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Interaction between an edge dislocation and a rigid hypotrochoidal inhomogeneity

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We study the plane elasticity problem associated with a rigid hypotrochoidal inhomogeneity embedded in an infinite isotropic elastic matrix subjected to an edge dislocation located at an arbitrary position. A closed-form solution to the problem is derived primarily with the aid of conformal mapping and analytic continuation. All of the unknown complex constants appearing in the pair of analytic functions characterizing the elastic field in the matrix are determined in an analytical manner. In addition, a simple method distinct from that by Santare and Keer (1986) is proposed to determine the rigid body rotation of the rigid inhomogeneity.

Key words: rigid hypotrochoidal inhomogeneity, edge dislocation, rigid body rotation, closed-form solution, conformal mapping, analytic continuation.

1. Introduction

The problem of an edge dislocation interacting with a circular or elliptical elastic inhomogeneity has been discussed in detail by several authors [1–5]. Closed-form solutions exist for the case of a circular elastic inhomogeneity, however only series-form solutions are available for the case of an elliptical elastic inhomogeneity. The problem of an edge dislocation near a rigid elliptical inhomogeneity was solved in closed-form by Santare and Keer [6] using Muskhelishvili's complex variable approach [7]. A closed-form solution is preferable when studying the interaction between a crack and an inhomogeneity via the use of a continuous distribution of dislocations to simulate the crack. The problem becomes almost analytically intractable when an edge dislocation is located near an elastic inhomogeneity of any shape, in which case only approximate solutions [8–10] or computational schemes [11] are available. In fact, even the case when an edge dislocation interacts with a rigid inhomogeneity of arbitrary shape presents considerable challenges. One apparent difficulty lies in the analytical determination of the rigid body rotation of the arbitrarily shaped inhomogeneity induced by the edge dislocation. Another difficulty lies in the complexity of the expressions for the pair of analytic functions characterizing the elastic field in the matrix following the introduction of the conformal mapping function which maps the exterior of the non-elliptical inhomogeneity onto the exterior of the unit circle in the image plane.

In this paper, a closed-form solution is derived for the interaction problem of an edge dislocation near a perfectly bonded rigid hypotrochoidal inhomogeneity using conformal mapping [7] and analytic continuation [12–14]. Moreover, the rigid body rotation of the rigid inhomogeneity and all of the unknown complex constants appearing in the pair of analytic functions characterizing the elastic field in the matrix are obtained in an analytical manner. Furthermore, there is no need to solve numerically the resulting set of linear algebraic equations. Here, a simple and effective method distinct from that by Santare and Keer [6] is proposed to determine the rigid body rotation of the rigid hypotrochoidal inhomogeneity. The solution obtained here can be conveniently employed as a Green's function to study the problem of a finite crack interacting with a rigid hypotrochoidal inhomogeneity. Previous studies on the crack-inhomogeneity interaction problem can be found in [15–17].

2. Complex variable formulation

We begin by establishing a Cartesian coordinate system $\{x_i\}$ $(i = 1, 2, 3)$. For the in-plane deformations of an isotropic elastic material, the three in-plane stresses $(\sigma_{11}, \sigma_{22}, \sigma_{12})$, two in-plane displacements (u_1, u_2) and two stress functions (φ_1, φ_2) are given in terms of two analytic functions $\phi(z)$ and $\psi(z)$ of the complex variable $z = x_1 + \mathrm{i}x_2$ as [7]:

$$
\begin{aligned}
&\sigma_{11} + \sigma_{22} = 2[\phi'(z) + \overline{\phi'(z)}],\\
&\sigma_{22} - \sigma_{11} + 2\mathrm{i}\sigma_{12} = 2[\bar{z}\phi''(z) + \psi'(z)],
\end{aligned}
\tag{2.1}
$$

