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# Least Support Orthogonal Matching Pursuit Algorithm with Prior Information

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**Abstract**

This paper proposes a new fast matching pursuit technique named Partially Known Least Support Orthogonal Matching Pursuit (PKLS-OMP) which utilizes partially known support as a prior knowledge to reconstruct sparse signals from a limited number of its linear projections. The PKLS-OMP algorithm chooses optimum least part of the support at each iteration without need to test each candidate independently and incorporates prior signal information in the recovery process. We also derive sufficient condition for stable sparse signal recovery with the partially known support. Result shows that inclusion of prior information weakens the condition on the sensing matrices and needs fewer samples for successful reconstruction. Numerical experiments demonstrate that PKLS-OMP performs well compared to existing algorithms both in terms of reconstruction performance and execution time.

**Key words:** Compressed sensing, Least Support Orthogonal Matching Pursuit, Partial Knowing Support, signal reconstruction, Restricted Isometry Property.

## 1 Introduction

Compressed sensing (CS) stands for a linear underdetermined problem, where the underlying sampled signal is sparse. The challenge in CS is to reconstruct this sparse signal from few measurements as possible as it could.

The standard CS theorem is based on a sparse signal model and uses an undetermined system of linear equations [1]. Linear Programming (LP) techniques are good for designing computationally CS decoders, but it shows kind of complexity for many applications. Therefore, the need for the faster decoding algorithms is necessary, even if a procedure increases the number of measurements. Several low complexity reconstruction methods are used nowadays as an alternative method for LP recovery. Some of these include: Convex

Optimization: like Basis Pursuit (BP) and Basis Pursuit De-Noising (BPDN), Iterative Greedy Algorithms like Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP), the Regularized OMP (ROMP), and Compressive Sampling Matching Pursuit (CoSaMP). Iterative greedy depends on its search method upon an estimation of the implicit support set of a sparse vector [2]. For example, CoSaMP algorithm is based on the idea of iteration to find the approximation of the original signal. In each iteration, the current approximation produces a residual, which it is the part of the treated signal that has not been approximated yet [3].

The simple idea behind the usage of greedy method is to find the support for unknown signal sequentially. The support set contains indices of non-zero elements of a sparse vector [4]. At each iteration, OMP greedy algorithm uses one or several coordinates of input signal vector x, that are selected using the maximum correlation value between the columns of $\Phi$ and the measurement vector. The candidates will be added to the currently estimate support set of x. The pursuit algorithm repeats this procedure several times until all the coordinates are arranged in the evaluated support set.

The computational complexity of the OMP depends on the number of iterations necessary for the exact signal reconstruction; simple OMP always runs through K iterations. Although complexity is smaller comparing with that of LP methods [5], sparsity of the signal plays an important role in the complexity of the OMP especially when the signal being recovered is not very sparse.

Recently, CS methods have been expanded by including partially known support in the recovery process. Theoretical and numerical experiments show that CS with partially known support can reduce the number of measurements required for exact or approximated recovery. Vaswani and Lu [6] used this technique for recovering of dynamic magnetic resonance imaging (MRI). Carrillo [7]-[8] exploited prior information for iterative algorithms.

In this paper, we focus on a fast greedy algorithm including partially known support. Exact reconstruction using fewer measurements is the purpose of the current work. The proposed algorithm has less computational complexity and faster than the standard OMP and Orthogonal Matching Pursuit with Partially Known Support [7] (OMP-PKS). We also derive conditions for stable sparse signal recovery with the partially known support. Theoretical analysis shows that exploiting prior information provides much weaker conditions for successful reconstruction. Simulations results demonstrate that the Partially Known Least Support Orthogonal Matching Pursuit (PKLS-OMP) outperforms commonly employed sparse reconstruction techniques and inclusion of the partially known support reduces the number of measurements for signal reconstruction.

The organization of the rest of the paper is as follows: Section 2 gives a brief review of the OMP and the OMP-PKS algorithms. In Section 3, the PKLS-OMP algorithm is proposed and its properties are analyzed. Numerical experiments evaluating the performance of the proposed algorithms are presented in Section 4. Finally, in Section 5, we give conclusions and future works.

## 2 Preliminaries

### 2.1 OMP algorithm

Notations: let x be a sparse signal, the arbitrary vector $x = \{x_1, x_2 \ldots\ldots, x_N\}^t$, let the support set $T \subset \{1,2 \ldots., N\}$ denotes the set of nonzero component indices of x (i.e $\mathrm{up}(x) = \{i|x_i \neq 0\}$ ), $A_I \in \mathbb{R}^{Mx|I|}$ consists of the columns of A with indices $i \in I$, $A^*$ denotes the transpose of A, and $A^\dagger$ denotes the pseudo-inverse $\{(A^*A)^{-1}A^*\}$.

Let us declare the standard CS problem, which achieves a signal $x \in \mathbb{R}^N$, have a K sparse input, via the linear measurements,

$$y = \Phi x \quad (1)$$

where $\Phi \in \mathbb{R}^{mxN}$ represents a random measurement (sensing) matrix, and $y \in \mathbb{R}^M$ represents the compressed measurement signal. A K sparse signal vector consists of most K nonzero indices. With the setup of $K < M < N$, the task is to reconstruct x from y (as $\hat{x}$ ) using a small number of measurements in addition to achieve good reconstruction qualification [5], [9].

We note that the compressed measurement signal y is the linear combination of at most K atoms (atom means a column of measurement matrix). One condition for the sparse signal recovery is to use the Mutual Incoherence Property (MIP) [10]. The MIP requires the correlations among the column vectors of $\Phi$ to be small.

The coherence parameter $\mu$ of sensing matrix is defined as,

$$\mu = \max_{i \neq j}\langle \varphi_i, \varphi_j \rangle, \quad (2)$$

where $\varphi_i, \varphi_j$ are two columns of $\Phi$ with a unit norm.

For the noiseless case when $\Phi$ is a series of two square orthogonal matrices, that

$$K < \frac{1}{2}\left(\frac{1}{\mu+1}\right) \quad (3)$$

guaranties the exact recovery of $\hat{x}$ when $\hat{x}$ has at most K nonzero entries (such a signal is called $K$-sparse) [11]-[13].

### 2.2 OMP-PKS

OMP-PKS is derived from the classical OMP. In this algorithm, some signal components are more important than the others are and should be kept as nonzero components. That is prior support information is available from the prior knowledge, e.g. the lowest sub band wavelet coefficients are selected as nonzero components without testing them for correlation [7], [13]-[14]. Compared with the OMP, the OMP-PKS can recover $y$ with the low measurement rate.

The OMP-PKS algorithm is shown in Algorithm 1 [13]-[14].

**Algorithm 1** OMP-PKS algorithm

*Input:*

- MxN measurement matrix $\Phi = [\varphi_1\ \varphi_2\ \ldots\ldots \varphi_N]$
- $M \times 1$ compressed measurement vector, y
- Set of indexes of $LL_n$ coefficients subband (i.e $n = 3$) , $\Gamma = \{\gamma_1\ \gamma_2\ \ldots\ldots \gamma_{|\Gamma|}\}$
- K Sparsity level

*Output:*

- Recovered signal, $\hat{s}$
- Set that includes k nonzero indices in $\hat{s}$, $\Lambda = \{\lambda_1\ \lambda_2\ \ldots\ldots \lambda_k\}$

