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STRENGTH PROPERTIES OF THIN-WALLED STRUCTURES DESIGNED WITH THE SADSF METHOD

Key words: design, thin-walled structures, limit analysis, SADSF, FEA.

Abstract: This study presents general conclusions resulting from elastic FEM analyses of several dozen of thin-walled structures designed with the use of the application version of the method of statically admissible discontinuous stress fields (SADSF).
The conclusions are illustrated with examples of two original structures designed to work under torsional load. The structures are based on I-sections, whose all surfaces are accessible from outside. Despite that fact, torsional rigidity of the structures is very high. Additionally, in these structures, we examined the effects caused by modifications consisting in local stiffening of some elements by applying edge stiffeners on their borders.

Sprężyste własności konstrukcji cienkościennych zaprojektowanych metodą SADSF

Słowa kluczowe: projektowanie, konstrukcje cienkościenne, nośność graniczna, analizy MES.

Streszczenie: Praca przedstawia generalne wnioski z analiz sprężystych MES kilkudziesięciu konstrukcji cienkościennych, które zostały zaprojektowane z wykorzystaniem wersji aplikacyjnej metody statycznie dopuszczalnych nieciągłych pól naprężeń (SADSF). Otrzymane wnioski są prezentowane na przykładzie dwóch oryginalnych konstrukcji, które zostały zaprojektowane na obciążenie momentem skręcającym. Są to konstrukcje oparte na profilu dwuteowym, których wszystkie powierzchnie są z zewnątrz dostępne, a mimo to są bardzo sztywne na zakładany rodzaj obciążenia. Dla konstrukcji tych badano dodatkowo efekty powodowane poprzez wprowadzanie modyfikacji polegających na lokalnym usztywnieniu elementów poprzez odwijanie ich swobodnych krawędzi.

Introduction

This work refers to the results included in the Polish-language monograph [1], and it presents a fragment of general conclusions of an extensive program aimed at identifying actual properties of elements and structures designed with the method of statically admissible discontinuous stress fields (SADSF, [2–6, 7–10]).

The aim of the program was to decisively verify the practical usefulness of the SADSF method in structure design. The investigations concentrated on thin-walled structures, whose properties predestine them for applications in practically all fields of mechanical engineering, where the methods appropriate for design of these structures have always been in the centre of engineers' attention.

The following investigations have been conducted in the framework of this program:

– Determining distribution of effort fields in the elastic range by FE method;
– Investigating the development of plastic zones in the range of elasto-plastic deformations with the use of thermovision, investigating actual mechanisms of collapse and paths of equilibrium in the whole range of applied loads; and,
– Evaluating fatigue life for time-variable loads by applying the local strain-life method.

In this work, we present conclusions formulated based on elastic FEM analyses carried out on several dozen structures designed with the SADSF method [1, 11–12]. Then, these conclusions can be treated, with a high degree of probability, as general ones pertaining to all thin-walled structures designed with the SADSF method. In all examined cases, good or very good load-carrying properties were found.

In designing such structures, one of the goals is to obtain an evenly distributed equivalent stress of material and low stress concentrations. This aim is usually achieved by correcting the preliminary project of the structure, which is created based on the intuition and experience of the designer. The consecutive corrections are introduced on the basis of numerical analyses results, usually cared out with the use of FEM. Nowadays, it is also possible to apply more advanced mathematical and numerical methods of the so-called topological optimization [13–14], which still rely on consecutive, iterative corrections.

Unfortunately, in the case of thin-walled structures, such an approach does not guarantee that an optimal solution is obtained. The characteristic feature of these structures is that they generally do not comply with de Saint Venant principle, and even a small change in either boundary conditions or structural details may lead to great and wide-spread changes in stress and strain fields (e.g., [3]).

Structures of this type are particularly sensitive to the changes of structural parameters, i.e. to the number of component elements, their spatial allocation, and the system of mutual connections between them [3]. Even small changes in the structure may cause significant changes in its carrying capacity. Errors of the structure cannot be rectified by increasing its dimensions. For this reason, FEM analyses and possible corrections of the boundaries can only be applied to the structures which have initially been correctly designed. This means that the constructed system must have the structure appropriate for the load exerted on it. The corrections themselves should only concern some geometric parameters whose modifications do not cause such radical changes in the structure's performance.

The SADSF method makes it possible to create, in a simple and effective way, preliminary projects of structures that are free of structural errors [3]. The method can be used already at the moment when only boundary conditions are known. It is only required that the designer should be familiar with elementary statics, be inventive, and know how to assemble ready-made particular fields. In the existing SADSF software packages, sets of these fields are given in the form of libraries [2–4].

Unfortunately, the SADSF method is an approximate one. The source of this imperfection lies in the conclusions from the lower-bound theorem, which can be summarized as follows [7]:

Safe estimation of shape and dimensions of the designed constructional element can be done by creating an adequate, statically admissible stress field, capable of carrying the given external load. The contour surfaces of such a field determine the shape of an element.

Statically admissible fields are the ones which satisfy equilibrium equations within the field, static conditions on the boundaries, and which do not exceed the assumed yield condition at any point. In general, such fields do not satisfy any kinematic conditions (compatibility conditions and kinematic boundary conditions). Such fields are then usually quite distinct from the real ones, and additionally, in the case of the SADSF method, they are discontinuous. It is assumed that the discontinuity lines are straight line segments which separate homogeneous stress regions.

In constructing such fields, one aims at satisfying the assumed yield condition in as many regions as possible. For a given set of boundary conditions, it is sometimes possible to construct several solutions of statically admissible stress fields. From among them, one can select those which satisfy additional criteria associated, for example, with minimized volume, reduced production costs, or increased fatigue life.

