Dynamic Characteristics of the Structure with Viscoelastic Dampers Combined with Inerters and Subjected to Temperature Changes

Zdzisław M. PAWLAK, Roman LEWANDOWSKI
Poznań University of Technology, ul. Piotrowo 5, 60-965 Poznań, Poland
zdzislaw.pawlak@put.poznan.pl, roman.lewndowski@put.poznan.pl

Abstract

The purpose of the work is dynamic analysis of passive dampers used in structural systems to reduce excessive vibrations caused by wind or earthquakes. Special systems are considered that contain inerter, i.e. device using rotational inertia, in combination with a viscoelastic damper. The so-called fractional models of viscoelastic dampers describe their dynamic behavior in a wide frequency range using a small number of model parameters. To describe material behavior over a wider frequency range, the time-temperature superposition principle is used. The shifting factor is calculated from the well-known William-Landel-Ferry formula. This allows for determination of damper parameters at any temperature based on the parameters obtained at the reference temperature. Laplace transformation of the derived equations of motion leads to the non-linear eigenproblem, which could be solved using the continuation method. The influence of temperature on the dynamic characteristics of the system is examined.

Keywords: Viscoelastic damper, inerter, temperature, fractional derivative, nonlinear eigenvalue problem

1. Introduction

One of the latest ideas to reduce excessive vibration is to use an inerter, i.e. a device capable of generating large inertia forces counteracting excitation forces. For the first time the concept of inerter was proposed by Smith [1]. In general, the inerter can have a different operating mechanism, e.g. gear system [2], screw mechanisms [3] or hydraulic device [4], which strengthens the rotational inertia and generates the resulting resistance force, which is proportional to the relative acceleration at the end terminals of the device.

The dynamic behavior of inerers with a rack and a ball screw is successfully described by a simple linear equation [5, 6]. In the case of hydraulic inerters, the linear description is inaccurate. One of the proposals for taking into account the non-linear damping effects is a model in which the non-linear damper is connected in parallel with the inerter [7].

In many cases, inerter is a part of passive damping system, e.g. it is used in combination with a tuned mass damper to improve the efficiency of the entire control system [8, 9]. Much research has been devoted to the optimal design of damping systems consisting of inerters, where various optimization and design strategies are proposed [7, 8, 9, 10]. However, there is little research analyzing the influence of the inerter on the dynamic characteristics of structures with embedded passive damping systems [11, 12]. The purpose of this work is to investigate the impact of various passive damping systems composed of viscoelastic (VE) dampers and inerters on the dynamic characteristics of the system under consideration, including the effect of temperature. An important feature of viscoelastic materials used in dampers (polymers) is that their damping properties depend on the temperature and frequency of vibrations. Studies on the impact of temperature on the efficiency of viscoelastic dampers installed in building structures are given in [13]. To describe the dynamic behaviour of the viscoelastic element, classical and fractional models are used. Fractional models describe well the dynamic behaviour of the damper with a small number of parameters necessary for identification [14]. Model parameters can be identified in experimental tests by determining the values of the complex modulus and creating the so-called master curves of its real part (storage modulus) and its imaginary part (loss modulus). The values of the complex module for other temperatures can be obtained by shifting the master curves [15], which is associated with a change in model parameters and may result in the need for re-identification of them. Similarly as in work [16], the influence of temperature on the dynamic properties of viscoelastic material was taken into account by changing the values of its model parameters. The dynamic characteristics of the system with viscoelastic dampers and inerters for different temperatures were determined after solving a properly formulated eigenvalue problem. Some observations regarding the influence of temperature on the damping properties of the considered systems were made on the basis of selected numerical examples.

2. Structure with VE dampers and inerters

The dynamic properties of the structure are analyzed, in which viscoelastic dampers and inerters are used in various configurations. To describe the dynamic behavior of the viscoelastic element, fractional models were used in which the Scott-Blair element described by derivatives of non-integer order are applied. In mechanical models of dampers, the Scott-Blairs element is marked with the rhombus symbol (Fig. 1), and described by two parameters: c_0 and $\propto$. The constitutive equation is:

$$U_d(t) = c_0 D_t^{\alpha} \Delta q(t), \tag{1}$$

where, $U_d(t)$ is a force acting at damper node, c_0 the damping parameter, $\Delta q(t)$ is the relative nodal displacement, i.e. $\Delta q(t) = q_j(t) - q_i(t)$ and $D_t^{\alpha}(*)$ is the Riemann-Liouville derivative of the non integer order $(0 < \propto \leq 1)$ with respect to time t.

