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MATHEMATICAL MODELLING THE CONDITIONS OF INTENSIFICATION OF THE RIVERBED LOCAL EROSION BEHIND OF OBSTACLE THAT DEVIATES FROM THE SHORE DOWNSTREAM

Abstract. The aim of this work is the mathematical modelling the conditions of intensification of local erosion of the river bed behind of a natural barrier that deviates from the shore downstream, due to the development of the local turbulent flow directed from the shore. There are given results of numerical simulation of the intensity of flow behind of the bottom obstacle, depending on its angle location and height. It was established that if the height of the obstacle equals to 1/3 of the stream depth and its angle location is between 35–40 degrees to the oncoming flow the most powerful local riverbed deformations can occur.

Key words: Intensity of the water flow, local erosion, mathematical modeling, numerical simulation, riverbed obstacle, turbulent flow.

INTRODUCTION

The monitoring the local erosion of channels of small and medium-sized rivers shows that the most severe erosion of the river bottom during floods occurs at places of locating of various bottom obstacles that deviate from the shore downstream [1]. In Ukraine, such riverbed deformations intensify particularly on the rivers of the Carpathian region, in places where rivers run in foothill areas [2].

Among such riverbed obstacles formed naturally, ridges of river sediments and trees which fall at river bottom deserve special attention. Local compression of river channel, that the barriers do, not only leads to an increase in the average velocity of transit flow in the wetted section, but during flood such obstacles can generate local turbulent flows, which trigger the movement of river sediments from the coast creating conditions for intensification of local erosion.

MATHEMATICAL MODELLING OF TURBULENT FLOW BEHIND OF OBSTACLE

To achieve the purpose of research we carry out numerical simulation of the dynamics of turbulent flow appearing behind of obstacle. The problem is solved in two stages [3-5].

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1. At first stage a problem of computing of the stationary velocity field of developed turbulent flow in a three-dimensional local field is being solved. The system of equations (1) for the Reynolds quasi laminar flow is solved, to which the continuity equations (2) are added:

$$\sum_j \overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} = \overline{X_i} - \frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + \gamma_T \sum_j \frac{\partial^2 \overline{u_i}}{\partial x_j^2}, \quad (1)$$

$$\sum_j \frac{\partial \overline{u_j}}{\partial x_j} = 0, \quad (2)$$

where $\overline{u_i}$ are components of the vector of mean velocity; x_i are coordinates of the point; $\overline{X_i}$ are components of the vector of mean mass force; ρ is density of fluid; $\overline{p}$ is averaged hydrodynamic pressure; γ_T is coefficient of turbulent viscosity; i, j are number equation and member number in the equation ($i, j = \overline{1,3}$), respectively.

For closure of the systems of equations (1), (2), the simplest turbulence model is adopted, according to which the coefficient of turbulent viscosity in the region of integration $\gamma_{\grave{O}} = Const$ is determined by the I. Rozovsky's formula:

$$\gamma_{\grave{O}} = 0{,}05\, u_0 \sigma, \quad (3)$$

where u_0 is average initial velocity; σ is height of bottom barrier.

Boundary conditions for the system of equations (1) are written as:

$$u_j \big|_{\Gamma} = \tilde{u}_j, \quad (4)$$

where $\tilde{u}_j$ is initial approximation of velocity.

With replace partial finite differences the transition from differential equations to sparse systems of linear algebraic equations as $A^{\bar{7}} x = b$ is performed, where $A^{\bar{7}}$ is sparse asymmetric square matrix, b is vector of right sides, $x = \{u_j^i, i = \overline{1,3}, j = \overline{1,\beta}\}$ is vector of unknown parameters (β is number of nodes sparse grid calculation). Nonzero elements of the matrix are seven diagonals. All elements of the main diagonal $A^{\bar{7}}$ is different from zero. The system of equations $A^{\bar{7}} x = b$ is solved using the direct method LU-decomposition. Since the determinant of sparse matrix $A^{\bar{7}}$ is different from zero, it is concluded that obtained by direct numerical approximate solutions of systems of linear algebraic equations to be resilient [6].

2. The values of the kinematics characteristics of flow, which were calculated in the previous step for the implicit scheme, is used as the initial approximations to solve a more complex problem, such as the system of Reynolds equations (5), to which the continuity equations (2) and equations of $k - \varepsilon$ turbulence model (6), (7) are added:

$$\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \left(\overline{u_i}\, \overline{u_j} \right)}{\partial x_j} = \overline{X_i} - \frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} - \frac{\partial \left(\overline{u_i' u_j'} \right)}{\partial x_j}, \quad (5)$$

$$\frac{\partial k}{\partial t} + \overline{u_j} \frac{\partial k}{\partial x_j} = \gamma_T \left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right) \frac{\partial \overline{u_i}}{\partial x_j} - C_D \frac{k^{3/2}}{L} + \frac{\partial}{\partial x_j} \left(\frac{\gamma_T}{\delta_k} \cdot \frac{\partial k}{\partial x_j} \right), \quad (6)$$