and

$$
\begin{aligned}
&2\mu(u_1 + \mathrm{i}u_2) = \kappa\phi(z) - z\overline{\phi'(z)} - \overline{\psi(z)},\\
&\varphi_1 + \mathrm{i}\varphi_2 = \mathrm{i}[\phi(z) + z\overline{\phi'(z)} + \overline{\psi(z)}],
\end{aligned}
\tag{2.2}
$$

where $\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress, μ and ν $(0 \le \nu \le 1/2)$ are the shear modulus and Poisson's ratio, respectively. In addition, the stresses are related to the two stress functions through [18]:

$$
\begin{aligned}
&\sigma_{11} = -\varphi_{1,2}, \qquad \sigma_{12} = \varphi_{1,1},\\
&\sigma_{21} = -\varphi_{2,2}, \qquad \sigma_{22} = \varphi_{2,1}.
\end{aligned}
\tag{2.3}
$$

3. Closed-form solution

As shown in Fig. 1, we consider a rigid hypotrochoidal inhomogeneity embedded in an infinite isotropic elastic matrix subjected to an edge dislocation with the Burgers vector (b_1, b_2) located at an arbitrary position $z = z_0$. Let S_1 and S_2 denote the rigid inhomogeneity and the elastic matrix, respectively, which are perfectly bonded across the hypotrochoidal interface L. The matrix is not subjected to any remote loading and the rigid body rotation at infinity is zero.

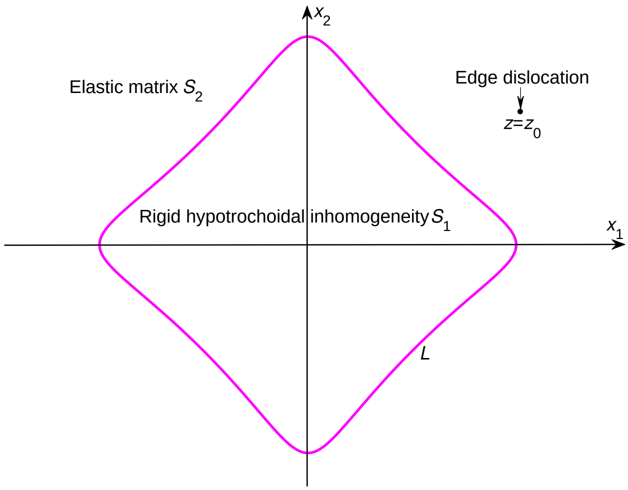

Fig. 1. A rigid hypotrochoidal inhomogeneity embedded in an infinite isotropic elastic matrix subjected to an edge dislocation located at an arbitrary position.

We first introduce the following conformal mapping function [7]:

$$z = \omega(\xi) = R\left(\xi + \frac{m}{\xi^N}\right), \quad \xi = \omega^{-1}(z), \quad R > 0, \quad 0 \le m \le \frac{1}{N}, \quad |\xi| \ge 1, \tag{3.1}$$

where N is an integer greater than 1.

As shown in Fig. 2, using the mapping function in Eq. (3.1), the exterior of the hypotrochoidal inhomogeneity is mapped onto $|\xi| \ge 1$, the hypotrochoidal interface L is mapped onto $|\xi| = 1$, and the position of the edge dislocation at $z = z_0$ is mapped onto the point $\xi = \xi_0$ with $\xi_0 = \omega^{-1}(z_0)$. When $mn(n+1) = 2$, the hypotrochoidal inhomogeneity resembles a regular polygonal inhomogeneity [12, 14].