The fractional viscous model or fractional Kelvin model in various configurations with inerter were used for the analysis. Inerter, regardless of the construction, generates a resistance force that is proportional to the relative acceleration at the terminal ends of the device

$$U_b(t) = b_1 D_t^2 \Delta q(t) \equiv b_1 \Delta \ddot{q}(t), \tag{2}$$

where, b_1 is the equivalent mass of the inerter, which, depending on the construction of the device, can be even several orders larger than its actual mass.

The passive damping system in which the fractional Kelvin model is connected in parallel with the inerter (Fig. 1), marked with the symbol KpB, generates a force expressed by the sum:

$$U_{KpB}(t) = k_0 \Delta q(t) + c_0 D_t^{\alpha} \Delta q(t) + b_1 \Delta \ddot{q}(t), \tag{3}$$

where, k_0 is the elasticity parameter.

If the spring element in the KpB model is omitted (Fig. 1), i.e. $k_0 = 0$ in Eqn. (3), a constitutive equation is obtained for the VpB model (a fractional viscous model connected in parallel with the inerter).

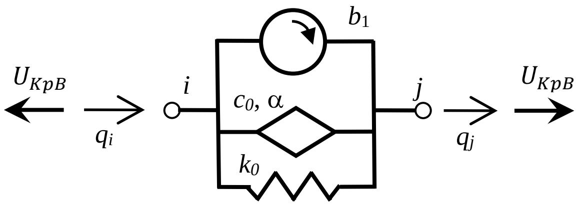

Figure 1. The fractional Kelvin model connected in parallel with the inerter (KpB model)

The system in which the fractional Kelvin model is connected in series with the inerter (Fig. 2) has been marked with the symbol KsB. In this case, the forces generated in the left and right nodes can be expressed by different equations with an additional internal variable q_d:

$$\begin{gathered} U_{KsB}^L(t) = k_0\big(q_d(t) - q_i(t)\big) + c_0 D_t^\alpha\big(q_d(t) - q_i(t)\big), \\ U_{KsB}^R(t) = b_1\left(\ddot{q}_j(t) - \ddot{q}_d(t)\right), \\ U_{KsB}^L(t) = U_{KsB}^R(t). \end{gathered} \tag{4}$$

The constitutive equation for the VsB model (a fractional viscous model connected in series with the inerter), can be obtained when the spring element in the KsB model is removed, i.e. $\mathrm{k}_0 = 0$ in Eqn. (4).

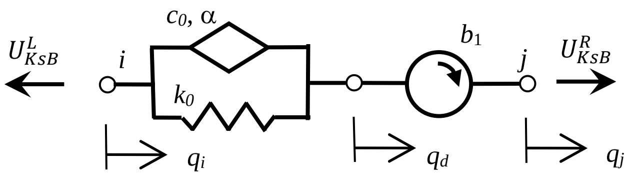

Figure 2. The fractional Kelvin model connected in series with the inerter (KsB model)

After applying the Laplace transformation, with zero initial conditions, the equations of motion 3 and 4 can be written as:

$$\bar{U}_{KpB}(s) = (k_0 + c_0 s^\alpha + b_1 s^2)\Delta\bar{q}(s) = B_{KpB}(s,\propto)\Delta\bar{q}(s) \tag{5}$$

for KpB model, and:

$$\bar{U}_{KsB}(s) = \frac{b_1 s^2 (k_0 + c_0 s^\alpha)}{k_0 + c_0 s^\alpha + b_1 s^2}\Delta\bar{q}(s) = B_{KsB}(s,\propto)\Delta\bar{q}(s) \tag{6}$$

for KsB model, where $\bar{U}(s)$ and $\Delta\bar{q}(s)$ denote the Laplace transforms of $U(t)$ and $\Delta q(t)$, respectively, and s is the Laplace variable. The above formulas describe the classic models, when $\alpha = 1$ is adopted.