$$\frac{\partial \varepsilon}{\partial t} + \overline{u_j} \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\gamma_T}{\delta_\varepsilon} \cdot \frac{\partial \varepsilon}{\partial x_j}\right) + \left(C_{1\varepsilon}\frac{p}{\varepsilon} - C_{2\varepsilon}\right)\frac{\varepsilon^2}{k}, \quad (7)$$

where $\overline{u_i}$ are components of the vector of mean velocity; t is time; x_i are coordinates of the point; $\overline{X_i}$ are components of the vector of mean mass force; ρ is density of fluid; $\overline{p}$ is averaged hydrodynamic pressure; $\overline{u_i^/ u_j^/}$ is correlation moment of velocity fluctuations; i, j are number equation and member number in the equation ($i, j = \overline{1,3}$), respectively; k is kinetic energy of turbulence; ε is dissipation rate of turbulent energy; L is path length of mixing; $C_D = 1$, $C_{1\varepsilon} = 1{,}44$, $C_{2\varepsilon} = 1{,}92$, $\delta_k = 1$, $\delta_\varepsilon = 1{,}3$ are model constants [7].

This approach solves the problem of stability of solutions in the calculation of kinematics characteristics of turbulent water flow.

In the $k-\varepsilon$ turbulence model values of turbulent stresses within the left side of equation (5) are taken according to gradient hypothesis [7]:

$$\overline{u_i^/ u_j^/} = \gamma_T\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) - \frac{2}{3}k \cdot \delta_j, \quad (8)$$

where $\delta_{ij} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$ is Kronecker's symbol, which is introduced in order to the sum of three normal turbulent stress was equal to twice the kinetic energy of turbulence $k = \frac{1}{2}\overline{u_i^/ u_j^/}$; $\gamma_T = C_\mu \frac{k^2}{\varepsilon}$ is coefficient of turbulent viscosity expressed in terms of value k and ε, where $k = \frac{1}{2}\left(\sum_j u_j'^2\right)$; C_μ is experimental coefficient (C_μ = 0,03...0,09).

During the calculation the integral test conditions of continuity of flow is performed, thickening computational grid is made and spline approximation of kinematics characteristics is performed. To construct the finite-difference analogue of the Reynolds equations in divergence form (5) cleavage method is used. After each time step of the numerical solution of equations by the McCormack's explicit scheme [8] on a condensed uniform grid box the field of averaged velocities are calculated and distributions of averaged values of the kinetic energy of turbulence k, of the rate of dissipation ε and of the coefficient of turbulent viscosity γ_T are determined.

Numerical simulation of the kinematics characteristics of the flow is produced in a three-dimensional local field $G = \{0 \le x \le N,\ 0 \le y \le M,\ 0 \le z \le L\}$ with shape of a parallelepiped located directly behind of obstacle which deviates from shore downstream.

THE WORK HYPOTHESIS, MODEL CHARACTERISTICS AND MAIN ASSUMPTIONS

The results of experimental studies of flow of bottom barriers that deviate from shore downstream (Schodro A.E. [1, 2]) indicate the dependence of the intensity of channel flow over an obstacle on the angle of its location and its relative height (to a depth of flow). As the main model characteristic, reflecting the intensity of the flow, we take the resistance of the bottom F_R in the field behind of bottom barrier. It is supposed the resistance forth of the bottom describes the course as a whole local channel deformations, and shear stress W_F/F_R (resistance force acting on the unit of the bottom, where W_F is surface area of swath turbulent flow over obstacle) describes intensity of local channel process, in particular, the intensity of local erosion, dredging and removal of particles from the zone of active leaching. Also, it is assumed the resistance of the bottom depends on two essential factors, namely the angle of location of the bottom obstacle and its relative height.

THE INPUT DATA AND MODEL DEPENDENCES

Mathematical modeling of the bottom resistance was carried to the straight channel conditions without significant deviation boundary, which has an average slope i_m = 0,0022 and is composed of gravel-pebble soils. The average diameter of the particles of the soil d_m and bed roughness coefficient n were taken for the three cases: 1) d_m = 0,0485 m, n = 0,0379; 2) d_m = 0,02 m, n = 0,0327; 3) d_m = 0,005 m, n = 0,02597. The values of flood hydrologic and hydraulic characteristics are given below in tabular form.