*Procedure:*

Phase1: selection without correlation test,

a- Choose all element in $LL_n$

$$t = |\Gamma|\ ,\quad \Lambda_t = \Gamma\ ,\quad \Phi_t = [\varphi_{\gamma_1}\ \varphi_{\gamma_2}\ \ldots\ldots \varphi_{\gamma_t}]$$

b- find least square for reconstructed signal $z_t$

$$z_t = \arg\min_z \|y - \Phi_t Z\|_2$$

c- find new approximation, $a_t$, where $a_t$ is the projection of $y$ on $\Phi_t$; calculate the residual $r_t$

$$a_t = \Phi_t z_t\ ;\quad r_t = y - a_t$$

Phase 2: reconstruct using the OMP,

a- increment $t$ (i.e $t = t + 1$), halt if $t > K$

b- calculate index $\lambda_t$ for basis $\varphi_j$ which has the highest correlation with residual of previous iteration ($r_{t-1}$)

$$\lambda_t = \arg\max_{j=[1,N], j\notin\Lambda_{t-1}} \left|\langle r_{t-1}, \varphi_j\rangle\right|$$

c- increase the index set of selected basis and also increase its matrix

$$\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}\ ;\quad \Phi_t = [\Phi_{t-1}\ \varphi_{\lambda_t}]$$

d- find least square for reconstructed signal $z_t$

$$z_t = \arg\min_z \|y - \Phi_t Z\|_2$$

e- find new approximation, $a_t$ , and calculate the residual $r_t$

$$a_t = \Phi_t z_t\ ;\quad r_t = y - a_t$$

f- return to a

***Lemma 1*** [11] Consequences of Restricted Isometry Property (RIP): $I \subset \Omega, if\ \delta_{|I|} < 1$ then for any $u \in R^{|I|}$,

$$\left(1-\delta_{|I|}\right)\|u\|_2 \le \|\Phi_I'\Phi_I u\|_2 \le \left(1+\delta_{|I|}\right)\|u\|_2, \tag{4}$$

$$\frac{1}{(1+\delta_{|I|})}\|u\|_2 \le \|(\Phi_I'\Phi_I)^{-1}u\|_2 \le \frac{1}{(1-\delta_{|I|})}\|u\|_2 \tag{5}$$

***Lemma 2*** [11] For disjoint sets $I_1, I_2 \subset \Omega, if\ \delta_{|I_1|+|I_2|} < 1$ then,

$$\left\|\Phi_{I_1}'\Phi \mathrm{v}\right\| = \left\|\left(\Phi_{I_1}'\Phi_{I_2}v_{I_2}\right)\right\| \le \delta_{|I_1|+|I_2|}\|v\|, \tag{6}$$

***Lemma 3*** [10] For all $x, x' \in R^n$ supported on disjoint subsets $I_1, I_2 \subset \Omega$,

$$|\langle \Phi \mathrm{x}, \Phi \mathrm{x}' \rangle = |\delta_{|I_1|+|I_2|}\|x\|\|x'\| \tag{7}$$

***Lemma 4*** [10] Consequence of restricted orthogonality constant: for two disjoint sets $I_1, I_2 \subset \Omega$ , let $\theta_{|I_1|,|I_2|}$ be the $|I_1|, |I_2|$ *-restricted orthogonality constant of* $\Phi$. If $|I_1| + |I_2| \le n$, $\theta_{|I_1|,|I_2|'}$, is the smallest number that satisfies

$$\left\|\Phi_{I_1}'\Phi_{I_2}\mathrm{x}_{I_2}\right\| \le \theta_{|I_1|,|I_2|}\|\mathrm{x}\|. \tag{8}$$

***Lemma 5*** [10] If $\Phi$ satisfies the RIP of both orders $K_1$ and $K_2$, then $\delta_{K_1} \le \delta_{K_2}$ for any $K_1 \le K_2$. This property is referred as the monotonicity of the isometry constant.

***Lemma 6*** [10] For two disjoint sets $\mathrm{I}_1, \mathrm{I}_2 \subset \Omega$ with $|\mathrm{I}_1| + |\mathrm{I}_2| \le \mathrm{n}$, $\theta_{|\mathrm{I}_1|,|\mathrm{I}_2|} \le \delta_{|\mathrm{I}_1|,|\mathrm{I}_2|}$

***Definition 1*** [5] Let $\mathrm{y} \in \mathrm{R}^{\mathrm{m}}$ and $\Phi_{\mathrm{I}} \in \mathrm{R}^{\mathrm{m}\times|\mathrm{I}|}$, let $\Phi_{\mathrm{I}}^*\Phi_{\mathrm{I}}$ be invertible matrix, the projection of y onto span $(\Phi_{\mathrm{I}})$ can be defined as,

$$\mathrm{y_p} = \mathrm{proj}(\mathrm{y}, \Phi_{\mathrm{I}}) = \Phi_{\mathrm{I}}{\Phi_{\mathrm{I}}}^{\dagger}\mathrm{y} \tag{9}$$

$$\Phi_{\mathrm{I}}^{\dagger} = (\Phi_{\mathrm{I}}^*\Phi_{\mathrm{I}})^{-1}\Phi_{\mathrm{I}}^*$$

where $\Phi_{\mathrm{I}}^{\dagger}$ represents the pseudo inverse of matrix $\Phi_{\mathrm{I}}$ and * denotes the transpose of $\Phi_{\mathrm{I}}$. Residue vector of the projection can be found as:

$$\mathrm{y_r} = \mathrm{resid}(\mathrm{y}, \Phi_{\mathrm{I}}) = \mathrm{y} - \mathrm{y_p}. \tag{10}$$

***Lemma 7*** [1] Residue Orthogonality : if a vector $\mathrm{y} \in \mathrm{R}^{\mathrm{m}}$ and $\Phi_{\mathrm{I}} \in \mathrm{R}^{\mathrm{m}\times\mathrm{K}}$ represents sampling matrix which has full column rank, and $\mathrm{y_r} = \mathrm{resid}(\mathrm{y}, \Phi_{\mathrm{I}})$, then

$$\Phi_{\mathrm{I}}^*\mathrm{y_r} = 0 \tag{11}$$

***Lemma 8*** [1] Approximation of Projection Residue: consider $\Phi_{\mathrm{I}} \in \mathrm{R}^{\mathrm{m}\times\mathrm{N}}$, if $\mathrm{I}, \mathrm{J} \subset \{1 \ldots\ldots \mathrm{N}\}$ are two disjoint set (i.e. $\mathrm{I} \cap J = \emptyset$) and let $\delta_{|I|+|J|} < 1$ suppose $y \in span\,(\Phi_I)$, $y_p = proj(y, \Phi_J)$ , $y_r = resid(y, \Phi_J)$ , then

$$\left\|\mathrm{y_p}\right\|_2 \le \frac{\delta_{|\mathrm{I}|+|\mathrm{J}|}}{1-\delta_{\max(|\mathrm{I}|,|\mathrm{J}|)}}\|\mathrm{y}\|_2. \tag{12}$$

## 3 PKLS-OMP

In this section, we introduce the proposed PKLS-OMP algorithm. Algorithm consists of two parts: LS-OMP and PKLS-OMP. Figure 1 shows a block diagram of the LS-OMP. The algorithm selects an atom for the current iteration by testing influence of this selection on the future iterations. An element is chosen at the beginning of the calculation by finding a set of maximum correlation between ϕ and whole signal matrix. This way is faster since it requires less number of iterations L<K. LS-OMP achieves better assessment for underlying support set through iterations without need to test each potential independently.

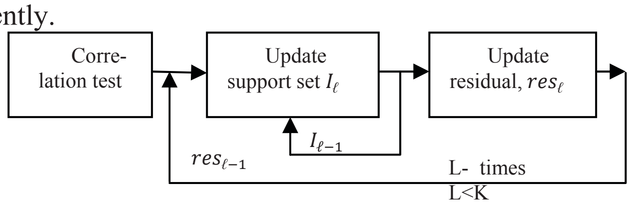


**Figure 1.** Block diagram of LS-OMP method

Theorem 1 below, shows theoretical stability guarantees of the LS-OMP.

***Theorem 1:***

If $x$ is a sparse signal and $x \in \mathbb{R}^N$, $y$ is measurement vector $y = \Phi x$, $\Phi$ is sampling matrix satisfies RIP condition, then $x$ can be recovered if :

$$\left\|y - y_r^{\ell}\right\|_2 \leq \frac{\delta_{2L}}{1-2\delta_{2L}} \left\|y_r^{\ell-1}\right\|_2 \quad (13)$$

where L represents a part of support that will be used as a least support to reconstruct the original signal its range is limited by the stopping condition suggested in eq.(13). $\ell$ represents the current iteration while $\ell - 1$ is the previous iteration ,assume $\delta_{2L} = 0.49$.