For a passively damped system, i.e. the structure equipped with viscoelastic dampers and inerters, the equation of motion can be written in the matrix form:

$$\mathbf{M}_0\ddot{\mathbf{q}}(t) + \mathbf{C}_0\dot{\mathbf{q}}(t) + \mathbf{K}_0\mathbf{q}(t) = \mathbf{p}(t) + \mathbf{f}(t) \tag{7}$$

where $\mathbf{M}_0$, $\mathbf{C}_0$ and $\mathbf{K}_0$ are respectively mass, damping and stiffness matrices of structure, that have the dimension $(n \times n)$ and n is the number of dynamic degrees of freedom of the structure. In addition, $\mathbf{q}(t)$, $\mathbf{p}(t)$, $\mathbf{f}(t)$ denote the displacement vector, the excitation force vector, and the vector of interaction forces that act between the structure and the dampers. Laplace transformation of the equation of motion (7) leads to the equation:

$$(s^2\mathbf{M}_0 + s\mathbf{C}_0 + \mathbf{K}_0)\bar{\mathbf{q}}(s) = \bar{\mathbf{p}}(s) + \bar{\mathbf{f}}(s) \tag{8}$$

In a system where p dampers are mounted on structure, the vector of interaction forces is in the form of sum:

$$\bar{\mathbf{f}}(s) = -\sum_{r=1}^{p} B_r(s,\propto)\mathbf{L}_r\bar{\mathbf{q}}(s) = -\mathbf{B}_d(s,\propto)\bar{\mathbf{q}}(s)\,, \tag{9}$$

where $\mathbf{L}_r$ is the location matrix that determines the position of the r-th damper in the structure, $B_r(s,\propto)$ is the function defined in Eqn. (5) or (6).

Finally, the equation of motion for the structure with VE dampers and inerters, written in the frequency domain, takes the form:

$$(s^2\mathbf{M}_0 + s\mathbf{C}_0 + \mathbf{K}_0 + \mathbf{B}_d(s,\propto))\bar{\mathbf{q}}(s) = \bar{\mathbf{p}}(s). \tag{10}$$

Assumption in Eqn. (10) $\bar{\mathbf{p}}(s) = 0$, leads to a non-linear eigenproblem, which can be solved using the continuation method, e.g. as described in [17]. As a solution, the n number of eigenvalues $\mathrm{s_i}$ and corresponding eigenvectors $\bar{\mathbf{q}}_\mathrm{i}$ are obtained. Then, in a similar way as in the case of small viscous damping, the obtained eigenvalues $\mathrm{s_i}$ allow to determine the dynamic properties of the considered structure equipped with VE dampers and inerters. Natural frequencies ω_i and non-dimensional damping ratios γ_i can be determined as follows:

$$\omega_i^2 = \mu_i^2 + \eta_i^2,\ \ \gamma_i = -\mu_i/\omega_i\,, \tag{11}$$

where, $\mu_i = \mathrm{Re}(s_i)$ and $\eta_i = \mathrm{Im}(s_i)$.

The non-dimensional damping ratio is a factor that allows to evaluate the damping properties of the considered system. Another parameter which allows to assess the effectiveness of the damping system is the so-called drift, which can be derived from the frequency response function. The frequency response function, or transfer function, describes the relationship between input and output in a linear system at steady-state vibrations. When the structure is excited by the harmonic force, i.e. $\mathbf{p}(t) = \bar{\mathbf{p}}e^{st} = \bar{\mathbf{p}}e^{\mathrm{j}\lambda t}$, where λ is the frequency of excitation, its response is in the form of the function $\mathbf{q}(t) = \bar{\mathbf{q}}e^{\mathrm{j}\lambda t}$, where $\bar{\mathbf{q}}$ is a frequency response vector of displacements and $j = \sqrt{-1}$. In the considered case, the solution of the equation of motion (10) is a vector:

$$\bar{\mathbf{q}}(s) = \mathbf{D}_H^{-1}(s)\bar{\mathbf{p}}(s) = \mathbf{H}(s)\bar{\mathbf{p}}(s) \tag{12}$$

where $\mathbf{H}(s) = \mathbf{D}_H^{-1}(s)$ is the frequency response function and $\mathbf{D}_H$ denotes the dynamic stiffness matrix of the considered system:

$$\mathbf{D}_H(s) = s^2\mathbf{M}_0 + s\mathbf{C}_0 + \mathbf{K}_0 + \mathbf{B}_d(s,\propto) \tag{13}$$