FLOOD HYDROLOGIC AND HYDRAULIC CHARACTERISTICS OF CHANNEL FLOW

The value of the average depth h_m and the value of the average velocity U_m of channel flow were calculated by the formulas (8), (9):

$$h_m = \left(\frac{Q \cdot n}{B\sqrt{i_m}} \right)^{0,6}, \quad (8)$$

$$U_m = \frac{Q}{B \cdot h_m}. \quad (9)$$

№	**Annual exceedance probabilities**, **%** Roczne prawdopodobieństwo przekroczenia normy, %	**Water discharge *Q*, m³/c** Natężenie przepływu Q, m³/s	**Width of the channel *B*, m** Szerokość cieku *B*, m	**Depth of flow h_m, m** Głębokość przepływu h_m, m	**Velocity U_m, m/c** Prędkość, U_m m/s
1	5	1793	145,3	3,98	3,1
2	10	1425	135,3	3,61	2,9
3	20	1057	126,5	3,15	2,65
4	30	832	121,2	2,79	2,45
5	50	538	87,5	2,62	2,34

Numerical simulation of the bottom resistance in place of the flow of obstacle was carried out for the two model cases: 1) angle α between transit flows and barrier axis takes values from 25 to 60 degrees; 2) obstacle height Z_b is 1/3 and 1/2 the depth of flow. Relative average speed and the relative width of the turbulent flow in the field behind of obstacle were taken depending on the angle of obstacle location and its height as recommended in [1, 2, 4].

According to [1, 2] the bottom resistance forth F_R will be:

$$F_R = F_P + F_T \,, \qquad (10)$$

where F_P is pressure drop; F_T is friction forth:

$$F_P = W_B \cdot \Delta h \cdot \rho \cdot g \,, \qquad (11)$$

$$F_T = \frac{W_F}{Z_b} \cdot k_T \cdot \rho \cdot \left(U_m \cdot \cos\alpha - V_m\right), \qquad (12)$$

$$\Delta h = i_T \cdot \left(L_b - 3Z_b\right), \qquad (13)$$

$$i_T = \frac{U_m{}^2}{R_b \cdot C^2} \,,\; R_b = h_m - Z_b \,,\; C = \frac{1}{n} R_b{}^{1/6} \,,\; L_b = B/\sin\alpha \,, \qquad (14)$$

where W_B is cross-sectional area of the investigated turbulent channel flow over the obstacle; Δh is drop of the free water surface in place of barrier flow; ρ is density of water; g is acceleration of gravity; W_F is surface area of the investigated turbulent flow over the obstacle (the surface area of water roll); Z_b is obstacle height; k_T is friction coefficient in the field behind of obstacle; U_m is average transit speed of channel flow; α is the angle between the transit flow and the axis of obstacle; V_m is average flow velocity in the field behind of obstacle; i_T is friction grade; R_b is hydraulic radius for the flow that moves on its border section of the flow behind of bottom obstacle (the interface is considered as "rough bottom" for the stream that runs on top); C is Chezy coefficient; n is roughness coefficient of the channel; L_b is obstacle length; B is width of the channel.

The value of the coefficient of friction k_T and the value of average speed V_m of flow in the field behind of obstacle were taken according to data of laboratory studies of water flow over bottom ledge, depending on the angle α, relative height of the ledge ($S_z = Z_b / h_m$), the ratio of the width of the stream B to its average depth h_m [1, 2].

Modeling of the bottom resistance F_R and shear stresses W_F / F_R was carried to for different values of the angle of placement of barrier to transit flow α, interrelation of the height barrier and depth of flow $S_z = Z_b / h_m$ and the relation U_m / W_K of the average flow velocity U_m and hydraulic size of sediments W_K, where:

$$W_K = \sqrt{\frac{2g(\rho_s - \rho) \cdot d_m}{1{,}75\ \rho}} \,, \qquad (15)$$

where ρ_s is density of the soil; d_m is average particle diameter of the soil of the river bed.

SIMULATION RESULTS AND CONCLUSIONS

The results of the numerical simulation of model characteristics F_R and W_F/F_R for assessing the intensity of the channel deformations behind of bottom obstacle for accepted data and modeling conditions are shown below on Fig. 1-5.

In general the obtained results of computer modeling correspond to natural and experimental data on the conditions of intensification of local erosion of the river bed behind of natural barriers that deviate from the shore downstream, due to the development of the local turbulent flow directed from the shore.

It is shown that shear stress W_F/F_R and overall resistance strength of bottom F_R can be used as representative model characteristic to assess the intensity of the flow channel deformations behind of the natural benthic barriers, to characterize both overall speed of river channel deformations behind of obstacles and intensity of local erosion, dredging of bottom behind of obstacle, depending only on two essential factors, namely angle location of benthic barrier on the transit flow and that the height of the obstacle and the average depth of flow are correlated with each other ($S_z = Z_b/h_m$).

It is established that the value of the relative height $S_z = Z_b/h_m$ of benthic barrier, where the most intense erosion behind of obstacle on straight sections of the river channel is expected, is 0,33 approximately. In addition, the angle between the axis of the obstacle and the direction of transit flow that characterizes the most intense local erosion may vary from 30 to 40 degrees. In particular, as the results of simulation modeling show, at the height of the obstacle, which is 1/3 of depth of flow, the most powerful local river channel deformations can occur if the angle between the obstacle and the transit river flow is from 35 to 40 degrees.