Our proof of Theorem 1 appears in the Appendix A.

The LS-OMP algorithm recovering an estimate of the signal x is specified in Algorithm 2.

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**Algorithm 2** *LS-OMP Algorithm for Signal Recovery*

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*Input:*

– NxM measurement matrix $\Phi$
– $N \times 1$ compressed measurement vector, $y$
– Sparsity level K of the sparse signal
– L Least Support Parameter

*Output:*

– An $\hat{X}_{1\times M}$ reconstructed signal, new set of nonzero $Aug\_p_{(1\times L)}$

*Procedure:*

1) Initialize the residual, $res_0 = y$, support set $I_0 = \phi$ , Least Support set $J_0 = \phi$ , and the iteration counter $\ell = 1$
2) find the maximum value of auto correlation between y and Φ
$J_\ell = \arg\max_{\ell=1\ldots L} |res_{\ell-1}\varphi_\ell|$
3) union the set of Φ matrix column indexed by J, and previous support of size l
$I_\ell = [I_{\ell-1} \cup \Phi_J]$
4) find the new augment value $Aug\_p = I_\ell^\dagger \times y$ ( $I_\ell^\dagger$ denotes the pseudo-inverse operators of set $I_\ell$)
5) find new residual value $res_\ell = y - Aug\_p \times I_\ell$
6) if stopping condition $\|y - y_r^\ell\|_2 \le \frac{\delta_{2L}}{1-2\delta_{2L}} \|y^{\ell-1}\|_2$, is correct then update position set from [1,L] to [1, ℓ ] and go to step (9) ,
7) upgrade the value of $res_{\ell-1} = res_\ell$ and $I_{\ell-1}$=$I_\ell$
8) increment ℓ = ℓ +1 and return to step (2)
9) the reconstructed sparse signal $\hat{X}_{1\times M}$ has nonzero indices at the indexes listed in $Aug\_p_{(1\times L)}$.

In the second part of the PKLS-OMP algorithm, the prior signal information is incorporated in the recovery process. A Discrete Wavelet Transform (DWT) is used to sparsify the signal and all the components in low sub band are selected as nonzero components. The PKLS-OMP algorithm for the data represented in the wavelet domain is shown in Algorithm 3:

**Algorithm3:** PKLS-OMP Algorithm for Signal Recovery

*Input:*
- NxM measurement matrix Φ
- $N \times 1$ compressed measurement vector, y
- Sparsity level K of the sparse signal
- L Least Support Parameter
- $T_0$ Set of indexes of $LL_3$

*Output:*
An $\hat{X}_{1\times M}$ reconstructed signal, new set of nonzero $Aug\_p_{(1\times L)}$

*Procedure:*
1) Initialize the residual , $res_0 = y$ ,
support set: $T_0 = [T_{01}, T_{02} \ldots T_{0|T_0|}]$
least Support set: $J_0 = \phi$
number of Iteration: ℓ =0
2) support size: Sup_size=$|T_0|$
3) $\varphi_j = [\varphi_1 \, \varphi_2 \ldots \varphi_{|T_0|}]$
4) find $res_\ell = y - \varphi_j{}^\dagger y$
5) index=$T_0$, $I_0 = \varphi_j$

6) increment $\ell = \ell + 1$
7) find the maximum value of auto correlation between $res_\ell$ and $\Phi$,
$J_\ell = \arg\max_{\ell=1\ldots L} \left| \varphi_\ell{}^{*} res_{\ell-1} \right|$
8) augment the index set and matrix of chosen atoms indexed by J, $I_\ell = [I_{\ell-1} \cup \Phi_J]$,
9) find the new augment value $Aug\_p = I_\ell^{\dagger} \times y$ ( $I_\ell^{\dagger}$ denotes the pseudo-inverse operators of set $I_\ell$)
10) find new residual value $res_\ell = y - Aug\_p \times I_\ell$
11) update index, $index(|T_0| + \ell) = J(\ell)$
12) if the termination condition $\|y_r\|_2 \le \frac{\delta_{2l}}{(1-\delta_{2l})} \|y\|_2$, update the position set from [1,L] to [1, $\ell$ ] and go to step (15 ),
13) upgrade the value of $res_{\ell-1} = res_\ell$ and $I_{\ell-1} = I_\ell$,
14) return to step (6) if iteration number $\ell < K$,
15) the reconstructed sparse signal $\widehat{X}_{1\times M}$ has nonzero indices at the index listed in $Aug\_p_{(1\times L)}$, arrange the value of Aug_p in the position listed by J. $\widehat{X}_{1\times M}(index\,[\,1 : |T_0| + \ell\,]) = Aug\_p_{(1\times L)}$

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We propose two different theorems, Theorem 2 and 3, to show the theoretical stability guarantees of the PKLS-OMP in terms of the RIP of $\Phi$.

***Theorem 2:***

If $x$ is sparse signal and $x \in \mathbb{R}^N$ , $y$ is measurement vector $y = \Phi x$ , $\Phi$ is sampling matrix satisfies RIP condition, then $x$ can be recovered if

$$\|y_r\|_2 \ge \frac{\delta_{2L}}{(1-\delta_{2L})} \|y_0\|, \tag{14}$$

for $0.005 \le \delta_{2L} \le 0.025$.

proof of Theorem 2 explain in the Appendix B.

***Theorem 3***

If $x$ is sparse signal and $x \in \mathbb{R}^N$ , $y$ is measurement vector $y = \Phi x$ , $\Phi$ is sampling matrix satisfies RIP condition, then $x$ can be recovered if :

$$\|y\|_2 \le \frac{1-\delta_{2L}}{\delta_{2L}(1-2\delta_{2L})} \|y_r\|_2 \tag{15}$$

for $0.005 \le \delta_{2L} \le 0.025$.

proof of Theorem 3 appears in the Appendix C.

A sufficient condition for stable recovery of the LS-OMP algorithm is $\delta_{2L} = 0.49$ (Theorem 1), which is a stronger condition than $0.005 \le \delta_{2L} \le 0.025$ required by PKLS-OMP with Theorem 2 and Theorem 3. Having a smaller RIP means that $\Phi$ requires fewer rows to meet the condition, i.e., fewer samples to achieve approximate reconstruction [8].

## 4 Experimental results

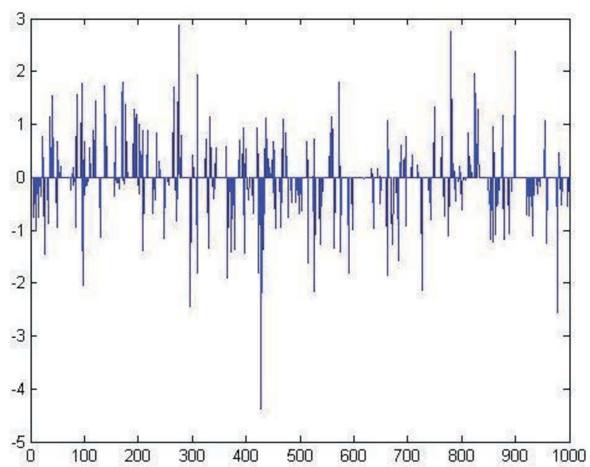
In this section, we present numerical experiments that explain the effectiveness of the PKLS-OMP for sparse and compressible signals.