For the system excited by the ground motion with a given acceleration $a_g(t)$, the inertia forces acting along the structural degrees of freedom depend on the mass distribution, i.e. the mass matrix:

$$\mathbf{p}(t) = -\mathbf{M}_0\mathbf{r}_p a_g(t), \tag{14}$$

where $\mathbf{r}_p$ is an allocation vector defining the position of inertia forces generated by the ground motion. For the shear frame, in which the most of the mass is concentrated at the level of floors, allocation vector has the form: $\mathbf{r}_p = col(1,1,\ldots,1)$. This means that ground vibrations induce inertial forces on each story along each direction of the degrees of freedom.

In order to estimate the damping efficiency, the response of the system excited with a frequency equal to the natural frequency ω_i and harmonic ground acceleration, i.e. $a_g(t) = A_g e^{\mathrm{j}\omega t}$, is derived:

$$\bar{\mathbf{q}}(\omega) = \mathbf{H}(\omega) A_g \tag{15}$$

For unit ground acceleration: $A_g = 1.0$, we obtain:

$$\bar{\mathbf{q}}(\omega) = \mathbf{H}(\omega) = -(-\omega^2 \mathbf{M}_0 + \mathrm{j}\omega \mathbf{C}_0 + \mathbf{K}_0 + \mathbf{B}_d(\omega, \propto))^{-1} \mathbf{M}_0 \mathbf{r}_p\ , \tag{16}$$

where, $\mathbf{H}(\omega) = \mathrm{col}(H_1, H_2, \ldots, H_n)$ is the vector of response transfer functions of displacements and the symbol H_i means displacement along the *i*th degree of freedom. In the case of shear frames, the first mode of natural vibrations definitely dominates in the dynamic response of the structure, which is why this case was selected for analysis. The measure of damping adopted in the work is the so-called sum of drifts of the system excited with a frequency equal to the natural frequency ω_1 and unit ground acceleration:

$$S_m(\omega_1) = \sum_{i=1}^{n} \Delta H_i(\omega_1)\ , \tag{17}$$

where, $\Delta H_i = |H_i - H_{i-1}|$ is a difference between displacements of two successive floors in a shear frame. Moreover, it is assumed that $H_0 = 0$, i.e. $\Delta H_1 = |H_1 - H_0| = |H_1|$.

3. Temperature influence

The dynamic behaviour of the viscoelastic damper harmonically excited, i.e. $q(t) = q_0 e^{\mathrm{j}\lambda t}$, can be described using a complex modulus $K(\lambda)$:

$$\bar{U}(\lambda) = K(\lambda)\Delta\bar{q}(\lambda) = [K'(\lambda) + jK"(\lambda)]\Delta\bar{q}(\lambda)\ , \tag{18}$$

where, $K'(\lambda)$ is the storage modulus, $K"(\lambda)$ is the loss modulus and the ratio between them is the loss factor $\eta(\lambda) = K"(\lambda)/K'(\lambda)$.

In order to determine the influence of temperature on the behavior of VE material, the temperature-frequency superposition principle [18] can be used:

$$K(\lambda, T) = K(\lambda_0, T_0) = K(\alpha_T \lambda, T_0)\ , \tag{19}$$

where, α_T is the so-called shift factor, λ_0 is the reference frequency and T_0 is the reference temperature. According to this principle, for a thermoreologically simple material, the master curve, i.e. the module function determined in the frequency domain for the reference temperature T_0 can be shifted horizontally to obtain module function for the actual temperature T. The horizontal shift factor α_T is most often determined empirically, e.g. using the William-Landel-Ferry formula:

$$\log \alpha_T = -C_1 \Delta T/(C_2 + \Delta T)\ , \tag{20}$$

where, C_1 and C_2 are the experimentally determined constants and $\Delta T = T - T_0$. It is assumed that the fractional free volume of the material increases linearly with respect to temperature and that the viscosity of the material decreases rapidly as its free volume increases.