The continuity of displacements across the perfect hypotrochoidal interface L can be expressed in terms of $\phi(\xi) = \phi(\omega(\xi))$ and $\psi(\xi) = \psi(\omega(\xi))$ for $|\xi| \ge 1$ as

$$\kappa\phi(\xi) - \frac{\omega(\xi)\overline{\phi'(\xi)}}{\overline{\omega'(\xi)}} - \overline{\psi(\xi)} = 2\mathrm{i}\mu\varpi_{21}\omega(\xi), \qquad |\xi| = 1, \tag{3.2}$$

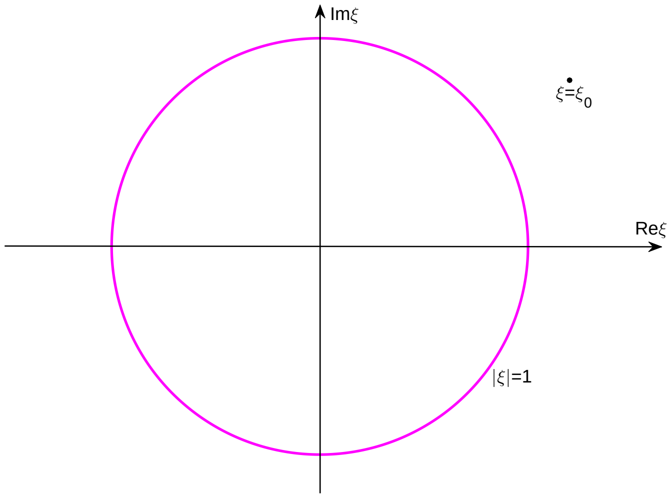

FIG. 2. The image ξ-plane.

where $\varpi_{21} = \frac{1}{2}(u_{2,1} - u_{1,2})$ is the unknown rigid body rotation of the hypotrochoidal inhomogeneity to be determined.

For the convenience of the ensuing analysis, we introduce the following analytic continuation

$$\phi(\xi) = \frac{\omega(\xi)\bar{\phi}'\left(\frac{1}{\xi}\right)}{\kappa\bar{\omega}'\left(\frac{1}{\xi}\right)} + \frac{1}{\kappa}\bar{\psi}\left(\frac{1}{\xi}\right), \qquad |\xi| \le 1. \tag{3.3}$$

In view of the analytic continuation in Eq. (3.3), the principal part of $\phi(\xi)$ for $|\xi| \le 1$, denoted as $\phi_{si}(\xi)$, and the principal part of $\phi(\xi)$ for $|\xi| \ge 1$, denoted as $\phi_{so}(\xi)$, are given, respectively, by:

$$\begin{aligned} \phi_{si}(\xi) &= \frac{A}{\kappa}\ln\frac{\xi - \bar{\xi}_0^{-1}}{\xi} - \frac{\bar{A}[\omega(\bar{\xi}_0^{-1}) - \omega(\xi_0)]}{\kappa\bar{\xi}_0^2\overline{\omega'(\xi_0)}(\xi - \bar{\xi}_0^{-1})} + \sum_{n=1}^{N-1}\lambda_n\xi^{-n}, \qquad |\xi| \le 1, \\ \phi_{so}(\xi) &= A\ln(\xi - \xi_0), \qquad |\xi| \ge 1, \end{aligned} \tag{3.4}$$

where λ_n $(n = 1, \ldots, N-1)$ are $N-1$ unknown complex constants to be determined and are attributed solely to the non-elliptical shape of the inhomogeneity, and

$$A = \frac{\mu(b_2 - \mathrm{i}b_1)}{\pi(\kappa + 1)}. \tag{3.5}$$

In view of the analytic continuation in Eq. (3.3), the interface condition in Eq. (3.2) can be written in the equivalent form:

$$\phi^-(\xi) - \phi^+(\xi) = \frac{2\mathrm{i}\mu\varpi_{21}}{\kappa}\omega(\xi), \qquad |\xi| = 1, \tag{3.6}$$

where the superscripts "+" and "−" indicate the values when approaching the unit circle from inside and outside, respectively. Considering the principal parts of $\phi(\xi)$ in Eq. (3.4), Eq. (3.6) can be rewritten in the form:

$$\begin{aligned}(3.7)\qquad \phi^+(\xi) - A\ln(\xi-\xi_0) - \frac{A}{\kappa}\ln\frac{\xi-\bar{\xi}_0^{-1}}{\xi} + \frac{\bar{A}[\omega(\bar{\xi}_0^{-1})-\omega(\xi_0)]}{\kappa\bar{\xi}_0^2\overline{\omega'(\xi_0)}(\xi-\bar{\xi}_0^{-1})} \\ + \frac{2\mathrm{i}R\mu\varpi_{21}}{\kappa}\xi - \sum_{n=1}^{N-1}\lambda_n\xi^{-n} \\ = \phi^-(\xi) - A\ln(\xi-\xi_0) - \frac{A}{\kappa}\ln\frac{\xi-\bar{\xi}_0^{-1}}{\xi} + \frac{\bar{A}[\omega(\bar{\xi}_0^{-1})-\omega(\xi_0)]}{\kappa\bar{\xi}_0^2\overline{\omega'(\xi_0)}(\xi-\bar{\xi}_0^{-1})} \\ - \frac{2\mathrm{i}Rm\mu\varpi_{21}}{\kappa}\frac{1}{\xi^N} - \sum_{n=1}^{N-1}\lambda_n\xi^{-n}, \quad |\xi|=1.\end{aligned}$$

The left-hand side of Eq. (3.7) is analytic and single valued everywhere within the unit circle, and the right-hand side of Eq. (3.7) is analytic and single valued everywhere outside the unit circle including the point at infinity. Using Liouville's theorem, we arrive at:

$$(3.8)\qquad \begin{aligned}\phi(\xi) &= A\ln(\xi-\xi_0) + \frac{A}{\kappa}\ln\frac{\xi-\bar{\xi}_0^{-1}}{\xi} - \frac{\bar{A}[\omega(\bar{\xi}_0^{-1})-\omega(\xi_0)]}{\kappa\bar{\xi}_0^2\overline{\omega'(\xi_0)}(\xi-\bar{\xi}_0^{-1})} \\ &\quad - \frac{2\mathrm{i}R\mu\varpi_{21}}{\kappa}\xi + \sum_{n=1}^{N-1}\lambda_n\xi^{-n}, \quad |\xi|\le 1, \\ \phi(\xi) &= A\ln(\xi-\xi_0) + \frac{A}{\kappa}\ln\frac{\xi-\bar{\xi}_0^{-1}}{\xi} - \frac{\bar{A}[\omega(\bar{\xi}_0^{-1})-\omega(\xi_0)]}{\kappa\bar{\xi}_0^2\overline{\omega'(\xi_0)}(\xi-\bar{\xi}_0^{-1})} \\ &\quad + \frac{2\mathrm{i}Rm\mu\varpi_{21}}{\kappa}\frac{1}{\xi^N} + \sum_{n=1}^{N-1}\lambda_n\xi^{-n}, \quad |\xi|\ge 1.\end{aligned}$$

The following remote asymptotic behavior of $\phi(\xi)$ can then be extracted from Eq. $(3.8)_2$:

$$\begin{aligned}(3.9)\qquad \phi(\xi) \cong \\ A\ln\xi + \sum_{n=1}^{N-1}\left[\lambda_n - \frac{A(\bar{\xi}_0^{-n}+\kappa\xi_0^n)}{\kappa n} - \frac{\bar{A}\bar{\xi}_0^{-(n+1)}\left[\omega(\bar{\xi}_0^{-1})-\omega(\xi_0)\right]}{\kappa\overline{\omega'(\xi_0)}}\right]\xi^{-n} \\ + O\left(\frac{1}{\xi^N}\right), \quad |\xi|\to\infty.\end{aligned}$$