Sensing matrice Φ has i.i.d. entries drawn from a standard normal distribution with normalized columns. The reconstructed signal to noise ratio (R-SNR) is used to measure performance of the reconstructed signal. R- SNR is defined as

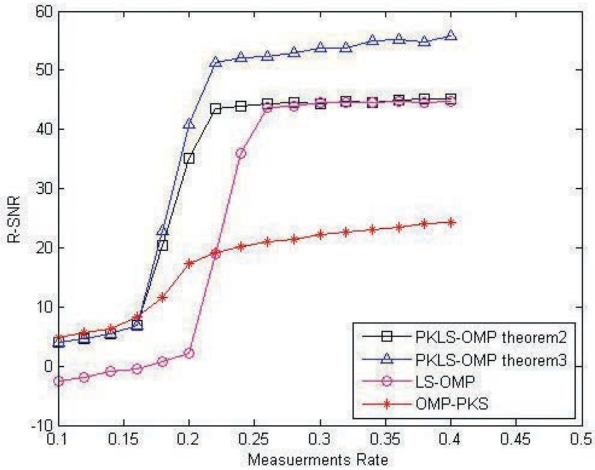
$$R - SNR = 10log10 \frac{\|x\|_2{}^2}{\|x-\hat{x}\|_2{}^2} \quad (16)$$

where $x$ and $\hat{x}$ denote the N-dimensional original and reconstructed signals, respectively.

In the first experiment, we generate a random sparse signal having length N=1024 with K=200 nonzero entries (Figure 2 (a)). The location of the nonzero entries are selected randomly using standard Gaussian distribution, the RIC value for LS-OMP=0.495, and for PKLS-OMP =0.002, the prior support information $T_0$=64. Figure 2 (b) compares the three proposed theorems and OMP-PKS for different number of measurements. Notice that inclusion of the prior information improves the reconstruction performance. That is the PKLS-OMP and the OMP-PKS perform better than the LS-OMP but it is obvious that the PKLS-OMP (Theorem 2 and 3) gives best result for all measurement rates.



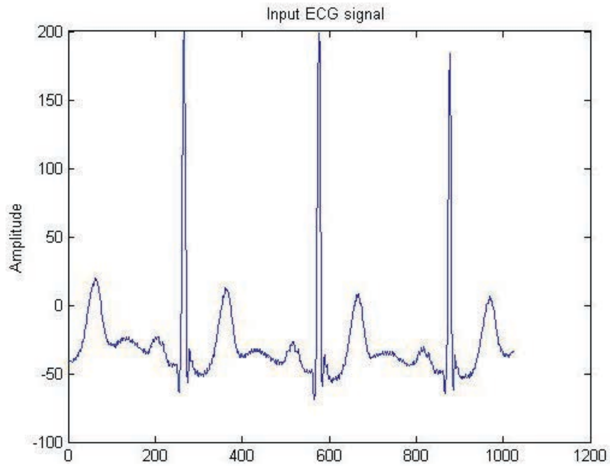
(a)



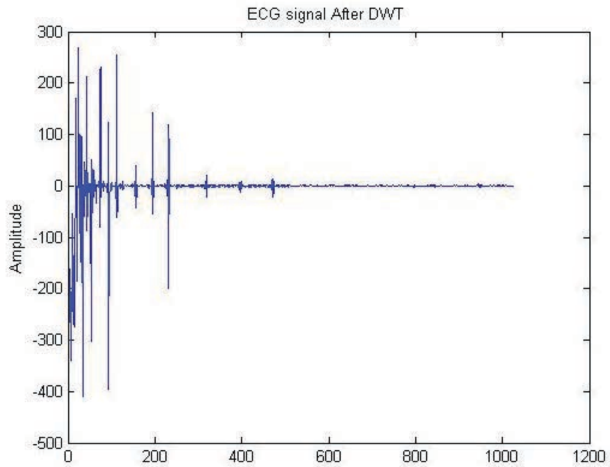

(b)

**Figure 2.** (a) A random sparse signal, (b) Reconstruction SNR versus measurement rate for the three proposed theorems and OMP-PKS.

The second experiment shows the effectiveness of PKLS-OMP to recover real compressible signals. We choose ECG signals as our simulation data used in [8]. A sparse signal approximation is determined by processing 1024 samples of ECG data with the four level discrete wavelet transform (DWT) filter type Symlets 8 and selecting the largest $Kmax = 128$ coefficients. The Least Support Parameter is set to L=60. Figure 3 (a)-(b) show an example of an input ECG signal and its decomposition using DWT. We use same ECG signal for all experiments below.




(a)




(b)

**Figure 3.** (a) input ECG signal, (b) decomposition of input EGC signal.

Figure 4 illustrates the effect of including prior information using Theorem 2 and 3. Partially known support is chosen as a low-pass approximation of the first sub band (LL) which corresponds to the first $T_0$=64 coefficients. The results demonstrate that the new method using both Theorem 2 and Theorem 3 gives perfect results to reconstruct the original signal as shown in Figure 4 (a) and (b).

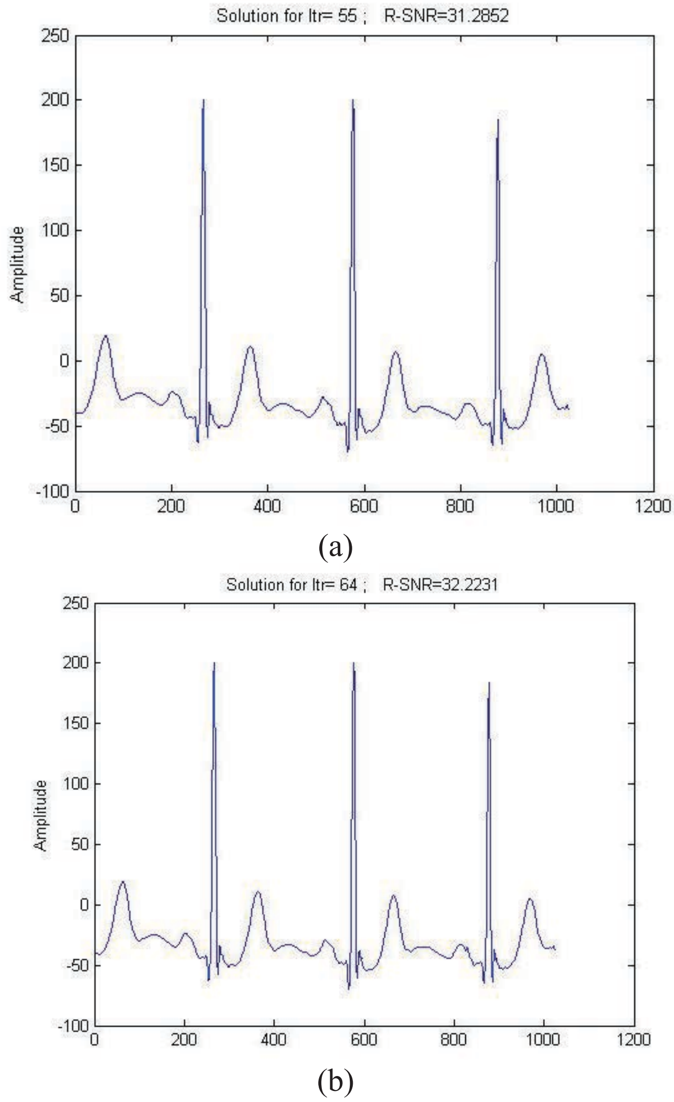



**Figure 4.** Reconstructed signal using the termination condition of (a) Theorem 2, (b) Theorem 3

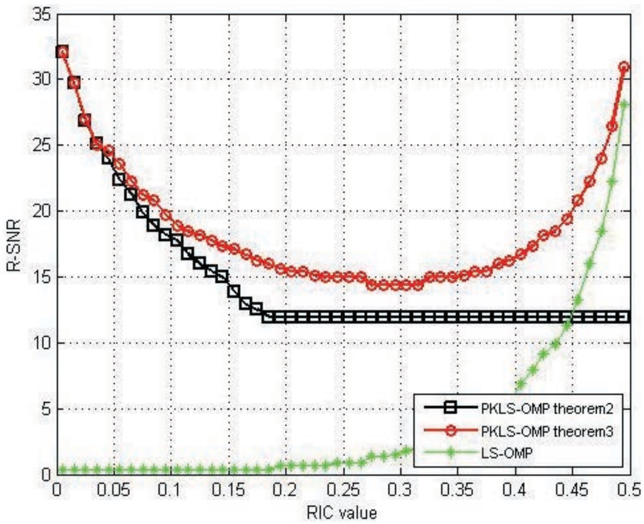



**Figure 5.** Best choice for RIC value for three theorem suggested above , using ECG signal with length=1024, T0=16.

Figure 5 shows the relation between R-SNR and RIC to choose the best RIC value, $\delta_{2L}$, for the suggested three theorems. As it can be seen the best value of $\delta_{2L,}$ for Theorem 2 and 3, is $0.005 \leq \delta_{2L} \leq 0.025$ and for Theorem 1, is $\delta_{2L} = 0.49$. Here, the partially known support is set to $T_0$=16.