In the analyzed systems, the viscoelastic material is described by the fractional Kelvin model:

$$\bar{U}_K(s) = (k_0 + c_0 s^\alpha)\Delta\bar{q}(s) \tag{21}$$

After entering $s = j\lambda$ in Eqn. (21) and assuming that $j^\alpha = \cos(\alpha\pi/2) + j\sin(\alpha\pi/2)$, the formulas for the above-mentioned modules can be written as:

$$\begin{aligned} K_p'(\lambda) &= k_0 + c_0\lambda^\alpha \cos(\alpha\pi/2) \\ K_p''(\lambda) &= c_0\lambda^\alpha \sin(\alpha\pi/2) \end{aligned} \tag{22}$$

The module functions specified at the reference frequency λ_0 and the reference temperature T_0 are:

$$\begin{aligned} K_p'(\lambda_0, T_0) &= k_0 + c_0\lambda_0^\alpha \cos(\alpha\pi/2) \\ K_p''(\lambda_0, T_0) &= c_0\lambda_0^\alpha \sin(\alpha\pi/2) \end{aligned} \tag{23}$$

The above functions can be determined for actual frequency λ and temperature T:

$$\begin{aligned} K_p'(\lambda, T) &= \widetilde{k_0} + \widetilde{c_0}\lambda^\alpha \cos(\alpha\pi/2) \\ K_p''(\lambda, T) &= \widetilde{c_0}\lambda^\alpha \sin(\alpha\pi/2) \end{aligned} \tag{24}$$

However, according to the relationship (19), the functions of the modules (24) are equivalent to the solutions obtained at the reference temperature T_0 and shifted frequency $\lambda_0 = \alpha_T\lambda$:

$$\begin{aligned} K_p'(\alpha_T\lambda, T_0) &= k_0 + c_0(\alpha_T\lambda)^\alpha \cos(\alpha\pi/2) \\ K_p''(\alpha_T\lambda, T_0) &= c_0(\alpha_T\lambda)^\alpha \sin(\alpha\pi/2) \end{aligned} \tag{25}$$

Comparing solutions (24) and (25), the following relationships can be formulated:

$$\widetilde{k_0} = k_0,\ \widetilde{c_0} = (\alpha_T)^\alpha c_0 \tag{26}$$

The above analyzes can be summarized that for a thermoreologically simple viscoelastic material the effect of temperature change can be taken into account by appropriate modification of the damping parameter c_0 in the considered model [16].

4. Numerical examples

In the numerical example, the dynamic response of a five-story building structure modeled as a shear frame was analyzed. It was assumed that the mass of the structure is concentrated at the floor levels and on each storey is the same, similarly, the stiffness parameter of each storey is the same, i.e. $m_i = 5000$ kg, $k_i = 2600$ kN/m, $i = 1, \dots, 5$. The test was carried out for structure with regularly distributed dampers and inerters (Fig. 3). The rheological properties of each considered damper model were: $k_0 = 400$ kN/m, $c_0 = 40$ kNs/m, and the equivalent mass of the inerter was $b_1 = 3\,000$ kg.

The Deltane 350 (Paulstra) polymer [15] was adopted as a viscoelastic material in the damper, for which the constants C_i appearing in the WLF formula (20) were determined, at the reference temperature $T_0 = 12°$, i.e. $C_1 = 6.71$ and $C_2 = 135.0$. The damping properties of the structure are neglected, and the parameter $\propto = 1.0$.