REFERENCES

1. Schodro A.E. River Bed Deformations Near Banks and Hydraulic Structures due to River Flow / A.E. Schodro // Proceedings of Fourth International Conference on River Basin Management Including all Aspects of Hydrology, Ecology, Environmental Management, Flood Plains and Wetlands. WIT-Press, Southampton, Boston, 2007. – P.381-387.
2. Shchodro A.E. Local Scour Investigation on Steep River Bends in Ukrainian Carpathian. Proceedings of the Tenth International Symposium on River Sedimentation. Effects of River Sediments and Channel Processes on Social, Economic and Environmental Safety. August 1-4, 2007.Moscow, Russia. V. 3. – P. 275-284.
3. Shchodro O.E. Fast Algorithm for the Simulation of Turbulent Flows in water Courses / O.E. Schodro, Ya.V. Khodnevich // Hydraulics and Hydraulic Engineering. Scientific Technical Collection. Issue 62. Kyiv, 2008. – P. 25-36 (in Ukrainian).
4. Khodnevich Ya.V. Water Flume Dynamics and Local Scores Modeling Near Obstacles With the Use of Poisson Equation/ Ya.V. Khodnevich, O.E. Schodro // Water management - state and prospects of development. Collected articles of young scientists. Part 1. Rivne, National University of Water Management and Nature Resources Use, 2010. – P. 55-57.
5. Stefanyshyn D.V. About one approach to the prediction of local erosion channels and banks of rivers in foothill areas using mathematical modeling capabilities / D.V. Stefanyshyn Stefa-

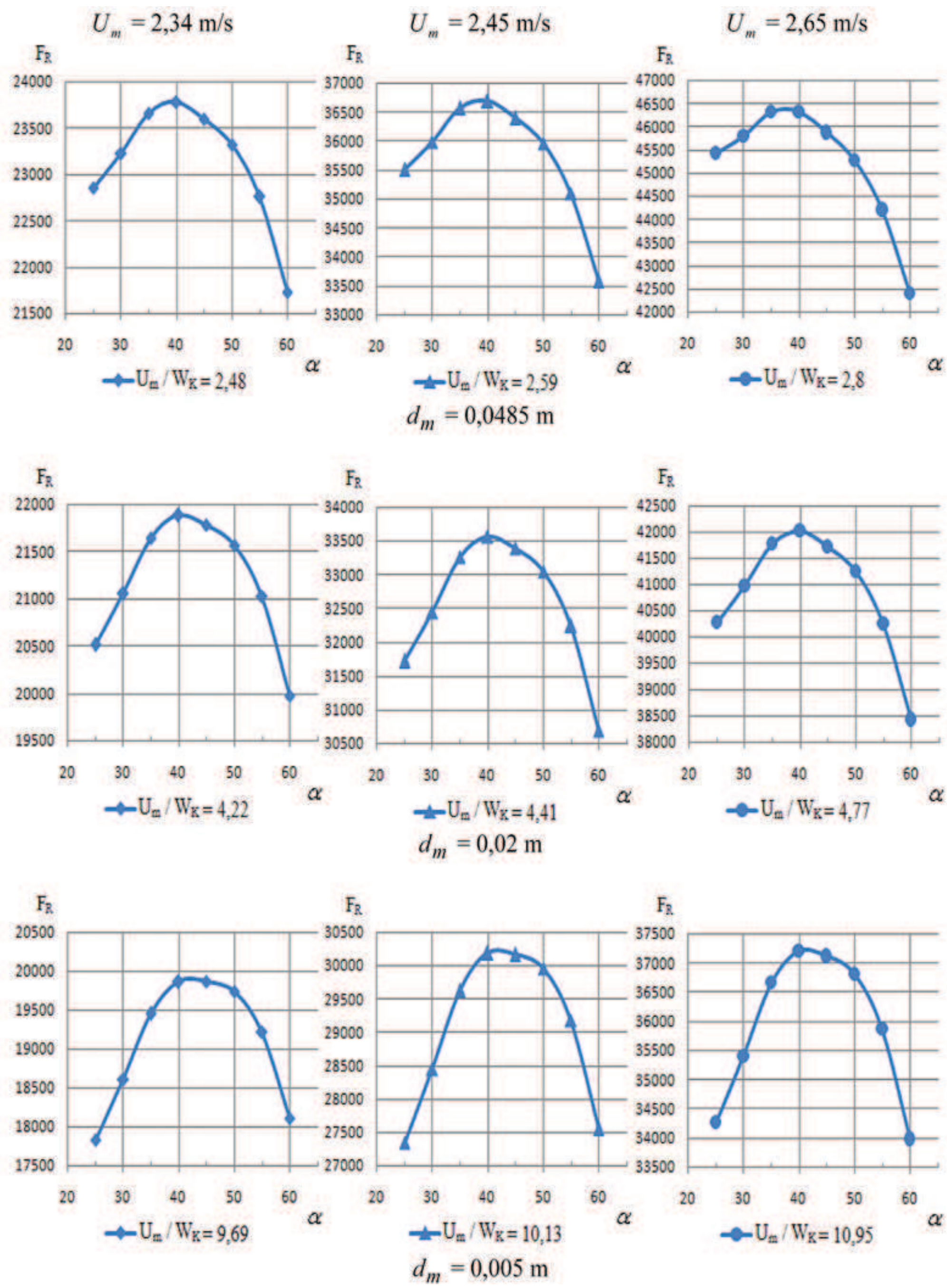

Fig. 1. Results of numerical simulation of the bottom resistance F_R if S_z = 0,33 (series 1)
Rys. 1. Wyniki numerycznej symulacji oporu dna F_R gdy S_z = 0,33 (seria 1)