The ECG reconstruction quality, using three proposed theorems, is evaluated as a function of the measurement rate $(M/N)$. Figure 6 shows the effect of introducing known support set ($T_0$=32) on measurement numbers. Inclusion of partially known support set in Theorem 2 and 3 improves the performance of the Theorem 1 (LS-OMP). Note that

PKLS-OMP using both Theorem 2 and 3 requires a fewer measurements than the LS-OMP and performs better R-SNR for all number of measurements.

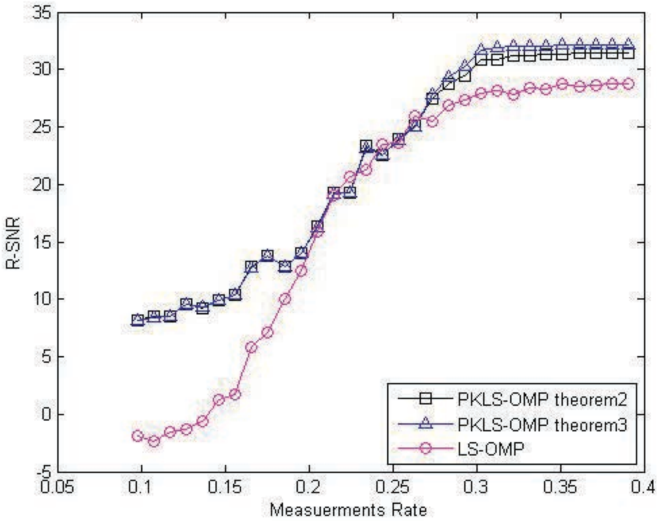



**Figure 6.** Reconstruction SNR versus measurement rate for the three proposed theorems.

In the next experiment, PKLS-OMP using the Theorem 2 and 3 is compared with the OMP-PKS as the size of the prior known support varies. The reconstruction SNR is used to evaluate the quality of the recovered signals. As shown in Figure 7, the PKLS-OMP using Theorem 2 and 3 gives best convergent results as they compared with the OMP-PKS for the measurement rate=0.4.

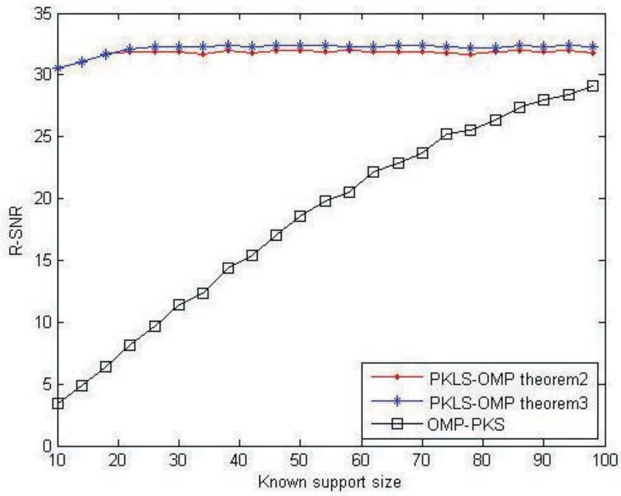



**Figure 7.** Reconstruction SNR as a function of known support size.

Changing the size of a known support set for Theorem 2 and Theorem 3 have no noteworthy effect on R-SNR but the OMP-PKS needs more known support to improve its performance.

Figure 8 shows the recovery time performance of the proposed PKLS-OMP and the OMP-PKS. As it is seen, PKLS-OMP using termination condition of both Theorem 2 and 3 achieves significantly less recovery time than that of the OMP-PKS. Moreover, as the measurement number increases our algorithm keeps its speed although OMP- PKS becomes slower.

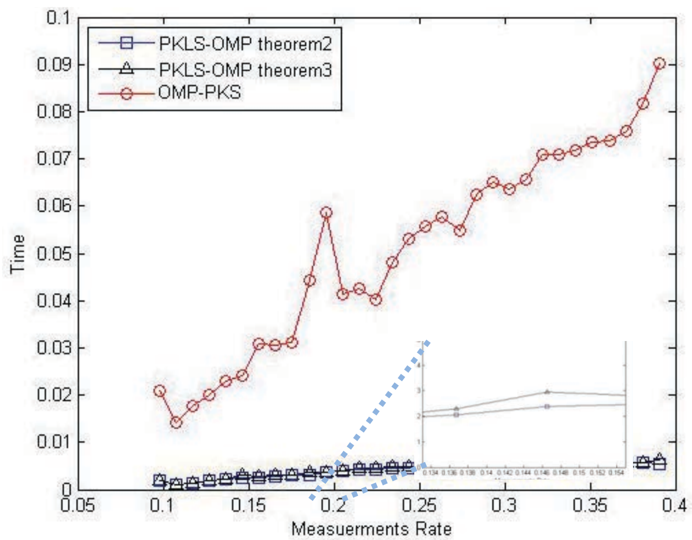



**Figure 8.** Recover time (in second) for the PKLS-OMP and OMP-PKS .

As a last experiment, the PKLS-OMP is compared with the OMP and the CoSaMP, as well as their partially known support versions (OMP-PKS, CoSaMP-PKS [7]). We also include Basis Pursuit with partially known support (BP-PKS) [7]. In all cases, partially known support is set to $T_0$=32. Note that in Figure 9, PKLS-OMP algorithm with Theorem 2 and 3 outperforms BP-PKS, CoSaMP-PKS and yields similar reconstruction with OMP-PKS for small number of measurement and performs better than the OMP-PKS when the number of measurements increases. Altough CoSaMP-PKS gives better results than CoSaMP, the algorithm needs much more measurements than the proposed algorithm to achieve accurate reconstructions.

Thus, the performance of the LS-OMP is improved and the number of measurements is reduced for exact reconstruction through the inclusion of prior information with Theorem 2 and 3. In addition, as it is seen in the previous example, PKLS-OMP is also more efficient than the OMP-PKS in terms of recovery time.

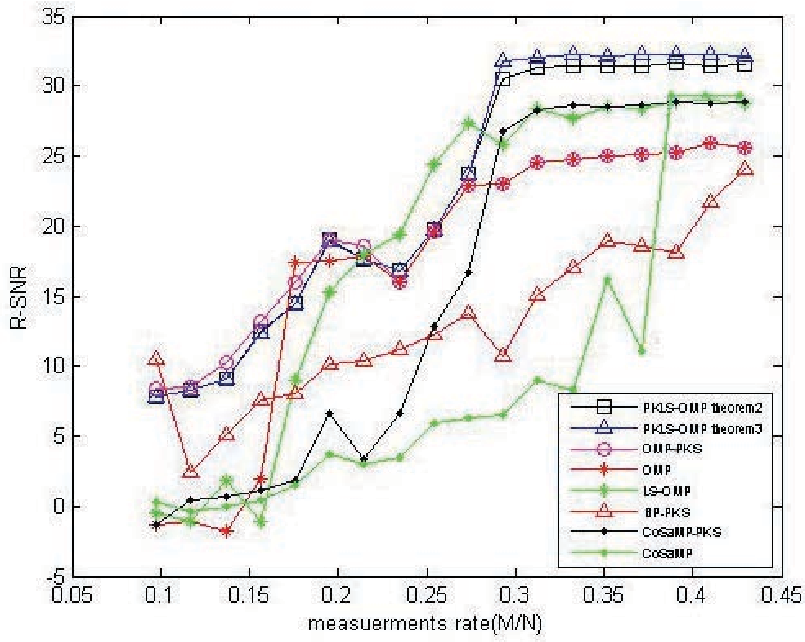



**Figure 9**. Comparison of PKLS-OMP and LS-OMP with OMP-PKS, CoSaMP-PKS, BP-PKS, OMP, and the COSAMP.

## 5 Conclusions

We introduce a new fast algorithm, named Partially Known Least Support Orthogonal Matching Pursuit (PKLS-OMP), for low-complexity recovery of sparse signals sampled by matrices satisfying the RIP with a constant parameter $0.005 \leq \delta_{2L} \leq 0.025$ .