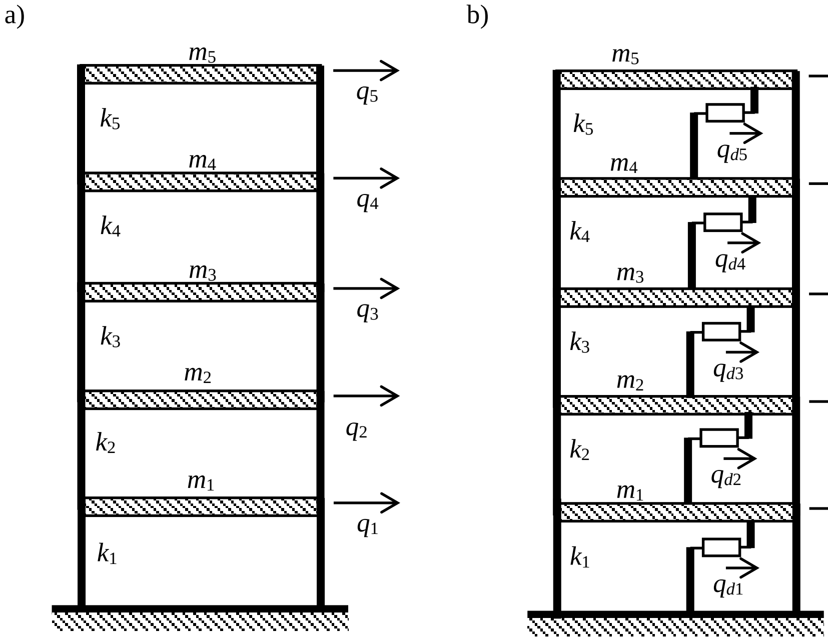

Figure 3. Five-story building structure: a) shear frame model, b) dampers distribution

The results of calculation are presented in the graphs (Fig. 4 and 5), for various models of dampers, i.e. Viscous model (V), Viscous model in series with inerter (VsB), Viscous model in parallel with inerter (VpB), Kelvin model (K), Kelvin model in series with inerter (KsB) and Kelvin model in parallel with inerter (KpB). In Fig. 4 the change of the first natural frequency ω_1 and the sum of drifts $S_m(\omega_1)$ caused by the temperature increase for various models is shown.

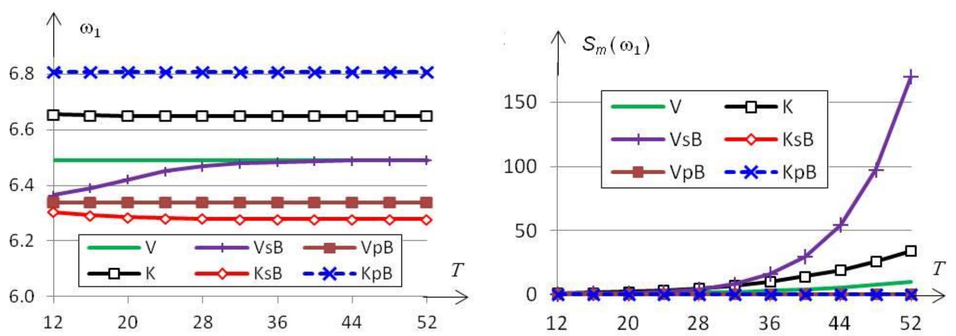

Figure 4. The natural frequency ω_1 and the sum of drifts $S_m(\omega_1)$ versus temperature for various models ($b_1 = 3000$ kg)

The relationships between the values of the non-dimensional damping ratios γ_1 and γ_2 and temperature for various models are shown in Fig. 5. Although for higher temperature values of the non-dimensional damping ratios have comparable values for each model (see Fig. 5), the sum of drifts definitely increases only for the VsB, K and V models (see Fig. 4). In addition, the influence of temperature on the values of damping ratios γ_1 and γ_2 for different masses of inerter b_1 is shown in Fig. 6.

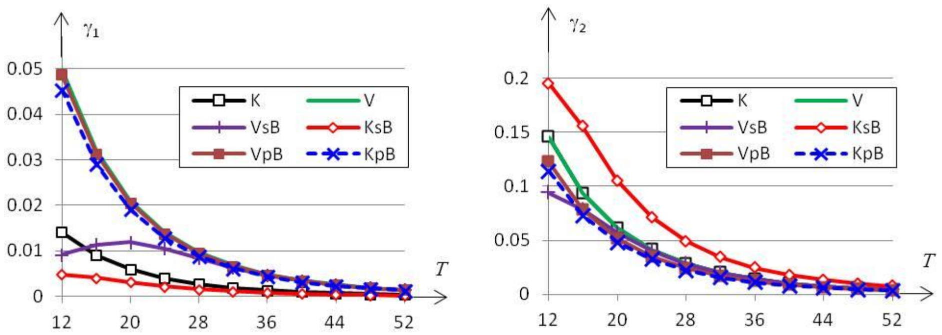

Figure 5. The non-dimensional damping ratios: γ_1 and γ_2 versus temperature for various models ($b_1 = 3000$ kg)