The SADSF method assumes the use of a rigid, ideally-plastic physical model of material, which is far from reality, and analyses only the limit state pertaining to the beginning of structure's collapse. In the case of thin-walled structures, it is additionally assumed that the collapse develops in a particular way, i.e. with maintaining plane (membrane) state of stress in every component element. One then assumes that the use of material is optimal and that the component elements behave like rigid blocks until the limit load is reached. A further increase of load leads to plasticization and collapse of the structure.

Despite these limitations, structure elements designed by the SADSF method exhibit good, sometimes surprisingly good, strength properties in the whole range of applied loads (also in elastic range) and have high fatigue life. Unfortunately, it is impossible to prove that this is the case in any system designed by this method. However, because we have at our disposal a great number of results confirming good properties of designed structures, we can confidently expect them in all cases (e.g., [1–3, 5–12]).

The results obtained within the framework of the FEM analysis are illustrated in this work by two interesting examples of original structures, which are based on an I-section, and are made of plane elements. These structures have proportions similar to those of cross-members of a vehicle frame. Torsional strength of such a structure is high despite the fact that all its surfaces are open to the outside. It shows that open cross-sections structures do not need to have low torsional rigidity, as it is suggested by simplified, one-dimensional theories. The example structures were designed by W. Bodaszewski with the use of the author's original software [3–4].

The reason for conducting analyses in elastic range was that this range is not considered in the SADSF method, but for most of the structures this is their operating range. The application of FEM made it possible to easily analyse – even before physical models were made – the effects caused by introducing modifications into the designed structures. These consisted in applying edge stiffeners on some elements to achieve the effect of local stiffening [1]. Such modifications were also introduced, among others, in order to experimentally compare load-carrying capacity of component elements.

The SADSF method was initiated by W. Szczepiński [7], and then developed mainly in Poland by the teams from the Institute of Fundamental Technical Problems of the Polish Academy of Sciences (IPPT PAN), Warsaw University of Technology and Kielce University of Technology.

However, the initial development of the method was impeded by very difficult problems of unknown discretization that appear in this method. For a long time, these problems remained unsolved, until a breakthrough was introduced by the works of W. Bodaszewski, which allowed for building effective algorithms. The present state of the development of the SADSF method fundamentals is presented in the monograph [2], and the application version of this method one can find in the monograph [3].

An extensive verification of properties of structures designed with the SADSF method, based on numerous examples, can be found in the above-mentioned monograph [1]. The conclusions presented there are consistent with those drawn from similar investigations that have been conducted (on a smaller scale) by other authors since the method came into existence (e.g., [5–10]).

1. Formulation and solution of design problem by means of SADSF method

The spatial structure, as well as the shape and dimensions of component elements of the analysed models of thin-walled structures, were determined using the software package SADSFaM [3–4], which operates on the basis of the Treska yield condition. The models were created by solving the problem whose formulation is illustrated in Fig. 1a.

The set of data consisted of the following [2, 3]: the limit load **P** applied on the segment S_p of the boundary S (here – it is reduced to the limit moment M_{gr}=P×h), the geometry of the segment S_p (here – the dimensions L,