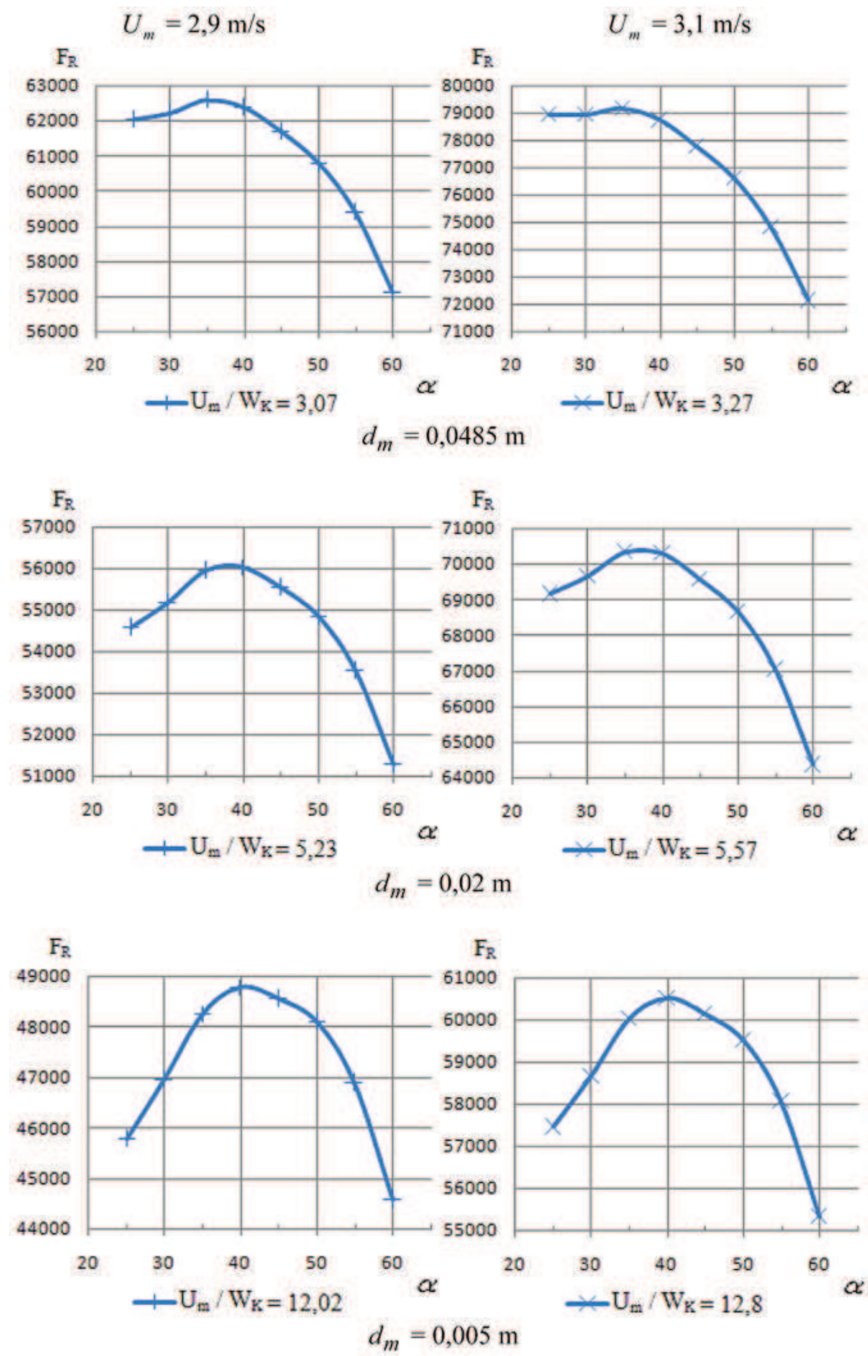

Fig. 2. Results of numerical simulation of the bottom resistance F_R if S_z = 0,33 (series 2)
Rys. 2. Wyniki numerycznej symulacji oporu dna F_R gdy S_z = 0,33 (seria 2)