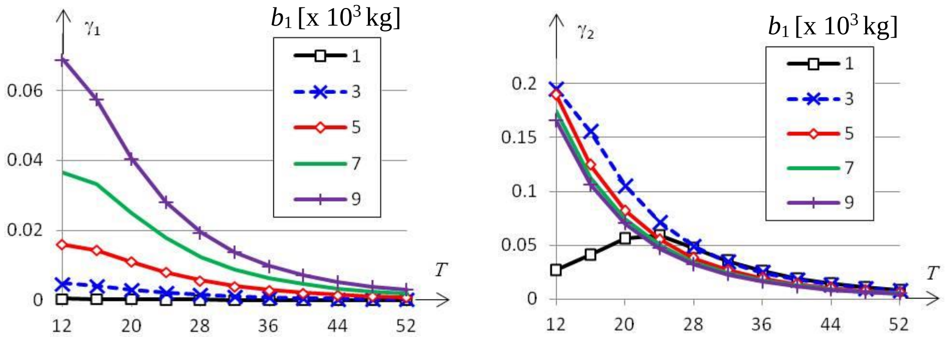

Figure 6. The non-dimensional damping ratios: γ_1 and γ_2 versus temperature for various mass of inerter b_1 (KsB model)

It is worth noting that for larger inerter mass values b_1, the damping properties are better for the analyzed models (see Fig. 6).

Figure 7 shows the change in the value of the non-dimensional damping ratio depending on the mass of the inerter. These functions are given for three different values of the α parameter, i.e. the order of the fractional derivative.

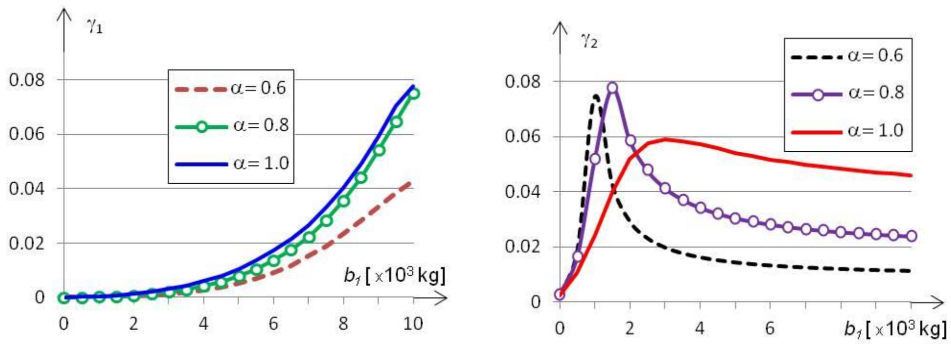

Figure 7. The non-dimensional damping ratios: γ_1 and γ_2 versus mass of inerter b_1 for various α values (KsB model)

5. Conclusions

The presented analysis concerns the study of the effect of temperature on the dynamic characteristics of structures with VE dampers and inerters. Thermoreologically simple materials are analyzed for which, according to the principle of temperature-frequency superposition, the master curve can be shifted horizontally. In this case, the change in temperature affects the value of only one parameter of the considered models, i.e. the damping factor $\boldsymbol{c_0}$. Based on numerical calculations, it can be concluded that the inerter mass and the way it is connected to the VE damper has a significant impact on the dynamic characteristics of the structure and on the sensitivity of these characteristics to temperature changes. The inerter has the greatest influence on the operation of the viscoelastic damper when it is connected in series with the damper (VsB and KsB models) and its mass is relatively high. In the case of a shear frame with an even distribution of mass and stiffness, the best damping effect is provided by the damper located on the lowest floor. In general, decreasing the α parameter, i.e. the order of the fractional derivative, reduces the non-dimensional damping ratio. However, in some cases (see Fig. 7), this ratio is greater with lower α values and low inerter mass.