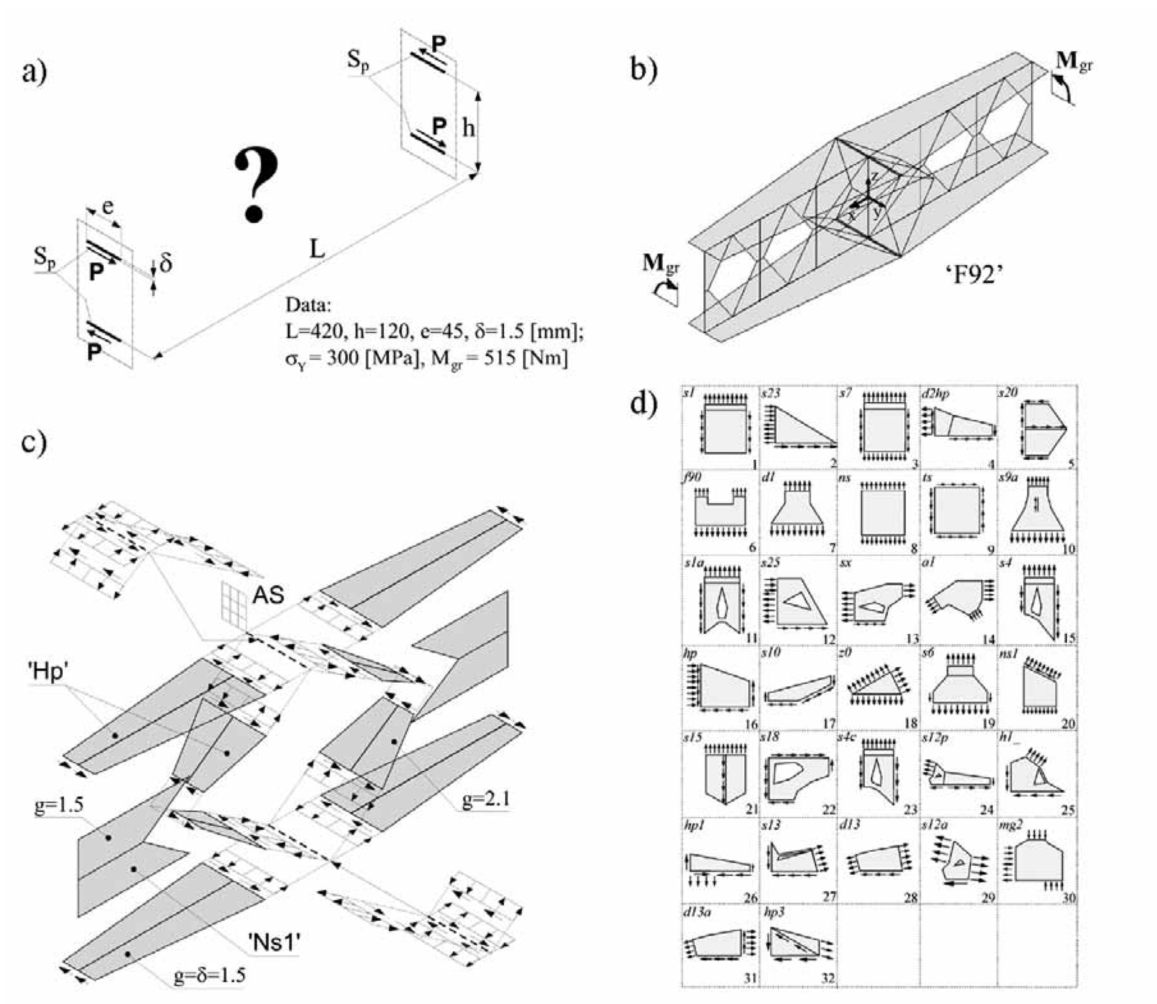


Fig. 1. Formulation and solution of design problem [3]: a) illustrative presentation of boundary conditions and problem formulation; b) statically admissible solution, complex spatial field that determines the structure; c) allocation of component library fields and interactions between them in the complex field; d) library of particular solutions from SADSFaM software package

h, e, δ), and the plastic properties of the material of the sought-after structure (yield point σ_y).

For the given boundary conditions, one can sometimes construct several solutions, because the assumed criterion (satisfying the yield condition in as many homogeneous regions as possible) is not unique. It allows us, already when assembling the fields, to take into account limitations imposed on the structure, and consequently select, from the whole set of obtained solutions, those which satisfy some additional criteria, i.e. ensure the lowest stress concentrations and the longest fatigue life, or the smallest volume. The number of solutions that are possible to construct primarily depends on the amount of fields available from the library and the designer's ingenuity.

In this work, we concentrate on two structures whose general forms are related to an I-section. Their solutions were constructed by selecting and assembling fields from the library that is presented in Fig. 1d.

The first of the considered solutions, denoted as 'F92', is shown in Fig. 1b, while allocation of component fields and their mutual interactions are shown in Fig. 1c. As one can see, the fields of type 'Hp' were used to design the flanges. In this way, the boundary conditions assumed on the boundaries S_p were satisfied. Satisfying boundary conditions on the connecting boundaries (broken lines in Fig. 1c) required introducing additional oblique elements in which the fields of type 'Hp' were also used, but with different boundary condition parameters. From equilibrium equations set up on the connecting boundaries, with assumed flange thickness g = δ = 1.5 [mm], one obtained the thickness of oblique elements equal to 2.1 [mm] (Fig. 1c). In the web of the section, one assumed the fields type 'Ns1' with a zero state of stress and the thickness of 1.5 [mm] – the only role of this element is 'to keep to the geometry'.

From the analyses of interactions between the component elements (see Fig. 1c), it follows that the bimoment appears there and arises with the distance from cross-sections of the external diaphragms, but it is enclosed within the system of additional oblique elements (look at the interactions on the boundaries marked with broken lines, which lie in the anti-symmetry plane AS). There, one can expect finding the regions of maximal equivalent stress, and this expectation is confirmed by FEM analyses. It can also be seen that, with the exception of the unloaded web, all other elements work similarly to plates subjected to non-uniform bending in their planes (without out-plane bending). Therefore, these are the elements in which, in the elastic range of operation, could appear the tendency of forming the states characteristic for bending axis, and deformations could arise that increase with the distance from this axis. Then, in such a structure, one can expect significant discrepancies between limit fields and elastic fields, and suppose that this could be one of the worse cases of the investigated structures.

The second example of a solution to the problem from Fig. 1a, denoted as 'F95', is presented in Fig. 2. This solution was obtained for slightly different values of parameters. The shapes and dimensions of the flanges were determined here by assembling the fields of type 's10'. The fulfilment of equilibrium conditions on the oblique boundaries of these fields required introducing additional elements in which one applied the fields of type 'Ts' that realize pure shear. Solving equilibrium equations set up on the boundaries of these fields, with the assumed flange thickness g= δ =2 [mm], we obtained the thickness of oblique elements equal to 1.8 [mm]. In the web, one assumed a zero state of stress, the fields type 'Ns1', and the thickness g=2 [mm].

There are no interactions in the symmetry plane of the structure (and anti-symmetry of internal forces, marked AS in Fig. 2b), which have a direction perpendicular to this plane, and could produce a bimoment. In this case, similarly as in the previous

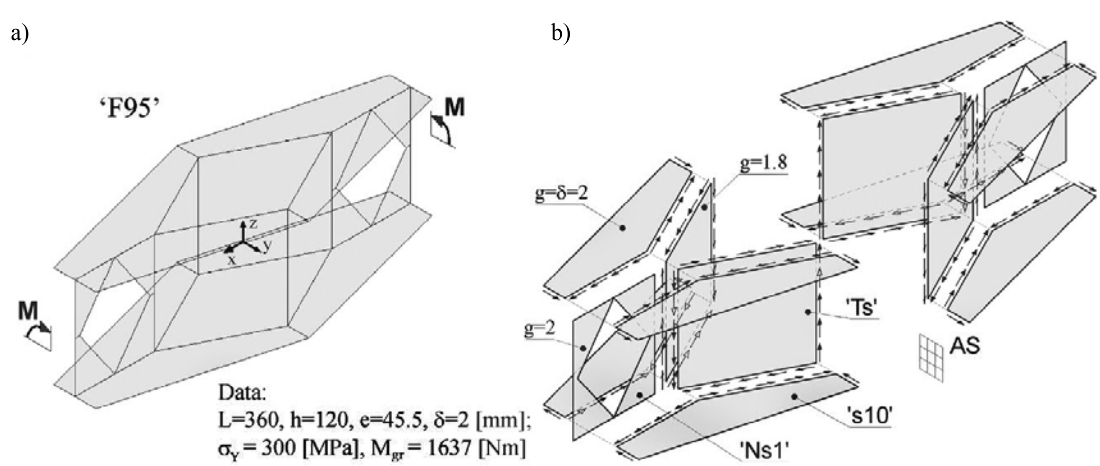


Fig. 2. Another example of solution to the problem from Fig. 1a [3]: a) contours of the model determined based on statically admissible stress field; b) spatial allocation and interactions between component library fields in the complex field

model, the bimoment increases with the distance from external cross-sections of the diaphragms, but it grows only to the point where the oblique elements appear. Then, the bimoment decreases and approaches zero in the cross-section AS. It is worth mentioning just now that, in the vicinity of cross-sections where the bimoment takes its greatest value, there also appears the highest stress concentrations in the elastic range.