The performance and effect of the prior information are studied through extensive simulations. Presented simulation results demonstrate that incorporation of partially known support improves their performance, thereby needing fewer samples to reconstruct sparse signals. In addition, the recovery performance of the algorithm outperforms that of the results of previously proposed CS-based methods.

As a feature work, we will extend our work to the reconstruction of sparse signals in the presence of noise and obtain the bound of the estimation error.

## Appendix A

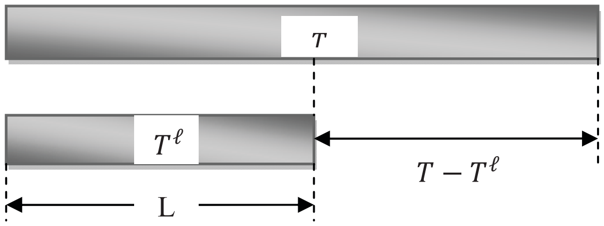
*PROOF OF THEOREMS*

***Theorem 1:***

If $x$ is a sparse signal and $x \in \mathbb{R}^N$ , $y$ is measurement vector $y = \Phi x$ , $\Phi$ is sampling matrix satisfies RIP condition, then $x$ can be recovered if :

$$\|y - y_r^\ell\|_2 \leq \frac{\delta_{2L}}{1-2\delta_{2L}} \|y_r^{\ell-1}\|_2$$

Figure 10. is use for Theorem 1 proof




In

**Figure 10.** Fictional diagrams for LS-OMP method

Figure 10, T represents the support of whole signal that contains the index of a coefficient that has maximum correlation between y and $\Phi$. L represents a part of support that will be used as a least support to re-construct the original signal. We have

$$\|y_r^\ell\|_2 = \|\text{resid}(y, \Phi_I)\|_2$$

$$\geq \|\text{resid}(\Phi_{T-T^\ell} x_{T-T^\ell}, \Phi_{T^\ell}\|_2 + \|\text{resid}(\Phi_{T^\ell} x_{T^\ell}, \Phi_{T^\ell})\|_2$$

Since $\text{resid}(\Phi_{T^\ell} x_{T^\ell}, \Phi_{T^\ell}) = 0$ , then

$$\|y_r^\ell\|_2 \geq \|\text{resid}(\Phi_{T-T^\ell} x_{T-T^\ell}, \Phi_{T^\ell}\|_2 \quad \text{(a-1)}$$

For $\ell - 1$, (a-1) become

$$\|y_r^{\ell-1}\|_2 \geq \|\text{resid}(\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}, \Phi_{T^{\ell-1}}\|_2 \quad \text{(a-2)}$$

$$\|\text{resid}\|_2 = \|y\| - \|y_p\| \quad \text{(a-3)}$$

From the definition of the residue,

$$\text{resid}(y, \Phi_I) = y - y_p \quad \text{(a-4)}$$

$$\|\text{resid}(\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}, \Phi_{T^{\ell-1}}\|_2 = \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2 - \|y_p\|_2 \quad \text{(a-6)}$$

We have $\|y_p\| \leq \frac{\delta_{|I|+|J|}}{1-\delta_{\max(|I|,|J|)}} \|y\|$ , since $|I|, |J|$ is the cardinality set then

$$\|y_p\|_2 = \frac{\delta_{2L}}{1-\delta_L} \|y\|_2$$

$$\therefore \|\text{resid}(\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}, \Phi_{T^{\ell-1}}\|_2$$

$$= \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2 - \frac{\delta_{2L}}{1-\delta_L} \|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\|_2$$

$$= (1 - \frac{\delta_{2L}}{1-\delta_L})\left\|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\right\|_2 \quad \text{(a-7)}$$

$$= \frac{1-\delta_L-\delta_{2L}}{1-\delta_L}\left\|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\right\|_2 \quad \text{(a-8)}$$

After the substitution of (a-8) into (a-2)we get,

$$\left\|y_r^{\ell-1}\right\|_2 \geq \frac{1-\delta_L-\delta_{2L}}{1-\delta_L}\left\|\Phi_{T-T^{\ell-1}} x_{T-T^{\ell-1}}\right\|_2 \quad \text{(a-9)}$$

Since we work in support set L and previous iteration is reduced in set L as $T - T^{\ell-1} = T^{\ell}$, then (a-9) become;

$$\left\|y_r^{\ell-1}\right\|_2 \geq \frac{1-\delta_L-\delta_{2L}}{1-\delta_L}\left\|\Phi_{T^\ell} x_{T^\ell}\right\|_2 \quad \text{(a-10)}$$

$$\|y\|_2 \geq \left\|\Phi_{T^\ell} x_{T^\ell}\right\|_2 \quad \text{(a-11)}$$

After the substitution of (a-11) into (a-10) we get

$$\left\|y_r^{\ell-1}\right\|_2 \geq \frac{1-\delta_L-\delta_{2L}}{1-\delta_L}\|y\|_2$$

or

$$\|y\|_2 \leq \frac{1-\delta_L}{1-\delta_L-\delta_{2L}}\left\|y_r^{\ell-1}\right\|_2 \quad \text{(a-12)}$$

$$y_p = y - y_r^{\ell} \quad \text{(a-13)}$$

since

$$\left\|y_p\right\|_2 = \frac{\delta_{2L}}{1-\delta_L}\|y\|_2 \quad \text{( a-14)}$$

After the substitution of (a-13)into (a-14) yields

$$\left\|y - y_r^{\ell}\right\|_2 = \frac{\delta_{2L}}{1-\delta_L}\|y\|_2 \quad \text{(a-15)}$$

And finally substitution of (a-13) into (a-15) yields

$$\left\|y - y_r^{\ell}\right\|_2 \leq \frac{\delta_{2L}}{1-\delta_{2L}} - \frac{1-\delta_{2L}}{1-2\delta_{2L}}\left\|y^{\ell-1}\right\|_2 \quad \text{(a-16)}$$

