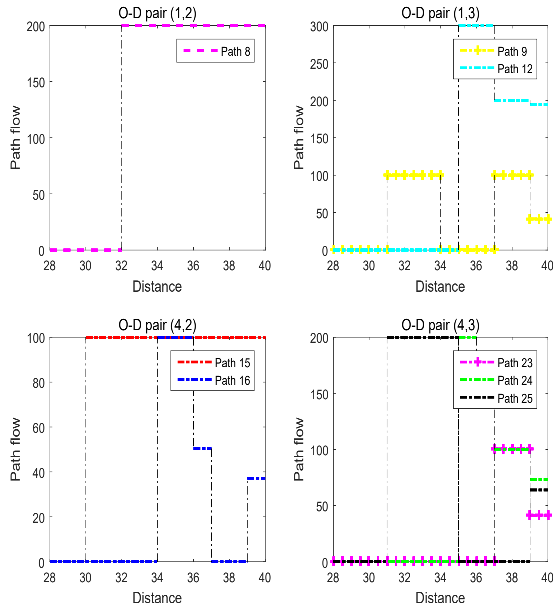

Figure 3. The path flows of gasoline vehicles for different O-D pairs

As can be seen from the Figure 2 and Figure 3, the path selection of the fixed O-D pair by the driver of an electric vehicle is closely related to the maximum driving distance. For example, based on

Table 2, we obtain that there is no feasible path for the electric vehicle for OD pair (1,2) when the maximum driving distance of the electric vehicle is 32, all the drivers choose gasoline vehicles to achieve their travel plans from origin 1 to destination 2 under Table 3. Specifically, the flow of electric vehicles on the link included in the path for O-D pair (1,2) becomes 0 while the flow of gasoline vehicles is 200, which is consistent with the results in Figure 2 and Figure 3.

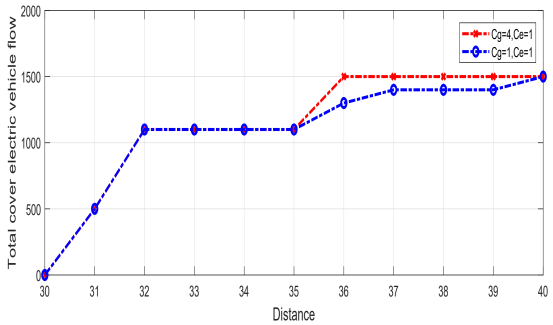

Figure 4. The relationship between the maximum travel distance of electric vehicle and total cover electric vehicle flow

Except for the above experimental results, we utilize Figure 4 to depict the relationship between the maximum driving distance of electric vehicles and its total covered flow of charging facilities. Intuitively, the greater the maximum range of electric vehicles, the greater the number of possible routes, thus increasing the utilization of each link over the entire network. Assuming that the charging facilities are installed in several high traffic links, the total covered traffic flow will inevitably increase or at least remain unchanged. Figure 4 illustrates the same phenomenon that the total covered electric vehicle flows remain unchanged or increase with the increase of maximum range for electric vehicles. In addition to the horizontal observation, it can also be seen from the vertical comparison in Figure 4 that when the maximum driving distance of electric vehicles is not considered, more drivers will choose to use electric vehicles if the cost of fuel vehicles is greater than that of electric vehicles. Beyond these, according to Figure 4, even though the cost of gasoline vehicles is greater than that of electric vehicles the total cover of electric vehicle flow remains unchanged due to the limit of the maximum driving distance if the maximum range of electric vehicles is less than 35. On the contrary, when the maximum range of electric vehicles is greater than 35, more drivers choose to travel by electric vehicle. Therefore, there is a close relationship between battery performance and whether users choose electric vehicles for travel.

Conclusion

This paper studies the location problem of electric vehicle charging facilities in a hybrid traffic network, and puts forward a bi-level mathematical model considering the constraints of maximum driving distance of the electric vehicle and link capacity for each link, which is more reasonable than the traditional traffic assignment problem. The hybrid traffic network in this paper refers to two types of vehicles, namely electric vehicles and gasoline vehicles. In view of the fact that many current user equilibrium traffic assignments do not consider the constraints of link capacity, based on the Framework of Frank-Wolf method, this paper makes a detailed analysis of this problem, i.e. the low-level model, from another perspective based on the Frank-Wolf method framework. The results of numerical examples show that the drivers' routing selection is closely related to the maximum driving distance of the electric vehicles and the maximum flow constraint of each link. In addition, the application of the proposed algorithm for the representative Nguyen-Dupius network shows that the solution process is applicable to the general network with link capacity constraints, which theoretically provides another perspective for the study of the charging position of electric vehicles over the mixed road network. In the future, considering that the actual traffic demand is time-varying, it is of great significance to determine the charging position of electric vehicles in the dynamic traffic network by taking this dynamic characteristic into account.

Acknowledgments

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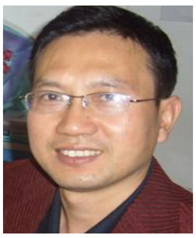

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