The solutions obtained by means of the SADSF method define the structure in a complete way. They not only provide information about spatial allocation of plane component elements, but also define their shapes and dimensions [2–3]. Interactions between the component fields, shown in Figs. 1c and 2b, indicate even the places where the elements must be connected.

It is worth mentioning that, when designing by means of the SADSF method, one assumes a priori that each component element of the structure would work in the membrane state. Such an assumption means that, in the real structure, there is a possibility to carry the applied load by the membrane forces.

2. Linear FEM analyses

2.1. Assumptions and method of result presentation

The analyses were carried out by means of the finite element method (FEM) using the system CosmosM. In the analyses, one assumes the following [1]:

– A linearly-elastic physical model of material and small strains;
– Triangular shell elements of 6 nodes and 6 degrees of freedom in a node type SHELL6;
– Loads introduced into the models by means of additional diaphragms, similarly as it was done in experimental investigations;
– The torsional moment load introduced into front diaphragms by means of the forces Fy applied to the nodes lying on the borders of holes (see Figs. 3a and 6a); in the rear diaphragms, the nodes lying on the borders of holes were deprived of the degrees of freedom Uy, and additionally – in order to exclude the possibility of rigid motion – the displacements Ux, Uz and rotations Ry and Rz of the node lying in the centre of the diaphragm were fixed; and,
– A load value equal to a half of that assumed in designing the limit load-carrying capacity.

Additionally, one assumes the following [1]:

– A yield point of $\sigma_Y = 300$ [MPa] is used for determining the limit load value, which means that, if one could obtain an ideal level of equivalent stress, the intensity of equivalent stress would be $\sigma_{eq} = 150$ [MPa] at each point of the analysed structure;
– The shape and dimensions of the analysed models are nearly the same as those of the contours obtained from the solutions to design problems; small corrections of external contours introduced only in the vicinity of corners by rounding them with arches drawn outside of external boundaries.
– The task of border correction was not undertaken; instead, the effects caused by border modifications consisting in applying edge stiffeners on external edges were analysed.

The applied shell model allows only for an approximate analysis of the local three-dimensional states that appear in the regions of connections between component elements and edge stiffeners. These states need additional, more precise investigations.

The results are presented in the form of illustrative drawings, which primarily depict important details of the model structure resulting from the solution to the design problems – or the details of their modified shapes, and then colour images of equivalent stress (σ_{eq}) distributions obtained for these models. In the calculations of equivalent stresses, one assumes the Huber-Mises yield criterion.

For each of the analysed models, its rigidity was determined from the formula: $k = M/\phi$ [Nm/degree]; where M – value of the applied torsion moment, ϕ – angle of rotation of the upper edge of membrane 'pl'; the angle was calculated based on the value of displacement of its extreme nodes (see Figs. 3 and 7).

2.2. General results of analyses

In order to facilitate reviewing the obtained results, let us first formulate a list of results which, because of their repeatability, seem to lead to general conclusions. We may then state that, in all of the analysed cases, we find the following [1]:

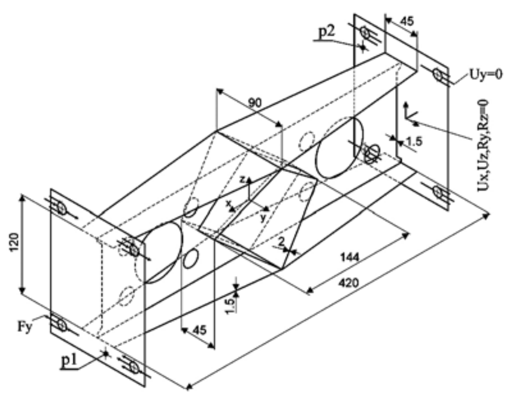
– The domination of membrane states – equivalent stress values are small in bending equivalent stress fields;
– Good equalization of elastic equivalent stress fields along free borders, and, in some cases, also in large fragments of the structure's volume;
– Relatively low concentrations of stress, and similar levels of maximal equivalent stress in component elements;
– Radically better properties of models designed with the SADSF method compared with the ones whose structures were not properly selected for the carried load, or whose structures that were designed intuitively;
– A worsening of elastic equivalent stress fields when one introduces modification to the designed structures, which consist in local stiffening of elements by applying edge stiffeners on their free borders. (In such cases, there appears a greater stress concentration and maximal equivalent stresses become greater; however, the modifications can also increase the values of statically-applied collapse load); and,

– The possibility of reducing the level of maximal equivalent stress in the vicinity of contour refractions by adding some extra material in these places.