Acknowledgments

This study has been supported by the Poznan University of Technology. Institute of Structural Analysis, carried out in the year 2020. This support is gratefully acknowledged.

References

1. M.C. Smith. *Synthesis of mechanical networks: the inerter*. IEEE Trans Automat Contr. 2002, 47(10), 1648–62.
2. R.M. Hessabi, O. Mercan, *Investigations of the application of gyro-mass dampers with various types of supplemental dampers for vibration control of building structures.* Engineering Structures, 2016, 126, 174-186.

3. P. Brzeski, M. Lazarek, P. Perlikowski, *Experimental study of the novel tuned mass damper with inerter which enables changes of inertance*. Journal of Sound and Vibration. 2017, 404, 47-57. https://doi.org/10.1016/j.jsv.2017.05.034
4. F.-C. Wang, M.-F. Hong and T.-C. Lin, *Designing and testing a hydraulic inerter,* Proc. Inst. Mech. Eng. C, J. Mech. Eng. Sci., 2011, vol. 225, no. 1, pp. 66–72.
5. N. Makris, G. Kampas, *Seismic protection of structures with supplemental rotational inertia,* J Eng Mech., 2016;142(11). 04016089-1-11
6. M. Wang, F. Sun, *Displacement reduction effect and simplified evaluation method for SDOF systems using a clutching inerter damper.* Earthq. Eng. Struct. Dynam. 2018, 47(7): 1651–72
7. D, De Domenico, G, Ricciardi, R. Zhang, *Optimal design and seismic performance of tuned fluid inerter applied to structures with friction pendulum isolators,* Soil Dynamics Earthquake Engineering, 132, (2020), 106099
8. D, Pietrosanti, M. De Angelis, M. Basili, *Optimal design and performance evaluation of systems with tuned mass damper inerter (TMDI)*. Earthq Eng Struct Dynam 2017; 46(8): 1367–88.
9. D. De Domenico, P. Deastra, G. Ricciardi, N.D. Sims, D.J. Wagg. *Novel fluid inerter based tuned mass dampers for optimised structural control of base-isolated buildings*. J. Franklin Inst. 2019, 356(14), 7626–49.
10. A. Radu, I.F. Lazar, S.A. Neild. *Performance-based seismic design of tuned-inerter dampers*. Structural Control and Health Monitoring, 2019, 26(5), [e2346]. DOI: 10.1002/stc.2346
11. M.Z.Q, Chen, Y, Hu, L. Huang, G. Chen, *Influence of inerter on natural frequencies of vibration systems.* Journal of Sound and Vibration, 2014, 333, 1874–1887.
12. Y, Hu, M.Z.Q. Chen, M.C. Smith, *Natural frequency assignment for mass-chain systems with ierters*, Mechanical Systems and Signal Processing, 2018, 108. 126-139. DOI: 10.1016/j.ymssp.2018.01.038
13. K.C. Chang, T.T. Soong, S.-T. Oh, M.L. Lai, *Effect of ambient temperature on viscoelastically damped structure*. J. Struct. Eng. 1992, 118(7), 1955–1973.
14. R. Lewandowski, A. Bartkowiak and H. Maciejewski, *Dynamic analysis of frames with viscoelastic dampers: a comparison of damper models*, Structural Engineering and Mechanics, 2012, 41(1), 113-137.
15. L. Rouleau, J.-F. Deu, A. Legay, F. Le Lay, *Application of Kramers–Kronig relations to time–temperature superposition for viscoelastic materials*. Mech. Mater. 2013, 65, 66–75. https://doi.org/10.1016/j.mechmat.2013.06.001.
16. R. Lewandowski, M. Przychodzki, *Approximate method for temperature-dependent characteristics of structures with viscoelastic dampers*. Arch Appl Mech 88, 2018, 1695–1711. https://doi.org/10.1007/s00419-018-1394-6
17. Z. Pawlak, R. Lewandowski, *The continuation method for the eigenvalue problem of structures with viscoelastic damper*. Computer and Structures, 2013, 125: 53-61.
18. R.A.S. Moreira, J.D. Corte-Real, J. Dias Rodrigues, *A generalized frequency–temperature viscoelastic model*. Shock Vib. 17 (2010), 407–418.