Using monotonicity of the isometry constant $\delta_L \leq \delta_{L+1}$

$$\left\|y - y_r^{\ell}\right\|_2 \leq \frac{\delta_{2L}}{1-2\delta_{2L}}\left\|y^{\ell-1}\right\|_2 \quad \text{(a-17)}$$

where $\delta_{2L} = 0.49$

## Appendix B

### ***Theorem 2:***

If $x$ is sparse signal and $x \in \mathbb{R}^N$ , $y$ is measurement vector $y = \Phi x$ , $\Phi$ is sampling matrix satisfies RIP condition, then $x$ can be recovered if

$$\|y_r\|_2 \geq \frac{\delta_{2l}}{(1-\delta_{2l})}\|y_0\|,$$

$$\text{for } 0.005 \leq \delta_{2L} \leq 0.025.$$

**Proof:**

Some important notation according to Figure 11

$T_0$ : The known part of support

$y_r$ : The residue of y into $\Phi_{T_0}$ where $y_r = \text{resid}(y, \Phi_{T_0})$
$x_r$ : The coefficient vector according to $y_r$, where $y_r = \Phi_{T-T_0} x_r$
$x_0$ : Unknown part of signal $x_0 = x_{T-T_0}$
$y_0$ : Measurement vector of unknown part $y_0 = \Phi_{T-T_0} x_0$
$y_{0,p}$ : The projection of $y_0$ in to $\Phi_{T_0}$ i.e $y_{0,p} = \text{proj}(y_0, \Phi_{T_0})$
$x_{0,p}$ : The projection coefficient vector according to $y_{0,p}$ i.e
$y_{0,p} = \Phi_{T_0} x_{0,p}$
$T_L$ : Least-Support part
$T_{L_0}$: Knowing part of $T_L$ where $T_{L_0} = T_0$
$\widehat{T}_0$ : Set of indices estimate in T after Iteration ended

$$y_r = y_0 - y_{0,p}$$
$$y_{0,p} = \Phi_{T_0} x_{0,p} \quad \text{then}$$
$$y_r = y_0 - \Phi_{\widehat{T}_0} x_{0,p} \qquad \text{(b-18)}$$

According to projection definition

$$x_{0,p} = \left(\Phi'_{\widehat{T}_0} \Phi_{\widehat{T}_0}\right)^{-1} \Phi'_{\widehat{T}_0} (\Phi_{T-T_0} x_0)$$

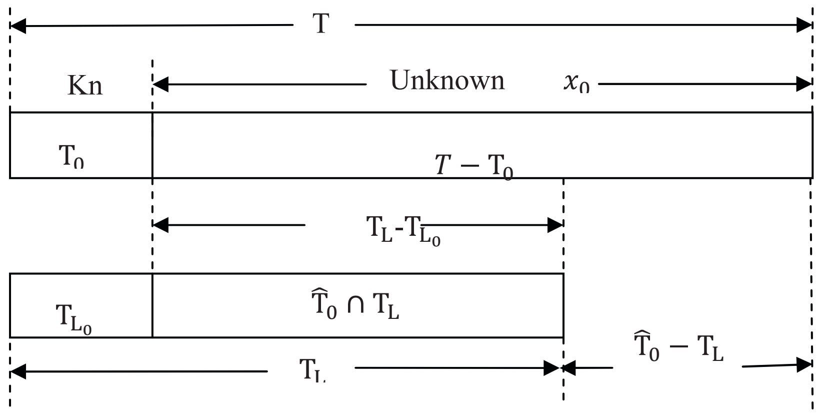



**Figure 11**. Illustration of support sets for Theorem 2 and Theorem 3.

$$\|x_{0,p}\|_2 = \left\|\left(\Phi'_{\widehat{T}_0} \Phi_{\widehat{T}_0}\right)^{-1} \Phi'_{\widehat{T}_0} (\Phi_{T-T_0} x_0)\right\|_2$$
$$\|x_{0,p}\|_2 \leq \frac{1}{1-\delta_K} . \delta_{2K} \|x_0\|_2$$
$$\|x_{0,p}\|_2 \leq \frac{\delta_{2K}}{1-\delta_{2K}} . \|x_0\|_2 \qquad \text{(b-19)}$$

We have

$$\|y_0\|_2 = \|\Phi_{T-T_0} x_0\|_2 \qquad \text{(b-20)}$$
$$\|\Phi'_{\widehat{T}_0} y_0\|_2 \leq \|\Phi'_{T_L - T_{L_0}} y_0\| + \|\Phi'_{\widehat{T}_0 - T_L} y_0\|$$

$$\leq \left\|\Phi'_{T_L-T_{L_0}} y_0\right\| + \left\|\Phi'_{\hat{T}_0-T_L}\Phi_{T-T_0}x_{T-T_0}\right\|$$

$$\leq \left\|\Phi'_{T_L-T_{L_0}} y_0\right\| + \delta_{2K}\left\|x_{T-T_0}\right\| \quad \text{(b-21)}$$

$$\left\|\Phi'_{T_L-T_{L_0}} y_0\right\| \leq \left\|\Phi'_{T_L-T_{L_0}}\Phi_{T_L-T_{L_0}} x_{T_L-T_{L_0}}\right\|_2 + \left\|\Phi'_{T_L-T_{L_0}}\Phi_{\hat{T}_0-T_L}x_{\hat{T}_0-T_L}\right\|_2$$

$$\left\|\Phi'_{T_L-T_{L_0}} y_0\right\| \leq (1+\delta_{2K})\left\|x_{T_L-T_{L_0}}\right\|_2 \quad \text{(b-22)}$$

Substitution (b-22) into (b-21) yields

$$\left\|\Phi'_{T_0} y_0\right\|_2 \leq (1+\delta_{2K})\left\|x_{T_L-T_{L_0}}\right\|_2 + \delta_{2K}\|x_0\| \quad \text{(b-23)}$$

To find the value of $\left\|\Phi'_{T_0} y_0\right\|_2$ we have

$$\left\|\Phi'_{T_0} y_0\right\|_2 = \left\|\Phi'_{T_0}\Phi_{T-T_0}x_0\right\|_2 \leq (1-\delta_{2K})\|x_0\|_2 \quad \text{(b-24)}$$

Substitution (b-24) into (b-23) gives

$$(1-\delta_{2K})\|x_0\|_2 \leq (1+\delta_{2K})\left\|x_{T_L-T_{L_0}}\right\|_2 + \delta_{2K}\|x_0\|$$

$$(1-2\delta_{2K})\|x_0\|_2 \leq (1+\delta_{2K})\left\|x_{T_L-T_{L_0}}\right\|_2$$

$$\left\|x_{T_L-T_{L_0}}\right\|_2 \geq \frac{(1-2\delta_{2K})}{(1+\delta_{2K})}\|x_0\|_2 \quad \text{(b-25)}$$

Substitution (b-25) into (b-22) gives

$$\left\|\Phi'_{T_L-T_{L_0}} y_0\right\| \geq (1+\delta_{2K})\frac{(1-2\delta_{2K})}{(1+\delta_{2K})}\|x_0\|_2$$

$$\|x_0\|_2 \leq \frac{1}{(1-2\delta_{2K})}\left\|\Phi'_{T_L-T_{L_0}} y_0\right\| \quad \text{(b-26)}$$

We have

$$y_r = \Phi_{T-T_0}x_0 - \Phi_{T_0}x_{0,p}$$

$$\left\|\Phi'_{T_L-T_0} y_r\right\|_2 \geq \left\|\Phi'_{T_L-T_0}\Phi_{T-T_0}x_0\right\|_2 - \left\|\Phi'_{T_L-T_0}\Phi_{T_0}x_{0,p}\right\|_2$$

Using (b-19) we obtain

$$\left\|\Phi'_{T_L-T_0} y_r\right\|_2 \geq \delta_{2K}\|x_0\|_2 - \frac{\delta_{2K}}{1-\delta_{2K}}.\delta_{2K}.\|x_0\|_2$$

$$\left\|\Phi'_{T_L-T_0} y_r\right\|_2 \geq \frac{\delta_{2K}(1-2\delta_{2K})}{1-\delta_{2K}}.\|x_0\|_2 \quad \text{(b-27)}$$

Substitution (b-26) into (b-27) yields

$$\left\|\Phi'_{T_L-T_0} y_r\right\|_2 \geq \frac{\delta_{2K}(1-2\delta_{2K})}{1-\delta_{2K}}.\frac{1}{(1-2\delta_{2K})}\left\|\Phi'_{T_L-T_{L_0}} y_0\right\|$$

thus

$$\|y_r\|_2 \geq \frac{\delta_{2K}}{(1-\delta_{2K})}.\|y_0\| \quad \text{(b-28)}$$

the cardinality length= L, finally we obtain

$$\|y_r\|_2 \geq \frac{\delta_{2L}}{(1-\delta_{2L})}.\|y_0\|$$

which completes the proof of Theorem 2.