2.3. Models based on solution ‘F92’

The shape and dimensions of the model that corresponds, almost identically, to the solution ‘F92’ from Fig. 1b, and the boundary conditions shown in Fig. 3a, were assumed in FEM calculations. The details of the structure of one of the analysed variants of shell model, in which one introduced modification consisting in applying edge stiffeners on free borders of the flanges and the oblique elements, are shown in Fig. 3b.

a)



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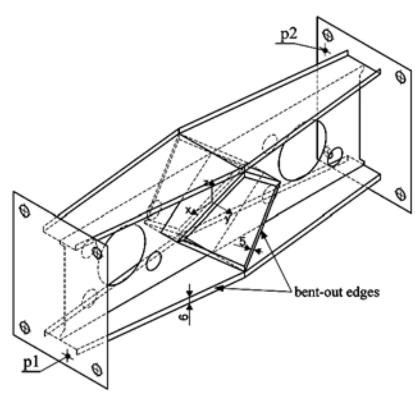


Fig. 3. Shape, dimensions and boundary conditions assumed for FEM calculations of analysed models based on solution from Fig. 1b [1]: a) model corresponding exactly to SADSF solution; b) model with introduced modifications – edge stiffeners on flanges and oblique elements

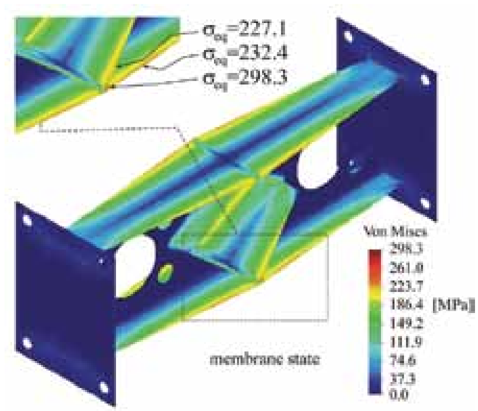
The introduced modification and the analysis of its effects in the stage of elastic operation are associated with the experimental investigations [1], which were carried out in the whole range of equilibrium paths, up to collapse. There, one expected that stiffening of the flanges and the oblique elements would result in a visible increase of the collapse load of such a structure. Indeed, a significant increase of collapse load was found in that case. However, both in the case presented in Fig. 3b and in all other analysed cases of modifications by edge stiffeners, elastic equivalent stress fields evidently worsened.

Distributions of equivalent stresses calculated for the model from Fig. 3a are shown in Fig. 4. Fig. 4a depicts the distribution of equivalent stress related to the membrane state component, while the distribution in Fig. 4b pertains to the bending state. As it was mentioned in Section 1, it is not possible to obtain equalized elastic equivalent stress in the whole volume of the flanges and oblique elements, because there also appear states characteristic for bending axes. However, as it turned out, even in such a case, the designed structure exhibits a number of good properties, like the following:

– Relatively good equalization of equivalent stress along the free boundaries, and similar levels of maximum equivalent stress in both the flanges and the oblique elements (see the values marked in Fig. 4a);
– No load carried by the central web (Fig. 4a), in which a zero state of stress was assumed during the design (see Fig. 1c);
– The domination of membrane states: the value of maximal equivalent stress originating from the bending state amounts to approx. 30% of the maximal value obtained for the membrane state (98.7/298.3 ≈ 0.3).

The greatest values of stress in bending states can be found in the vicinity of the diaphragms. It follows from

a)



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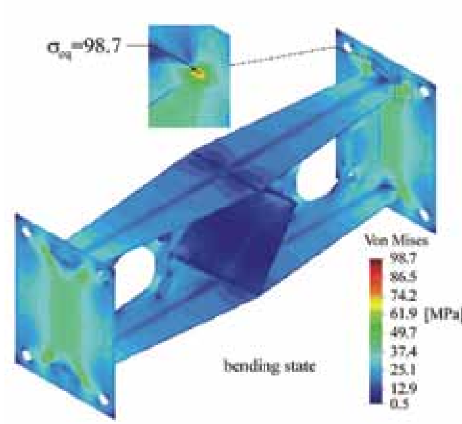


Fig. 4. Distributions of equivalent stresses in model from Fig. 3a calculated according to Huber-Mises yield criterion

the fact that some small bimoment, within the range of bending stiffness, is taken over by the diaphragms.

Rigidity of this model equals k ≈ 1183 [Nm/degree].

The distributions of equivalent stress fields for the model with edge stiffeners are shown in Fig. 5. It is visible that this modification caused the following:

– A decrease in the equivalent stress level along the edge stiffeners in the membrane state; however, local concentrations appear, in which maximal equivalent stress is greater by approx. 21% than that in the model without modifications (see Fig. 4a); and,
– Over a one-and-a-half-fold increase in maximal bending-related equivalent stress, compared to the result obtained for the model without modification, shown in Fig. 4b.

The rigidity of this model equals k ≈ 1492 [Nm/degree] and is approx. 26% greater than that of the model without edge stiffeners.

For comparison and a better evaluation of the quality of the obtained SADSF solution, we also carried out FEM calculations for a model of a simple I-section, for which we assumed qualitatively the same boundary conditions, thicknesses of metal plates, as well as dimensions of outer contours of the web as those in the SADSF solution. The width of flanges was assumed constant and equal to the maximum width determined from the statically admissible fields. However, in order to obtain the level of equivalent stress in the model of I-section close to that existing in the SADSF model, one must apply to it a 19-times smaller load.

The distributions of fields of total equivalent stresses on the surface of the SADSF model and the simple I-section are presented in Fig. 6. Such a great difference between load levels results, first of all, from the fact that the structure from Fig. 6a, designed by SADSF method, carries the applied load mainly in the membrane state, while in the case of structure from Fig. 6b, load transmission must take place in the conditions where the bending state is the dominant one, and this means low rigidity and a generally high equivalent stress level. In this model, maximal stress related to membrane state is equal to 122.2 [MPa], while that related to bending state reaches 205.9 [MPa].