By substituting the remote asymptotic behavior of $\phi(\xi)$ in Eq. (3.9) into Eq. (3.3) and making use of the singular behavior of $\phi(\xi)$ for $|\xi|\le 1$ in Eq. $(3.4)_1$, we arrive at the following set of $N-1$ linear algebraic equations:

$$
\begin{aligned}
&\kappa\lambda_n + m(N-n-1)\bar{\lambda}_{N-n-1} \\
&\quad = m(N-n-1)\left[\frac{\bar{A}(\xi_0^{-(N-n-1)} + \kappa\bar{\xi}_0^{N-n-1})}{\kappa(N-n-1)} + \frac{A\xi_0^{-(N-n)}[\overline{\omega(\bar{\xi}_0^{-1})} - \overline{\omega(\xi_0)}]}{\kappa\omega'(\xi_0)}\right], \\
&\qquad\qquad n = 1,\ldots,N-2, \\
&\kappa\lambda_{N-1} = m\bar{A}.
\end{aligned}
\tag{3.10}
$$

The set of linear algebraic equations in Eq. (3.10) does not contain the unknown ϖ_{21}. The $N-1$ complex constants λ_n $(n = 1,\ldots,N-1)$ can be uniquely determined by analytically solving the linear algebraic equations in Eq. (3.10) as:

$$
\begin{aligned}
&\lambda_n = \\
&\frac{m(N-n-1)\left(\begin{aligned}&-\frac{Am(\bar{\xi}_0^{-n}+\kappa\xi_0^{n})}{\kappa} + \frac{A[m\bar{\xi}_0^{-N}(|\xi_0|^{2N}-1) - \xi_0^{-1}(|\xi_0|^2-1)]}{\xi_0^{N-n} - mN\xi_0^{-(n+1)}} \\ &+\frac{\bar{A}(\xi_0^{-(N-n-1)}+\kappa\bar{\xi}_0^{N-n-1})}{N-n-1} - \frac{\bar{A}mn[m\xi_0^{-N}(|\xi_0|^{2N}-1) - \bar{\xi}_0^{-1}(|\xi_0|^2-1)]}{\kappa(\bar{\xi}_0^{n+1} - mN\bar{\xi}_0^{n-N})}\end{aligned}\right)}{\kappa^2 - m^2 n(N-n-1)}, \\
&\qquad\qquad n = 1,\ldots,N-2, \\
&\lambda_{N-1} = \frac{m\bar{A}}{\kappa}.
\end{aligned}
\tag{3.11}
$$

The remaining original analytic function $\psi(\xi)$ can be obtained from the analytic continuation in Eq. (3.3) as:

$$
\psi(\xi) = \kappa\bar{\phi}\left(\frac{1}{\xi}\right) - \frac{\bar{\omega}\left(\frac{1}{\xi}\right)\phi'(\xi)}{\omega'(\xi)}, \qquad |\xi| \geq 1. \tag{3.12}
$$

At this stage, the original pair of analytic functions $\phi(\xi)$ and $\psi(\xi)$ for $|\xi| \geq 1$ still contains the single unknown ϖ_{21}. The elastic field in the matrix is known once ϖ_{21} is determined. The remote asymptotic behavior of $\psi(\xi)$ is:

$$
\psi(\xi) \cong \bar{A}\ln\xi + \frac{c}{\xi} + O\left(\frac{1}{\xi^2}\right), \qquad |\xi| \to \infty, \tag{3.13}
$$

where c is real-valued (i.e., $\mathrm{Im}\{c\} = 0$) in order to satisfy the condition that the resultant moment about the origin on the disk $|z| = \infty$ is zero.

Substituting the following asymptotic behaviors of $\phi(\xi)$ extracted from Eq. (3.8) into Eq. (3.12):

$$
\begin{aligned}
\phi(\xi) \cong{}& A\ln\xi \\
&+ \sum_{n=1}^{N-1}\left[\lambda_n - \frac{A(\bar{\xi}_0^{-n}+\kappa\xi_0^{n})}{\kappa n} - \frac{\bar{A}\bar{\xi}_0^{-(n+1)}[\omega(\bar{\xi}_0^{-1}) - \omega(\xi_0)]}{\kappa\overline{\omega'(\xi_0)}}\right]\frac{1}{\xi^n} \\
&+ \left[\frac{2\mathrm{i}Rm\mu\varpi_{21}}{\kappa} - \frac{A(\bar{\xi}_0^{-N}+\kappa\xi_0^{N})}{\kappa N} - \frac{\bar{A}\bar{\xi}_0^{-(N+1)}[\omega(\bar{\xi}_0^{-1}) - \omega(\xi_0)]}{\kappa\overline{\omega'(\xi_0)}}\right]\frac{1}{\xi^N} \\
&+ O\left(\frac{1}{\xi^{N+1}}\right), \qquad \xi \to \infty;
\end{aligned}
\tag{3.14a}
$$