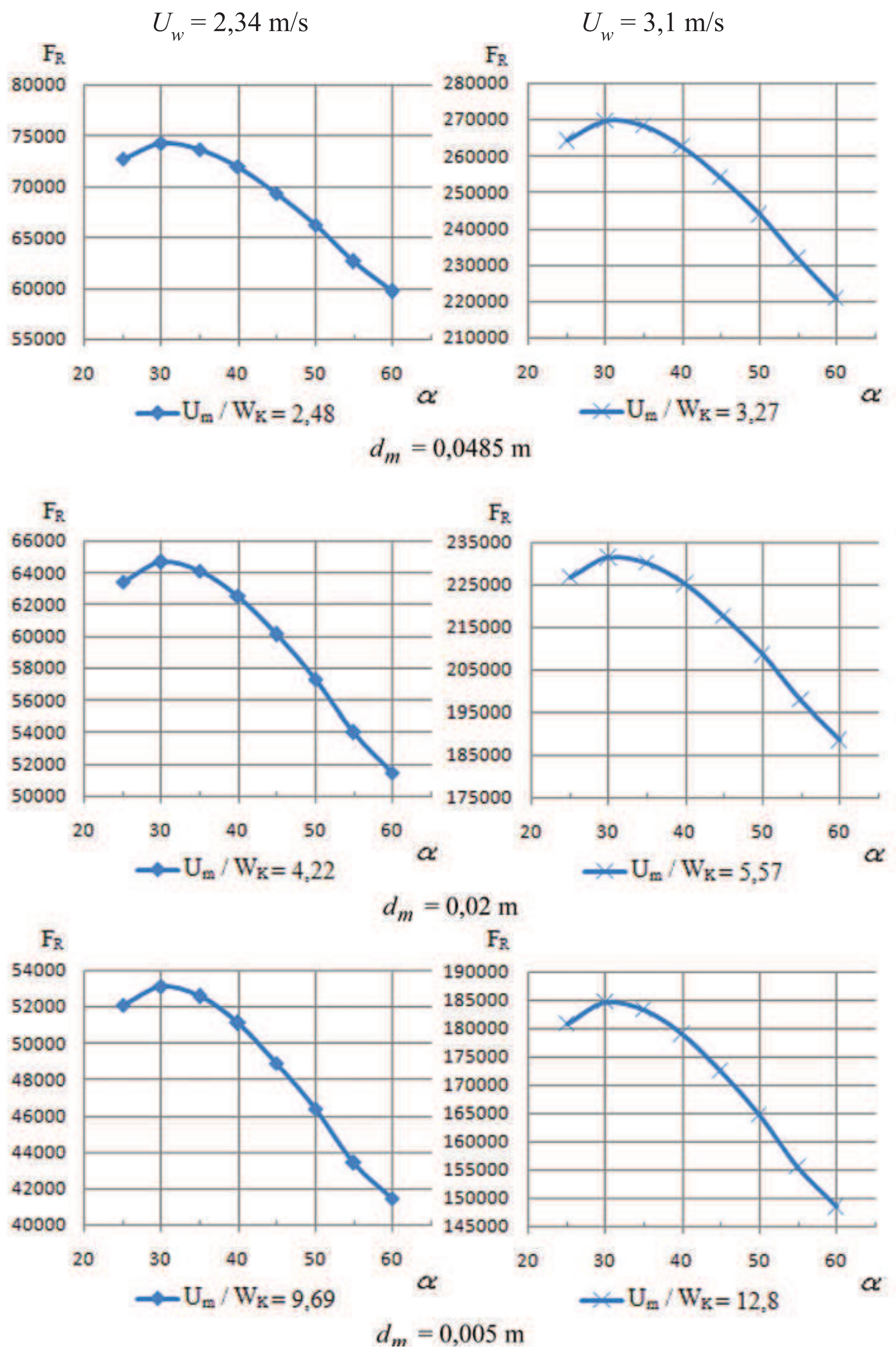

Fig. 3. Results of numerical simulation of the bottom resistance F_R if $S_z = 0{,}5$
Rys. 3. Wyniki numerycznej symulacji oporu dna F_R gdy $S_z = 0{,}5$