By applying the SADSF method, one can not only determine the shape and dimensions of elements, but also – as it was already mentioned – the system of their

a) b)

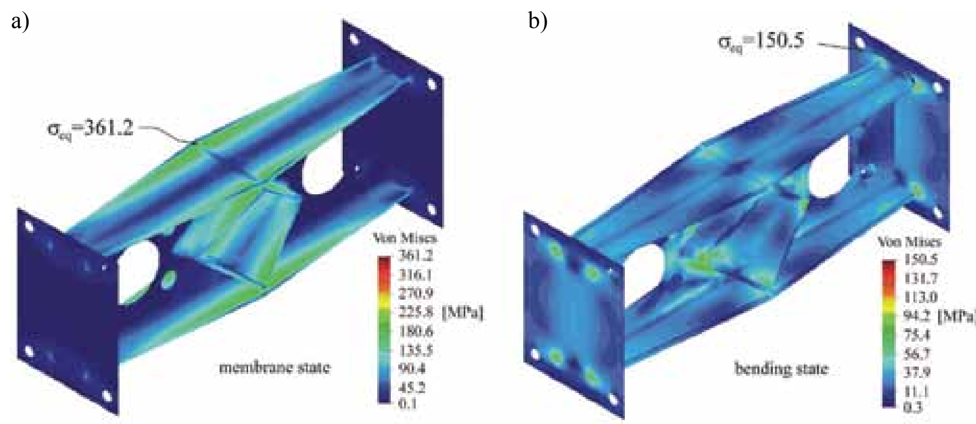


Fig. 5. Distributions of equivalent stresses obtained for the model of torsion-loaded shell with edge stiffeners on flanges and oblique elements shown in Fig. 3b (calculated according to Huber-Mises yield criterion)

a) b)

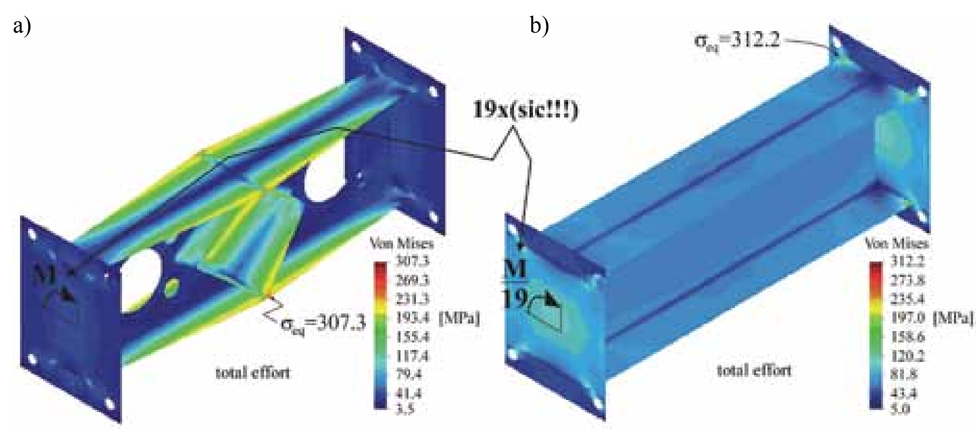


Fig. 6. Exemplary comparison between distributions of total equivalent stress in the model designed by SADSF method and the model of simple I-section

mutual connections and the spatial allocation, which guarantee the domination of the membrane state in real thin-walled structures. This is the source of the most important benefits achieved when applying this method to the design of thin-walled structures.

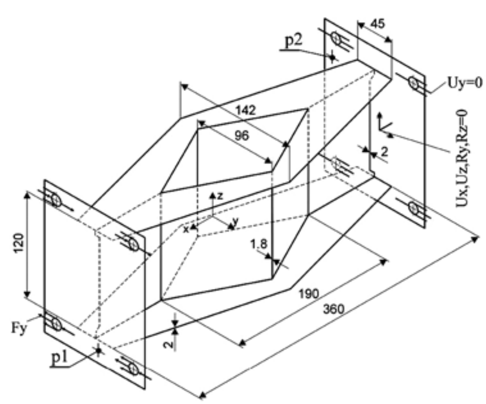
Rigidity of the model with the simple I-section equals k ≈ 21 [Nm/degree] and is as much as 56 times (1183/21 ≈ 56) smaller than that of the model designed with the SADSF method.

2.4. Models based on solution 'F95'

The shape and dimensions of the model represented by the solution 'F95' from Fig. 2a, and the boundary conditions assumed in FEM analysis, are presented in Fig. 7a, while construction details of one of the analysed models, the one with introduced edge stiffeners on flanges, are shown in Fig. 7b. The edge stiffeners were introduced for the same reasons as in the case of structure from Fig. 3b. In the latter case, however, in experimental investigations, we did not found any increase in collapse load.

The distributions of equivalent stresses, calculated for the model from Fig. 7a, are presented in Fig. 8. Fig. 8a depicts the distribution of equivalent stress for

a)



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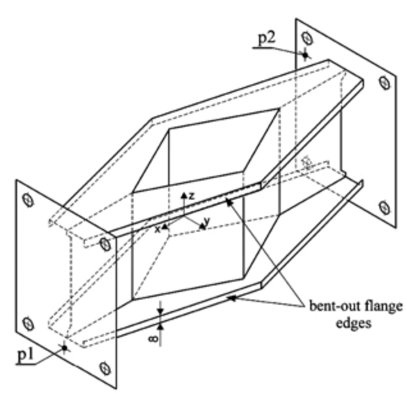
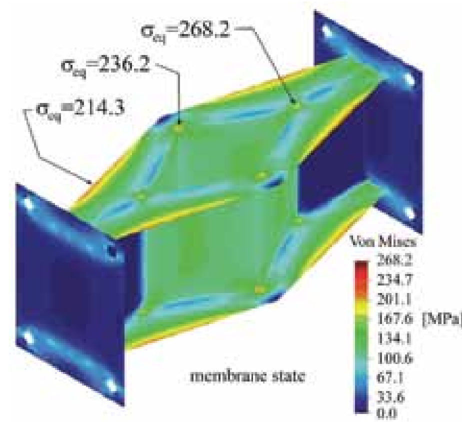