$$
\begin{aligned}
\text{(3.14b)} \quad \phi(\xi) \cong & \sum_{n=1}^{N-1} \lambda_n \xi^{-n} - \frac{A}{\kappa} \ln \xi \\
& - \left[A\xi_0^{-1} + \frac{A\bar{\xi}_0}{\kappa} - \frac{\bar{A}\left[\omega(\bar{\xi}_0^{-1}) - \omega(\xi_0)\right]}{\kappa\overline{\omega'(\xi_0)}} + \frac{2\mathrm{i}R\mu\varpi_{21}}{\kappa} \right] \xi \\
& + O(\xi^2), \qquad \xi \to 0,
\end{aligned}
$$

and imposing the condition that $\mathrm{Im}\{c\} = 0$, we finally obtain

$$
\text{(3.15)} \quad \varpi_{21} = \frac{\mathrm{Im}\left\{ (b_2 - \mathrm{i}b_1) \left[\begin{array}{l} m\bar{\xi}_0^{-N} + \kappa(m\xi_0^N - \bar{\xi}_0) - \kappa^2 \xi_0^{-1} \\ -\frac{(\kappa\xi_0^{N+1} + mN)[m\bar{\xi}_0^{-N}(|\xi_0|^{2N} - 1) - \xi_0^{-1}(|\xi_0|^2 - 1)]}{\xi_0^{N+1} - mN} \end{array} \right] \right\}}{2\pi R(\kappa + 1)(\kappa + m^2 N)},
$$

which is independent of the shear modulus of the matrix and λ_n ($n = 1, \ldots, N-1$). As a check, when $N = 1$ for a rigid elliptical inhomogeneity, Eq. (3.15) simply recovers the result by SANTARE and KEER [6]. Here we have adopted a different method from that used by SANTARE and KEER [6] to determine the rigid body rotation of the rigid hypotrochoidal inhomogeneity.

When the edge dislocation lies on the x_1-axis with $\xi_0 = \bar{\xi}_0$, Eq. (3.15) reduces to

$$
\text{(3.16)} \qquad \frac{\varpi_{21} R}{b_1} = \frac{\xi_0^{N+1}(\kappa + m^2 N) - \xi_0^2 m(N+1) - mN(\kappa - 1)}{2\pi\xi_0(\xi_0^{N+1} - mN)(\kappa + m^2 N)},
$$

which is unaffected by b_2 and which is illustrated in Figs. 3 and 4 for different values of ξ_0, N and κ. It is seen from Figs. 3 and 4 that: (i) ϖ_{21} and b_1 always have the same sign; (ii) the magnitude of ϖ_{21} decreases as ξ_0 increases (i.e., the edge dislocation is further away from the interface); (iii) the magnitude of ϖ_{21} decreases as the Poisson ratio of the matrix increases (i.e., κ decreases) for a finite value of N; (iv) the magnitude of ϖ_{21} increases as N increases; (v) as $N \to \infty$, ϖ_{21} becomes

$$
\text{(3.17)} \qquad \frac{\varpi_{21} R}{b_1} = \frac{1}{2\pi\xi_0},
$$

which is independent of the elastic property of the matrix and which is simply the result for a rigid circular inhomogeneity with $m = 0$.