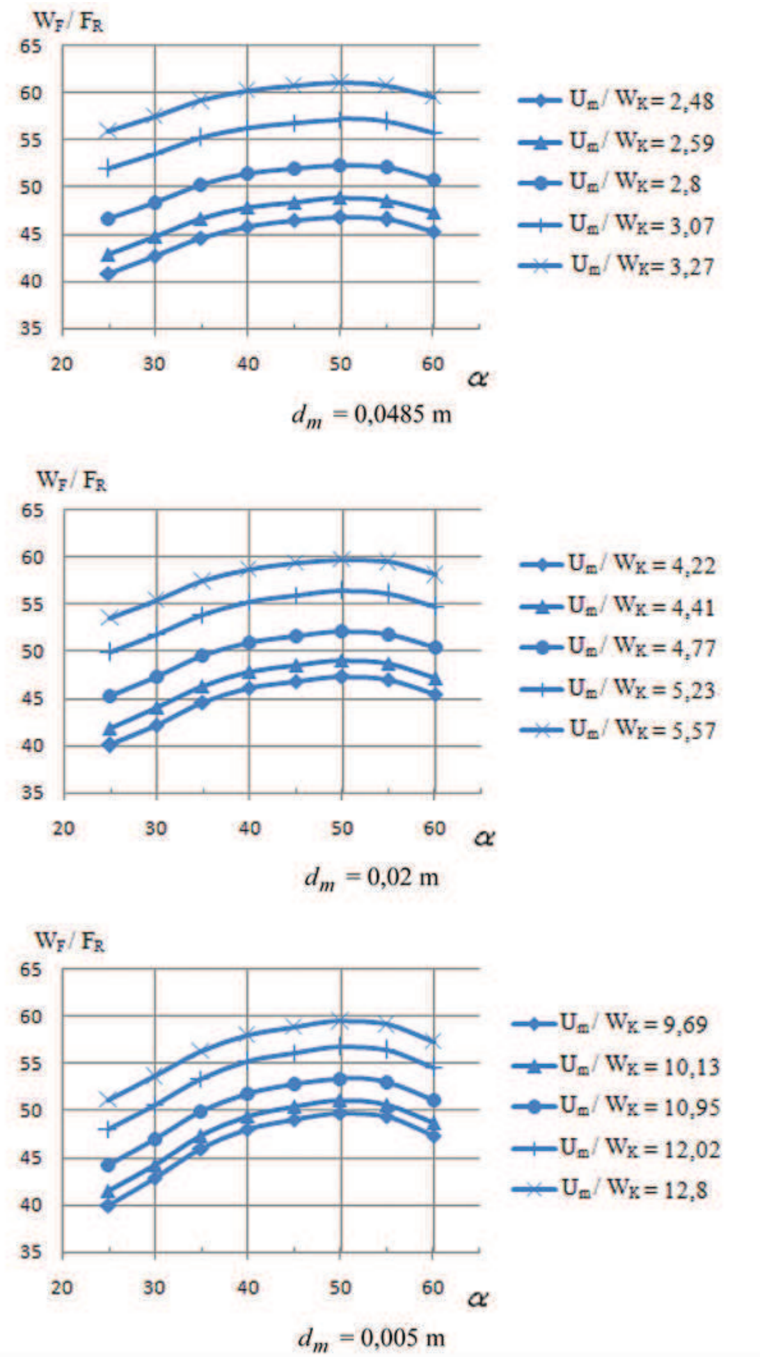

Fig. 4. Results of numerical simulation of the shear stress W_F / F_R if $S_z = 0{,}33$
Rys. 4. Wyniki numerycznej symulacji naprężenia stycznego W_F / F_R gdy $S_z = 0{,}33$