Fig. 7. Shape and dimensions, and boundary conditions assumed for FEM calculations of analysed models based on solution 'F95' from Fig. 2a [1]: a) model corresponding exactly to SADSF solution; b) model with edge stiffeners

the membrane state component, while Fig. 8b shows similar distribution for the bending state. In Fig. 8a, one can see the following:

– The appearance of small, local stress concentrations in various places in the structure (i.e. at the points indicated by arrows), but equivalent stress values in these places are similar;
– Good equalization of equivalent stress along the free borders of the flanges;
– Almost ideal level of equivalent stress equalization in oblique webs, in which pure sheer was assumed (the fields 'Ts' in Fig. 2b);
– No load on webs adjacent to extreme diaphragms, where a state of zero stress was assumed when designing the structure (see Fig. 2b).

a)



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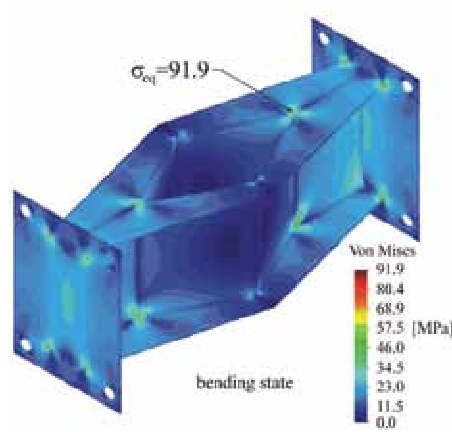

Fig. 8. Distributions of equivalent stress in the model from Fig. 7a calculated according to Huber-Mises yield criterion

Consequently, in Fig. 8b, one can notice that the equivalent stress associated with the bending state of stress assumes small values, and its maximal value equals only 34% (91.9/268.2 ≈ 0.34) of the maximum value of equivalent stress associated with the membrane state.

Rigidity of this model equals k ≈ 8686 [Nm/degree].

Distributions of equivalent stresses calculated for the model with edge stiffeners on flanges are presented in Fig. 9. When we compare these distributions with those presented in Fig. 8, we can conclude the following in the case of membrane states:

- The introduction of edge stiffeners caused that maximal equivalent stress level along these edges dropped by approx. 25% (from 214.3 to 163 [MPa]); and,
- The level of maximal equivalent stress increased insignificantly, from 268.2 to 343.1 [MPa], and the point of maximal stress can now be found in a different place.

Similarly, for the bending states we can notice the following:

- The introduction of edge stiffeners caused that maximal equivalent stress associated with bending increased by approx. 16 % (from 91.9 to 107 [MPa]); and,
- A small increase in bending-related equivalent stress level appeared along the border connecting the flange with the oblique webs.

Rigidity of this model equals k ≈ 9810 [Nm/degree] and is approx. 13% greater than that of the model without edge stiffeners.

Similarly as in the previous case, we also compared this solution with the model of a simple I-section, for which we assumed boundary conditions and outer contours of flanges as the same as those in the SADSF solution. In order that the amount of material was approximately equal in both models, we assumed a thickness of all elements 1.5-times greater than that in the SADSF solution. This time, however, in order to obtain the level of maximal equivalent stress in the I-section model similar to that of the SADSF solution, the value of torsional moment load had to be decreased as much as 29 times. The calculated distributions of fields of total equivalent stresses on the surfaces of the SADSF model and the I-section are shown in Fig. 10.

Rigidity of this model equals k ≈ 140 [Nm/degree] and is as much as 62 times (8686/140 ≈ 62) lower than rigidity of the model designed with the SADSF method.

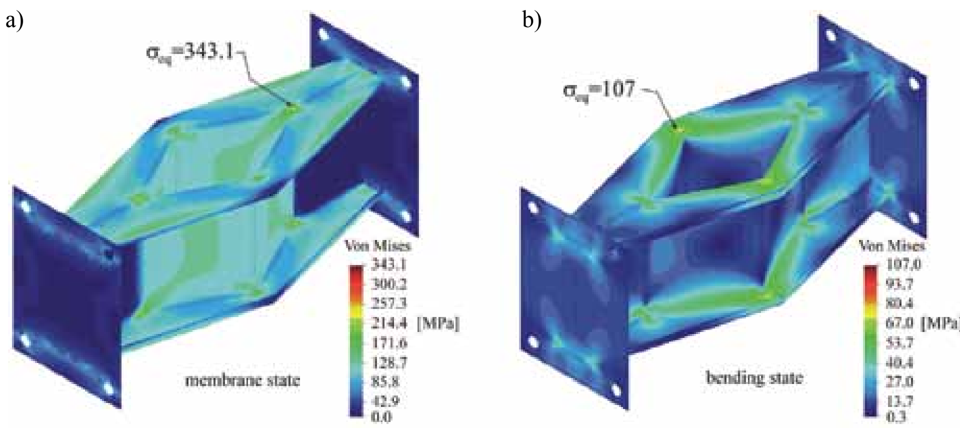

Fig. 9. Distributions of equivalent stress in the model with edge stiffeners on flanges shown in Fig. 7b

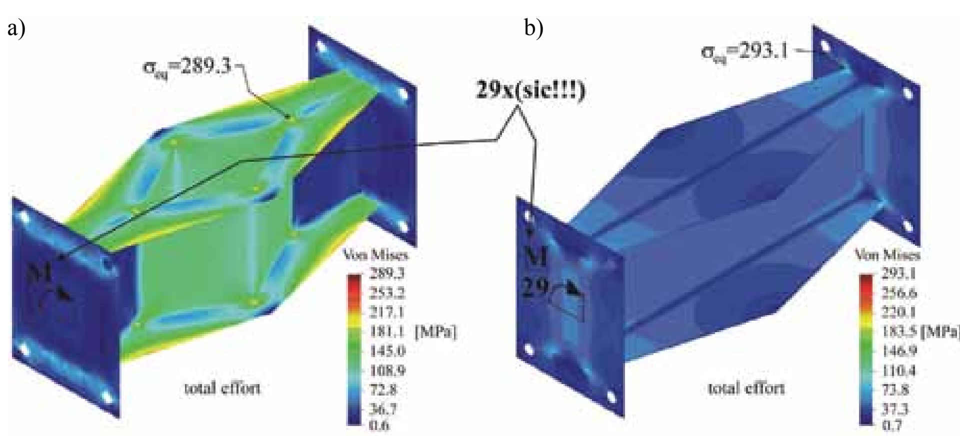

Fig. 10. Comparison of distributions of total equivalent stress in the model designed by SADSF method and the model of simple I-section