When the edge dislocation lies on the hypotrochoidal interface with $\xi_0 = e^{\mathrm{i}\gamma}$, Eq. (3.15) becomes

$$
\text{(3.18)} \qquad \frac{\varpi_{21} R}{b_1} = \frac{\kappa\cos\gamma - m\cos(N\gamma) + \rho[\kappa\sin\gamma + m\sin(N\gamma)]}{2\pi(\kappa + m^2 N)},
$$

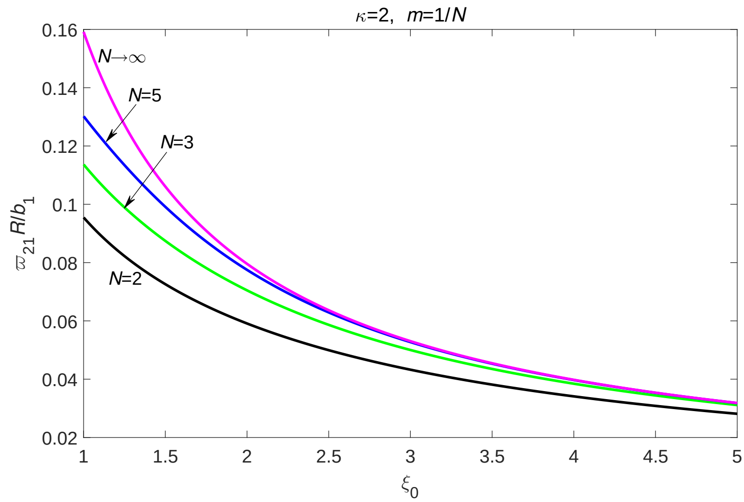

FIG. 3. Variations of the rigid body rotation for different values of $\xi_0 = \bar{\xi}_0 \geq 1$ and N with $\kappa = 2$, $m = 1/N$ when the edge dislocation lies on the positive x_1-axis.

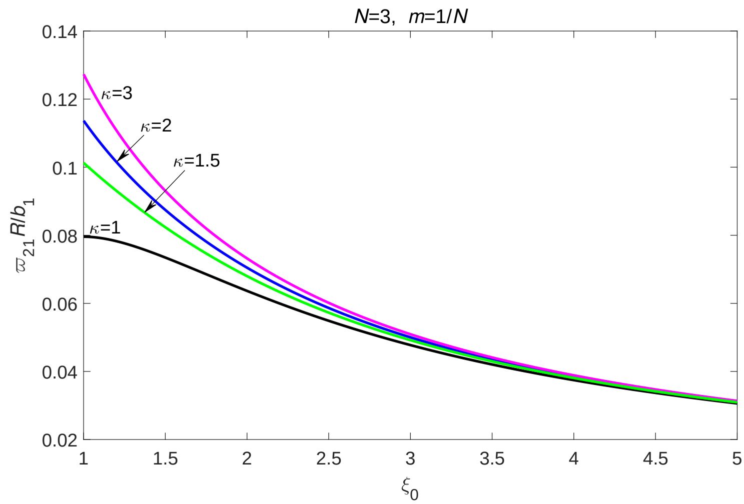

FIG. 4. Variations of the rigid body rotation for different values of $\xi_0 = \bar{\xi}_0 \geq 1$ and κ with $N = 3$, $m = 1/N$ when the edge dislocation lies on the positive x_1-axis.

where

$$\rho = \frac{b_2}{b_1}. \tag{3.19}$$

We illustrate in Fig. 5 the variations of ϖ_{21} induced by an edge dislocation lying on the hypotrochoidal interface. It is seen from Fig. 5 and Eq. (3.18) that:

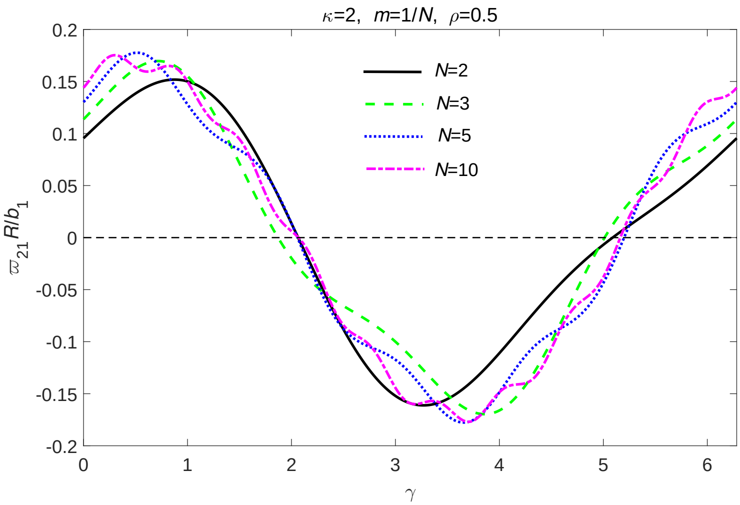

FIG. 5. Variations of the rigid body rotation for different values of γ and N with $\kappa = 2$, $m = 1/N$, $\rho = 0.5$ when the edge dislocation lies on the hypotrochoidal interface L.

(i) $\varpi_{21} = 0$ when the edge dislocation is located at two particular positions of the hypotrochoidal interface; (ii) the curve becomes more wavy when N increases; (iii) as $N \to \infty$, ϖ_{21} becomes

$$\varpi_{21} = \frac{b_1 \cos\gamma + b_2 \sin\gamma}{2\pi R}, \tag{3.20}$$

which is independent of the elastic property of the matrix and which is simply the result for a rigid circular inhomogeneity with $m = 0$.

We have now completely determined the pair of analytic functions $\phi(\xi)$ and $\psi(\xi)$ for $|\xi| \geq 1$ characterizing the elastic field of stresses and strains in the matrix. The elastic field of stresses, strains and rigid body rotation in the matrix can be determined by substituting the obtained $\phi(\xi)$ and $\psi(\xi)$ for $|\xi| \geq 1$ into Eqs. (2.1) and (2.2). For example, the mean stress $\sigma_{11} + \sigma_{22}$ and the rigid body rotation $\varpi_{21}^{(\mathrm{m})} = \frac{1}{2}(u_{2,1} - u_{1,2})$ within the matrix induced by the edge dislocation are concisely and explicitly given by:

$$\begin{aligned} &\frac{1}{4}(\sigma_{11}+\sigma_{22}) + \mathrm{i}\frac{2\mu\varpi_{21}^{(\mathrm{m})}}{\kappa+1} \\ &= \frac{\frac{A\kappa}{\xi-\xi_0} + \frac{A}{\xi(\bar{\xi}_0\xi-1)} + \frac{\bar{A}[\omega(\bar{\xi}_0^{-1})-\omega(\xi_0)]}{\bar{\xi}_0^2\overline{\omega'(\xi_0)}(\xi-\bar{\xi}_0^{-1})^2} - 2\mathrm{i}NRm\mu\varpi_{21}\xi^{-(N+1)} - \kappa\sum\limits_{n=2}^{N}(n-1)\lambda_{n-1}\xi^{-n}}{R\kappa(1-mN\xi^{-(N+1)})}, \\ &\qquad |\xi| \geq 1. \end{aligned} \tag{3.21}$$

Finally, we can further determine the image force acting on the edge dislocation using the Peach–Koehler formula [6, 19].

4. Conclusions

We have solved the interaction problem of an edge dislocation near a rigid hypotrochoidal inhomogeneity. A closed-form solution is derived. All of the $N-1$ complex constants λ_n $(n = 1, \ldots, N-1)$ and the rigid body rotation of the rigid inhomogeneity are determined analytically in Eqs. (3.11) and (3.15), respectively. The determination of λ_n $(n = 1, \ldots, N-1)$ is uninfluenced by ϖ_{21}; conversely, the determination of ϖ_{21} is also unaffected by λ_n $(n = 1, \ldots, N-1)$.

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