## Appendix C

### *Theorem 3*

If $x$ is sparse signal and $x \in \mathbb{R}^N$ , $y$ is measurement vector $y = \Phi x$ , $\Phi$ is sampling matrix satisfies RIP condition, then $x$ can be recovered if :

$$\|y\|_2 \leq \frac{1-\delta_{2L}}{\delta_{2L}(1-2\delta_{2L})}\|y_r\|_2$$

for $0.005 \leq \delta_{2L} \leq 0.025$.

Proof:

We have

$$y_r = y_0 - y_{0,p}$$

$$y_{0,p} = \Phi_{\widehat{T}_0} x_{0,p} \quad \text{then}$$

$$y_r = y_0 - \Phi_{\widehat{T}_0} x_{0,p} \tag{c-29}$$

Substitution (b-19) into (c-29) we obtain

$$\|y_r\|_2 \geq \left\|\Phi_{T-T_0} x_0\right\|_2 - \left\|\Phi_{\widehat{T}_0} . \frac{\delta_{2K}}{1-\delta_{2K}} . x_0\right\|_2$$

$$\|y_r\|_2 \geq \left\|\Phi_{T-T_0} x_0\right\|_2 - \frac{\delta_{2K}}{1-\delta_{2K}} . \left\|\Phi_{\widehat{T}_0} x_0\right\|_2$$

$$\|y_r\|_2 \geq \sqrt{1-\delta_K}\|x_0\|_2 - \sqrt{1-\delta_K}\, \frac{\delta_{2K}}{1-\delta_{2K}}\|x_0\|_2$$

$$\|y_r\|_2 \geq \frac{1-2\delta_{2K}}{\sqrt{1-\delta_{2K}}}\|x_0\|_2 \tag{c-30}$$

$$y_0 = \Phi_{T-T_0} x_0$$

From Figure 2 we have

$$\left\|\Phi'_{T_L-T_{L_0}} y_0\right\|_2 = \left\|\Phi'_{\widehat{T}_0 \cap T_L} y_0\right\| \tag{c-31}$$

$$\leq \left\|\Phi'_{\widehat{T}_0 \cap T_L} \Phi_{\widehat{T}_0 \cap T_L} x_{\widehat{T}_0 \cap T_L}\right\|_2 + \left\|\Phi'_{\widehat{T}_0 \cap T_L} \Phi_{\widehat{T}_0 - T_L} x_{\widehat{T}_0 - T_L}\right\|$$

$$\leq (1-\delta_{2K})\left\|x_{\widehat{T}_0 \cap T_L}\right\| + \delta_{2K}\left\|x_{\widehat{T}_0 - T_L}\right\| \tag{c-32}$$

Since the set $\widehat{T}_0 - T_L$ is out of our working set as shown in Figure 2, then its value will be equal zero. Since $\widehat{T}_0 \cap T_L = T_L - T_0$ , then (c-32)becomes

$$\left\|\Phi'_{T_L-T_{L_0}} y_0\right\|_2 \leq (1-\delta_{2K})\left\|x_{T_L-T_0}\right\|_2 \tag{c-33}$$

$$\left\|\Phi'_{\widehat{T}_0} y_0\right\|_2 = \left\|\Phi'_{\widehat{T}_0 \cap T_L} y_0\right\| + \left\|\Phi'_{\widehat{T}_0 - T_L} y_0\right\|$$

$$= \left\|\Phi'_{\widehat{T}_0 \cap T_L} y_0\right\| + \left\|\Phi'_{\widehat{T}_0 - T_L} \Phi_{T-T_0} x_{T-T_0}\right\|$$

$$= \left\|\Phi'_{\widehat{T}_0 \cap T_L} y_0\right\| + \delta_{2K}\left\|x_{T-T_0}\right\| \tag{c-34}$$

$$\left\|\Phi'_{\widehat{T}_0 \cap T_L} y_0\right\| \leq \left\|\Phi'_{\widehat{T}_0 \cap T_L} \Phi_{\widehat{T}_0 \cap T_L} x_{\widehat{T}_0 \cap T_L}\right\|_2 + \left\|\Phi'_{\widehat{T}_0 \cap T_L} \Phi_{\widehat{T}_0 - T_L} x_{\widehat{T}_0 - T_L}\right\|_2 \tag{c-35}$$

Substitution (c-35) into (c-34) and removing the set that is out of working area we get

$$\left\|\Phi'_{T_0} y_0\right\|_2 \leq (1-\delta_{2K})\left\|x_{\widehat{T}_0 \cap T_L}\right\|_2 + \delta_{2K}\left\|x_{T-T_0}\right\| \tag{c-36}$$

To find the value of $\left\|\Phi'_{T_0} y_0\right\|_2$ we have

$$\left\|\Phi'_{T_0} y_0\right\|_2 = \left\|\Phi'_{T_0}\Phi_{T-T_0} x_0\right\|_2 \geq (1+\delta_{2K})\|x_0\|_2 \qquad \text{(c-37)}$$

Substitution (c-37)into (c-36) yields

$$(1+\delta_{2K})\|x_0\|_2 \leq (1-\delta_{2K})\left\|x_{\hat{T}_0\cap T_L}\right\|_2 + \delta_{2K}\|x_0\|$$

$$\|x_0\|_2 \leq (1-\delta_{2K})\left\|x_{\hat{T}_0\cap T_L}\right\|_2 \qquad \text{(c-38)}$$

$$\left\|x_{\hat{T}_0\cap T_L}\right\|_2 \geq \frac{1}{(1-\delta_{2K})}\|x_0\|_2 \qquad \text{(c-39)}$$

Substitution (c-39)into (c-33) yields

$$\left\|\Phi'_{T_L-T_0} y_0\right\| \leq (1-\delta_{2K})\frac{1}{(1-\delta_{2K})}\|x_0\|_2 \qquad \text{(c-40)}$$

We have

$$\|x_0\|_2 \geq \frac{1}{\sqrt{(1-\delta_{2K})}}\|y_0\| \qquad \text{(c-41)}$$

we have

$$y_r = \Phi_{T-T_0} x_0 - \Phi_{T_0} x_{0,p}$$

$$\left\|\Phi'_{T_L-T_0} y_r\right\|_2 \leq \left\|\Phi'_{T_L-T_0}\Phi_{T-T_0} x_0\right\|_2 - \left\|\Phi'_{T_L-T_0}\Phi_{T_0} x_{0,p}\quad\right\|_2$$

Using Lemma 2, we get

$$\left\|\Phi'_{T_L-T_0} y_r\right\|_2 \leq \delta_{2K}\|x_0\|_2 - \frac{\delta_{2K}}{1-\delta_{2K}}.\delta_{2K}.\|x_0\|_2$$

$$\left\|\Phi'_{T_L-T_0} y_r\right\|_2 \leq \frac{\delta_{2K}(1-2\delta_{2K})}{1-\delta_{2K}}.\|x_0\|_2 \qquad \text{(c-42)}$$

$$\|x_0\|_2 \geq \frac{1-\delta_{2K}}{\delta_{2K}(1-2\delta_{2K})}\left\|\Phi'_{T_L-T_0} y_r\right\|_2 \qquad \text{(c-43)}$$

Substitution of (c-43) into (c-40) yields

$$\left\|\Phi'_{T_L-T_0} y_0\right\|_2 \leq \frac{1-\delta_{2K}}{\delta_{2K}(1-2\delta_{2K})}\left\|\Phi'_{T_L-T_0} y_r\right\|_2$$

thus

$$\|y\|_2 \leq \frac{1-\delta_{2K}}{\delta_{2K}(1-2\delta_{2K})}\|y_r\|_2 \qquad \text{(c-44)}$$

Setting the cardinality length= L, we get

$$\|y\|_2 \leq \frac{1-\delta_{2L}}{\delta_{2L}(1-2\delta_{2L})}\|y_r\|_2$$

which completes the proof of Theorem 3.

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