Conclusions

In this study, the author has presented only a small fragment of his FEM analyses carried out on the thin-walled structures designed by the SADSF method. In all those cases – similarly as in these presented in this study – good and very good load-carrying properties were obtained, i.e. good equalization of elastic effort – also along free borders, the domination of membrane states, low concentrations of stress, etc. In consequence, these structures also had high fatigue life. Similar conclusions following on the analyses of other structures designed by the SADSF method appear in all publications on this subject [2–3, 5–10, 14]. Therefore, we can treat such properties as the expected ones.

Constructional modifications consisting in the application of edge stiffeners introduce significant local stress concentrations, both in membrane and bending state, the latter being even several times higher than in unmodified structures. As it was shown, such modifications could lead to decreasing the structure's fatigue life even several dozen times. Among analysed cases of over a dozen of variants of edge stiffeners, we could hardly find one which would ensure a decrease in the level of maximal equivalent stress along with an increase in fatigue life. It might suggest that the best overall elastic properties one can obtained in the structures whose contours result exactly from the SADSF solution with minor modifications consisting in smoothing the border lines.

To the mentioned properties, one should also add those resulting from the SADSF method itself, such as the flow of big fragments of the structure that takes place when the limit load is reached [1, 11]. This phenomenon can be associated with almost complete absorption of the collapse energy and resistance to impacts.

The SADSF method then allows us to create preliminary projects of structures characterized by a high level of quality, to which it is worth applying detailed analyses. In the case of thin-walled structures, these projects are free of structural faults, to which the class of thin-walled structures is particularly sensitive. As shown in the above examples (Fig. 6, 10), the existence of such faults can significantly worsen the structure's load-carrying capacity by decreasing its strength even several dozen times. Application of the FE method makes it possible to detect such faults, but it becomes possible only after complete analyses are performed. Even then, the FE method would not provide any hints of how to introduce the necessary corrections [3].

Moreover, the fact that SADSF solutions, in many cases, are not unique, which results from the assumed criterion of structure designing, can be utilized in searching for such solutions that fulfil some additional criteria, e.g., technological limitations, production cost, etc. These limitations can be taken into account by the designer already at the stage of selecting and assembling the library fields. From among the obtained solutions, one can consequently select those which satisfy some additional criteria, concerning, e.g., structure strength or fatigue life.

The results obtained so far confirm the enormous usefulness of the SADSF method in designing thin-walled structures and justify the necessity of popularizing this method among engineers.

References

1. Markiewicz I.: *Badania własności konstrukcji zaprojektowanych metodą SADSF*. Monografie, Studia, Rozprawy (M47). Kielce: Wydawnictwo Politechniki Świętokrzyskiej, 2013 (in Polish).
2. Bodaszewski W., Szczepiński W.: *Kształtowanie elementów konstrukcji metodą nieciągłych pól naprężeń*. BEL Studio, 2005 (in Polish).
3. Bodaszewski W.: *Analizy statyczne i kształtowanie brył cienkościennych*. Warszawa: BEL Studio, 2013 (in Polish).
4. Bodaszewski W.: Algorithms of the method of statically admissible discontinuous stress fields (SADSF), Part I-IV. *Engineering Transactions*, 2004, 52(3), pp. 175–193 (Part I); 2004, 52(4), pp. 231–252 (Part II); 2005, 53(1), pp. 15–30 (Part III); 2005, 53(2), pp. 119–131 (Part IV).
5. Dietrich L., Miastkowski J., Szczepiński W.: *Nośność graniczna elementów konstrukcji*. Warszawa: PWN, 1970 (in Polish).
6. Frąckiewicz H., Szczepiński W., Tereszkowski Z., Szlagowski J., Bodaszewski W., Trela S., Barchan A.: *Węzły i połączenia konstrukcyjne*. Warszawa: WNT, 1985 (in Polish).
7. Szczepiński W.: *Projektowanie elementów maszyn metodą nośności granicznej*. Warszawa: PWN, 1968 (in Polish).
8. Szczepiński W., Szlagowski J.: *Plastic design of complex shape structures*. Warszawa – Chichester: Ellis Horwood & PWN, 1990.
9. Szlagowski J.: *Metodyka kształtowania wytrzymałościowego elementów konstrukcji wg kryterium nośności granicznej*. Prace IPPT PAN 25/1990 (in Polish).
10. Zowczak W.: *Projektowanie wytrzymałościowe metodą linii poślizgu*. Monografie, Studia, Rozprawy (M41). Kielce: Wydawnictwo Politechniki Świętokrzyskiej, 2004 (in Polish).
11. Markiewicz I., Bodaszewski W., Glinka G.: Global and Local Design Method for Fatigue Resistant Structures. *SAE Transactions*, 2003, 112(5), pp. 467–477.
12. Markiewicz I.: Elastic stresses in thin-walled torsional structures designed with SADSF method. *Engineering Transactions*, 2013, 61(2), pp. 137–150.
13. Bendsoe M.P., Sigmund O.: *Topology Optimization: Theory, Methods, and Applications*. Springer, 2003.
14. Mróz Z., Bojczuk D.: Finite topology variations in optimal design of structures. *Structural and Multidisciplinary Optimization*, 2003, 25(3), pp. 153–173.