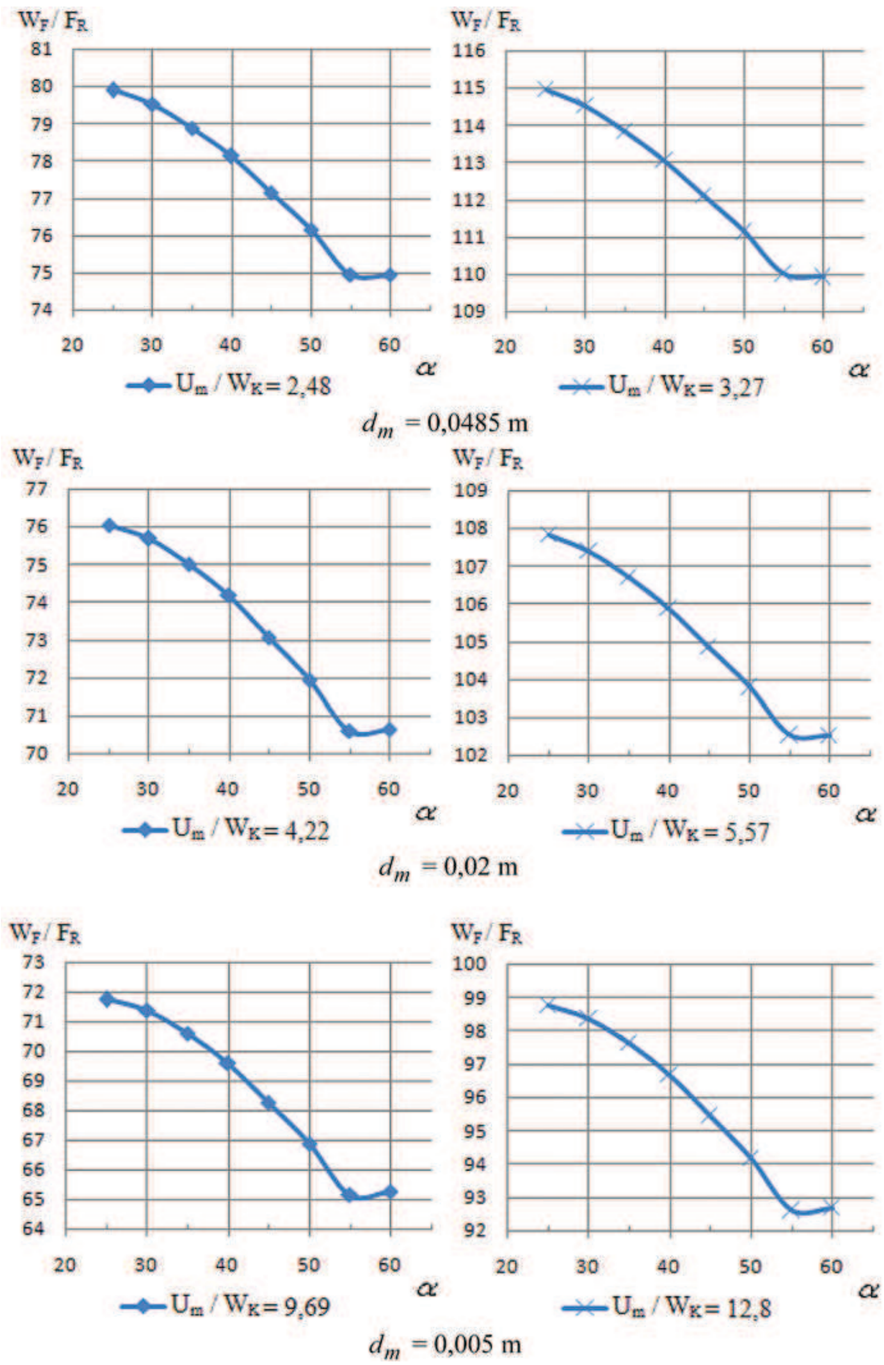

Fig. 5. Results of numerical simulation of the shear stress W_F/F_R \if $S_z = 0{,}5$
Rys. 5. Wyniki numerycznej symulacji naprężenia stycznego W_F/F_R gdy $S_z = 0{,}5$

nyshyn, Ya.V. Khodnevich, O.E. Schodro // Environmental Safety and Environmental Sciences. Collected Works. Issue 7. Kyiv National University of Construction and Architecture, The Institute of Telecommunications and Global Information Space of the National Academy of Sciences of Ukraine. Kyiv, 2011. – P. 64-72 (in Ukrainian).

6. Khodnevich Ya.V. Efficient solving sparse systems of linear equations for the problem of modeling the water flow / Ya.V. Khodnevich // Proceedings of Sixth VI International Conference named I.I. Lyashko "Computational and Applied Mathematics". 5, 6 September, 2013. Kyiv National Taras Shevchenko University. Kyiv, Ukraine. – P. 207 (in Ukrainian).
7. Wilcox D.C. Turbulence Modeling for CFD / D.C. Wilcox // DCW Industries, Inc. La Canada, California, 1994. – 456 p.
8. Tseng M.H. Two-dimensional shallow water flows using TVD-MacCormack scheme / M.H. Tseng, C.R. Chu // Journal of Hydraulic Research. 2000. Vol. 38. No. 2.

MODELOWANIE MATEMATYCZNE WARUNKÓW INTENSYFIKACJI MIEJSCOWEJ EROZJI KORYTA RZEKI ZA PRZESZKODĄ, KTÓRA ODCHYLA SIĘ OD BRZEGU W DÓŁ RZEKI

Streszczenie

Celem niniejszej pracy jest matematyczne modelowanie warunków intensyfikacji lokalnej erozji koryta rzeki za naturalną barierę , która odchyla się od brzegu w dół rzeki z powodu rozwoju miejscowego gwałtownego prądu skierowanego od brzegu. W pracy podane są wyniki numerycznej symulacji intensywności przepływu za przeszkodą na dnie, w zależności od jej wysokości i kąta pod jakim jest umiejscowiona. Ustalono, że najsilniejsze deformacje koryta rzeki mogą wystąpić jeśli wysokość przeszkody równa jest 1/3 głębokości strumienia a kąt umiejscowienia wynosi między 35 a 40 stopni w stosunku do nadchodzącego przepływu.

Słowa kluczowe: Intensywność przepływu wody, miejscowa erozja, matematyczne modelowanie, symulacja numeryczna, gwałtowny prąd

MATHEMATISCHE MODELLIERUNG DER INTENSIVIERUNGSBEDINGUNGEN DER ŐRTLICHEN FLUSSBETTSEROSION HINTER DEM HINDERNIS, DAS STROMMABWÄRTS VOM FLUSSUFER ABFÄLLT

Zusammenfassung

Zum Ziel der vorliegenden Bearbeitung wurde die mathematische Modellierung der Intensivierungsbedingungen der örtlichen Flussbetterosion hinter dem Hindernis, das stromabwärts vom Flussufer wegen der örtlichen, heftigen Uferströmung abfällt. Es wurden hier die Ergebnisse der nummerischen Simulation der Intensivierungsbedingungen der örtlichen Flussbetterosion hinter dem Hindernis am Grund bekanntgegeben im Bezug auf die Hőhe und Winkel der Platzierung. Es wurde festgestellt, die größten Deformationen des Flussbetts auftreten können, wenn die Hindernishőhe 1/3 der Stromtiefe gleicht und der Winkel selbst zwischen 35-40 Grad gegenüber dem kommenden Wasserdurchfluss beträgt.

Schlüsselworte: Intensivierung des Wasserdurchflusses, örtliche Erosion, mathematische Modellierung, nummerische Simulation, heftige Strömung